Implicit Representation of Inscribed Volumes

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ABSTRACT

We present an implicit approach for constructing smooth 3-D inscribed volumes intended for modeling porous structures, such as volcanic rocks, foam, radiolarians, and Swiss cheese. Positive inscribed volumes can model natural pebbles, and negative volumes can model porous structures. We introduce two techniques for blending and creating interconnections between these inscribed volumes to adapt our approach to both regular and irregular. We begin with a set of convex polytopes such as 3-D Voronoi diagram cells and compute inscribed volumes bounded by the cells. The cells can be irregular in shape, scale, and topology, and this irregularity transfers to the inscribed volumes, producing natural-looking spongy structures. Describing the inscribed volumes with implicit functions gives us the freedom to exploit volumetric surface combinations and deformation operations effortlessly.

CCS CONCEPTS

• Computing methodologies → Shape modeling;

KEYWORDS

volumetric modeling, implicit curves and surfaces modeling, inscribed volume, spongy structures, radiolarian, computer aesthetic

1 INTRODUCTION

Modeling porous objects including radiolarians, foams, volcanic rocks, or similar volumes is a challenging task. We introduce a consistent approach to modeling such natural or man-made forms while conserving their smoothness and natural aesthetics, as well as preserving the complex geometry of interconnected structures containing inscribed volumes. The problem is described by Bear et al. [2]: it is difficult to formulate a general approach, as there are wide morphological and structural distinctions between the above examples. Porous materials – such as volcanic rocks, sponges, or Swiss cheese – are volumes with pores that may or may not interconnect. To achieve a general representation of such objects, volumetric modeling is required.

We aim to model a general form of porous structures that are not only natural looking but aesthetically satisfying, with smooth surfaces inside the inscribed volumes. We use an implicit volume modeling approach [17], as this technique produces smooth and aesthetic shapes that can be seamlessly modified at the local or global level while maintaining the consistency of the model to produce the desired target results [15, 21]. In addition, their function definition is compact and usually requires fewer high-level primitives to construct a model compared with other methods [3].

The contributions of this work are summarized as follows:

1. A general approach that can represent many types of spongy structures.
2. A base method that produces convex blobby volumes with aesthetic curvature, representing either holes or solids.
3. Two techniques for making connections between the chambers of porous substances, achieving different sets of properties.

2 RELATED WORK

For surface modeling and rendering of objects with porous or mesoscopic details, texture synthesis approaches have been suggested. Tong et al. [19] proposed a bidirectional texture function synthesis considering both surface geometry and surface details to achieve realistic porous materials. The synthesis is done by examining distances between 2-D surface texons derived from the 3-D texton method [8]. The drawback with textons is that the clustering introduces discretization errors, and the distance metric requires costly access to a large inner-product matrix which makes this method computationally slow [6]. Tong et al. proposed Quasi-Homogeneous materials [18] which produced more realistic surface materials with a complex capture and rendering process. These methods are basically focused on representing porous surface materials with 2-D texture synthesis and rendering techniques for objects such as the outer surface of bread showing the mesoscopic scale pores.

Chen et al. [4] used 3-D texture mapping employing complex microstructures to model porous volumes for the purpose of product design and manufacturing. The final structures made with regular geometries could be warped to some extent and are more suited for industrial purposes and less beneficial for natural spongy structures due to a lack of the required stochastic properties.

Magda and Kriegman [10] described a method for acquiring volumetric surface textures which separated geometric information from reflectance data. Since their algorithm offers high compression of the original data, it is especially suitable for the textures with complex, opaque microstructure, shadowing, and unknown reflectance properties that are difficult to compress. Their method also offers fully rendered textures at any orientation to the viewer at extra memory and computation cost.
Baravalle et al. [1] presented an interesting method for modeling porous structures by a particle system generation of 3-D textures using a dynamical system. They introduced a 2-D sliced based texture mapping method, applying a 3-D texture to a specific slice of an object; making this method computationally efficient. This method is not suitable for modeling of smooth or round macro-pores as is found in some structures, however it is able to represent mesostructure details of bread or similar textures on the outer shell of a surface.

Various techniques have been used for modeling specific structures, but are not intended as generic methods. These include a technique by Martinez et al. for open-celled foam, representing dry foams with hollowed internal structures [12]. Their technique uses a 3D Voronoi approach and was used for studying properties of foam microstructure in the context of industrial manufacturing of synthetic foams. In other methods, a microstructure model based on the combination of 3-D image analysis and random Laguerre tessellations is presented which captures the basic features of real solid foams [9, 14]. A similar cell foam modeling method based on Voronoi tessellations is also presented by Maliaris et al. [11].

Since we are using an implicit volume modeling approach, we discuss the most relevant background work in this field. Implicit modeling gained attention due to its intuitive mathematical representation of surfaces. Implicit volume modeling [17] produces smooth and aesthetic shapes that can be seamlessly modified at the local or global level while maintaining a consistent modeling structure [15, 20]. In addition, their function definition is compact and they require few high-level primitives to construct a model [3].

Some 3-D implicit modeling and design tools software packages were developed by employing skeletal-based modeling, such as BlobTree.NET [5] and ShapeShop [16].

The inscribed curves method by Wyvill et al. [21] is the 2-D foundation of the work introduced in this paper. This earlier method is based on 2-D implicit models, and we extend the technique to 3-D and generalize to include connected inscribed volumes crucial for 3-D porous volume construction.

In many applications (such as Swiss cheese) interconnections occur between inscribed volumes. The style of these connections, and whether they are regular or chaotic, can be captured using two different techniques. We demonstrate that the techniques can be adapted to many different types of porous structures.

The inscribed volumes (IVs) in their positive form can be used to model natural pebbles, liquid foams, or similar objects, however, to achieve complex models, such as sponge structure models, they should be defined in a negative form by hollowing out other solid primitives using Constructive Solid Geometry (CSG) or a negative blending operation.

We use 3-D Voronoi cells to produce convex, stochastic, and natural looking geometry with the ability to control the topology of the inscribed volumes and infer the adjacency of neighboring cells. The Voronoi diagram is a robust tool for representing many of the inscribed volumes and infer the adjacency of neighboring cells. The Voronoi diagram is a robust tool for representing many of the inscribed volumes and infer the adjacency of neighboring cells.

3 BACKGROUND

Wyvill et al. [21] conducted a user survey to determine whether parametric or implicit curves are more appealing. The user study showed a strong preference for implicit curves over B-spline curves. Figure 1 shows an example of the favored curve type.

![Figure 1: The inscribed curves method. This image received the most votes from users for containing the most aesthetically pleasing curves.](image)

The idea behind inscribed curves (IC) in 2-D is to define a set of aesthetically pleasing curves in the interior of each bounded convex polygon of a 2-D Voronoi diagram such that the curves approximate their bounding region. In this context, the aesthetic aspect refers to the smoothness of the closed curves. For each convex polygon \( P \) the following data is available: a vertex set consisting of \( p_0, p_1, \ldots, p_{k-1}; n_i \) as the inward unit normal for \( l_i \), where \( l_i \) is the edge connecting \( p_i \) and \( p_{i+1} \).

The signed distance of an arbitrary point \( x \in \mathbb{R}^2 \) to the edge \( l_i \) is

\[
 d_i(x) = (x - p_i) \cdot n_i, \tag{1}
\]

where \((\cdot)\) denotes the dot product.

For each point located on the edge \( l_i \) we have

\[
 l_i = \{ x \in \mathbb{R}^2 | d_i(x) = 0 \}. \tag{2}
\]

Wyvill et al. sought to define a function \( F \) that was positive inside \( P \) and increased with distance from the boundary. Such a function can be obtained by taking the product of the distances:

\[
 F(x) = \prod_{0 \leq i < k} d_i(x), \tag{3}
\]

where \( k \) is the total number of vertices of \( P \). Any point positioned on any edge of \( P \) has \( F(x) = 0 \).

In practice, the product of distances can become unmanageable by creating enormous values. Instead, consider the field \( f \), where

\[
 f(x) = \log F(x), \quad x \in P. \tag{4}
\]

Combining 3 and 4, we have

\[
 f(x) = \log\left(\prod_{0 \leq i < k} d_i(x)\right) = \sum_{0 \leq i < k} \log d_i(x), \quad x \in P. \tag{5}
\]

Given the field \( f \), we can extract a curve: let \( S_a \) be the points on the surface defined by \( S_a = \{ x | f(x) = a \} \). That is, \( S_a \) is the implicit surface in \( f \) with isovalue \( a \).

The above method for inscribed curves has two drawbacks. First, it is defined only in 2D; we need 3D volumes. Second, the curves...
within cells are isolated; we want the cells to communicate, with fields that represent interconnections between the cells allowing us to create sponge-like structures. We address both of these in the following section.

4 INScribed VOLUMES

We start by describing a basic 3D method which considers each cell independently, defining a distance field function for every cell. Subsequently, we introduce two methods for creating interconnections between neighboring cells. Defining precise shapes in the interconnection volume between regions gives us the flexibility to support a great variety of porous structures.

4.1 Isolated Inscribed Volumes

The idea is to have a distance function to generate a set of smooth inscribed volumes where each volume is approximated from the walls of a bounded convex region and must be defined in the interior of the region. The smoothness and roundness of the inscribed volumes where each volume is approximated from the vertex of the face or even the face centroid. For each point located on the face itself, the distance is zero.

Just as in 2D, the field function $f$ can be defined as

$$f(x) = \log \prod_{0 \leq i < k} d_i(x) = \sum_{0 \leq i < k} \log d_i(x), \quad x \in \mathbb{C}.$$  

where $k$ is the total number of faces of $\mathbb{C}$.

The smoothness and size of inscribed iso-surfaces depend on the choice of the value of the iso-value $a$. If $a$ approaches zero then $S_a$ approximates the boundary of the enclosing polytope $\mathbb{C}$. The larger the value of $a$, the rounder and smoother the resulting iso-surface.

As written, the field value of $f$ is derived from its distance to each face of $\mathbb{C}$, with each face weighted equally. However, equal weighting yields undesirable results. Consider polytopes $\mathbb{C}_A$ and $\mathbb{C}_B$ with the same geometry. Now, add a face of infinitesimal size to $\mathbb{C}_B$. The isosurfaces within $\mathbb{C}_B$ and $\mathbb{C}_A$ will be quite different, even though the geometries are almost identical. We can address this in a manner analogous to the approach of Wyvill et al., adding a coefficient for each face. Let

$$f(x) = \sum_{0 \leq i < k} \lambda_i \log d_i(x),$$

where $\lambda_i$ is given by $\lambda_i = A_i / A_C$ where $A_i$ is the area of the $i$th face and $A_C$ is the total area of the polytope. Thus the faces with bigger area have more effect on the inscribed surface than the smaller ones. For better control over the influence of the face areas, $\lambda_i$ can be modified to:

$$\lambda_i = A_i / A_C, \quad 0 < c \leq 1.$$  

The value of $c$ can affect the overall size of the inscribed surfaces as well. Therefore, depending on the chosen modeling object, the choice of $c$ can be adjusted to obtain the desired smoothness and approximation of $\mathbb{C}$. For the results we show, we typically used a value of $c$ around 0.8.

4.2 Interconnected Inscribed Volumes

In order to construct a wider range of natural substances, we need to create interconnections between the generated volumes within cells. We distinguish between *fully-connected* and *partially-connected* structures. Fully connected porous structures are entirely interconnected; an example is the spongy structure of the Cancellous bone, shown in Figure 11. Partially connected structures have sporadic connectivity; an example is the bubbles found in some types of cheese.

4.2.1 The Fully-Connected Method. The fully-connected method forms links across every shared face. The aim of this algorithm is to produce a network of IVs. For an internal cell $\mathbb{C}$ of a 3-D Voronoi tessellation with $i$ faces, there will be $i$ passageways through the faces, one for each face. We achieve this by placing additional *connection fields* across each face. The connection fields are new primitives that should smoothly blend with the initial inscribed volume within $\mathbb{C}$. Our connection fields will pass through the centre of each face; we constrain them to neither break the shared face edges nor pass through other faces.

To construct interconnections, we first divide the cell $\mathbb{C}$ into $k$ sectors where $k$ is the number of faces of $\mathbb{C}$. Each sector is constructed from the vertices of the shared face and the centroid of the cell it lies in. We then create a *cage* by linking the two adjacent sectors: i.e., the cage is the polyhedron created by linking the vertices of the face with the centroids of the two neighbouring cells.
This new cage is a bounded region in which we introduce a new field primitive. The size of this new primitive and its roundness are given by the function \( f \) in the same way as the initial IV. Finally, the new primitive field blends with the original IVs. We chose the simple Ricci blending operator to control the smoothness of the final shape. However, using other blending operators, e.g., that of Gourmel et al. [7], with better control of the blending surfaces, can be considered as well.

### 4.2.2 The Partially-Connected Method

The partially-connected technique is used to develop a random and natural link between the IVs, not necessarily connecting all cells. In this method, a spherical booster field is generated around each cell’s centroid. These booster fields then blend with the initial IVs within the cells. The resulting shape becomes connected where centroids are close together, but the IVs remain disconnected for more distant centroids; the irregular pattern of connectivity gives the method the name partially-connected.

Each cell’s booster field blends with all its neighbors’ booster fields. Finally, we blend the result with the initial inscribed field. For this variation, it would be helpful to have an accurate blending operator such as gradient-based blending by Gourmel et al. [7] to avoid unwanted bulging artifacts. Incorporating this blending method is future work.

Adjusting the radius for the booster fields is purely optional and depends on what type of object is being modeled. In general, if the sizes and shapes of the bounded cells are irregular, some geometric restriction can be arbitrarily applied to the radius of the booster fields to prevent the final shape (i.e., after blending with initial IVs) from going beyond the cell boundaries. Moreover, restricting the radius of the booster fields naturally helps to produce a more random and partially connected network of IVs. This is mainly because when two cell centroids are far from each other, booster fields may not blend with each other which results in unconnected neighboring cells as well.

Another consideration about choosing the sizes of the booster fields can be obtained by the actual cell sizes and their irregularities of shape. In general, if the cells’ tessellations are randomly selected, it is possible to have thin cells along with fairly huge cells. This requires us to pick a smaller radius for a smaller cell so as not to overwhelm the initial IV shape and also to select a bigger radius for larger cells to avoid the booster field getting masked or being ignored by its corresponding initial IV size. In each cell, we used the distance of the closest face of the cell to the centroid point as our guidance for determining the radius of the booster field.

### 5 RESULTS

The techniques described in this paper provide control over the volume’s size, smoothness, and blending operator. The first decision to make is determining the distribution and density pattern of IVs; for example, uniform patterns could be used for synthetic foams or natural honeycombs, while random distributions are suitable for many natural porous structures. The proper point distribution varies from one usage to another; our approach can be used regardless of the distribution.

The size and smoothness of IVs can be adjusted by changing the isovales (see Figure 3), or by changing the weights of function \( f \) as explained in the previous section. In general, as the IVs approach the boundaries their shapes appear less smooth and rounded and therefore less aesthetic.

Another factor is the blending operator, which not only affects the smoothness of the chambers in a porous structure, but also has a crucial role when the IVs blend with the augmenting fields in either the fully- or partially-connected method. Figure 4 shows a 2-D slice of IV field values using the fully-connected method. In this figure, we adjusted the size of the cages by moving the centroid vertex of the bigger cell toward the shared face to better balance the contribution of the interconnection primitives to each neighboring cell.

Variations of the partially-connected method depend chiefly on the size of the booster fields before and after blending, as well as the method used to blend between them and the initial IVs. As the size of the booster fields increases, the likelihood that neighboring cells will connect increases as well.

We have primarily used IVs as negative space, subtracting them from other geometry to produce porous structures. We can also interpret them as positive space, and when we do so, the resulting shapes resemble pebbles, as seen in Figure 6. Figure 7 shows porous triple torus shells using negative IVs. To create this figure, we first...
Figure 6: Inscribed volumes in positive; the resulting shapes resemble pebbles smoothed by the action of water.

Figure 7: An aesthetic object: porous triple torus shells.

A type of radiolarian called a polycystine, with its main spines and round pores on the outer and central shells. Real-life radiolarian (left), image credit Jane K. Dolven; our model (right).

Figure 8 shows a simple model inspired by a type of radiolarians called polycystines. Fossils of these marine plankton can be found in most oceans. They usually have hollowed skeletons made of opaline silica, spines on the outer shells, and a central capsule which separates their inner and outer portions.

For those structures whose pores are not completely smooth or round, or which have short tunnels connecting pore chambers, the regular or partial connection techniques described in section 4.2 are used. These algorithms are flexible enough to construct natural looking sponges such as dry foams, spongy bones, aquatic sponges, some kinds of cheese, and volcanic rocks including pumice.

Figure 8 shows a piece of cheese constructed from a sector of a cylinder and cut from the top and one side (right) compared to a real-life cheese (left). The IVs are naturally connected with each other using the partial-connecting algorithm; the sizes of holes and the amount of blending power between the booster fields inside each cell and the IVS, were manually chosen to give a plausible effect. These parameters can be easily adjusted in the user interface implementation. Figure 9 shows a real-life rock (left), and two porous rocks, both created using the partial connecting algorithm (right). The left rock in the right-side figure is a sphere modified by 3-D Perlin noise [13], where the value of noise is clamped between [0,1] and then combined with the original sphere distance field. Due to the implicit definition, other methods of surface displacement could be employed instead if desired.

Figure 9: A real-life piece of cheese (left), image credit Getty Images-Creativ Studio Heinemann; synthetic cheese created with partial interconnections between holes, rendered with subsurface scattering (right).

Figure 10: A real-life volcanic rock (left), and two porous rocks generated by our partially-connected method (right).

Figure 11 depicts a cross-section of a normal Cancellous bone, modeled inside a solid outer layer of Cortical bone. We generated
this model using the fully-connected method, as the bone structure has smoothed chambers connected with each other through wide openings.

![Figure 11: A cross-section of Cancellous (spongy) bone inside a simplified Cortical bone (right); a real-life spongy bone (left), image credit Steve Gschmeissner and Getty Images.](image)

5.1 Discussion

Natural modeling of porous or spongy substances is a combination of art and different sciences including biology, geology, and computer graphics. In computer graphics, we use several techniques where each of them helps to mimic one or more characteristics of porous structures to achieve similar results. Our approaches are able to construct several types of porous structures, giving the flexibility of choosing and adjusting the right settings for each model.

The mathematical definition of implicit inscribed volumes offers remarkable flexibility and accuracy when it comes to surface alteration, to customize the modeling construction process. This is particularly important when a natural and irregular surface displacement or a seamless surface combination is required. To guarantee that the resulting shapes have smooth curvature and are C\(^2\) continuous, an appropriate fall-off filter function (Wyvill function) is performed which is also useful to get aesthetic results.

Using the Voronoi diagram not only helps produce naturally shaped IVs, but the topological distribution of its seeds assists with customized or random topological patterns of IVs as well. Nevertheless, for a photo-realistic visualization of natural materials an alternative tessellation methods so that we can further increase the variety of shapes.

6 CONCLUSIONS AND FUTURE WORK

In this work, we presented a procedural approach to creating porous structures using inscribed volumes. The first step was constructing 3-D IVs approximated by a set of bounded and convex regions such as a Voronoi diagram. The IVs in their negative (inverted) form can be used for generating porous structures and in their positive (normal) form are smooth rounded shapes resembling pebbles. Some modern furniture designs, exhibiting smooth and aesthetic curved surfaces, might also lend themselves to the design framework presented here.

We introduced two simple, flexible interconnection techniques (fully-connected and partially-connected) to embrace morphologically different spongy structures. The flexibility of our framework gives the freedom of adjusting size and smoothness of the pores as well as choosing a number of properties for modeling a specific type of sponges where connecting IVs are required. We demonstrated that various natural spongy structures such as Cancellous bone, cheese, rocks, and radiolarians can be constructed using our technique. Synthetic porous materials with uniform pore distributions can also be generated from our techniques with ease, as they usually do not have the complex and unpredictable characteristics of natural substances. The results can be deformed or effortlessly blended or combined, using BlobTree [20] operations such as CSG and blending, with other skeletal primitives to construct more complex objects. These models can be used in different applications such as animations, and medical visualizations.

The definition of IVs was limited to a set of boundaries that were convex and were made of polygonal walls. Nevertheless, the IVs can be defined over any bounded convex region as long as the region is represented by its faces and vertices. This means that in a polytope curved faces can replace polygonal faces. The function must be changed and we have not tested it in our framework yet, but this would be a nice feature which can produce interesting results in the future.

Although our implementation can deploy both random and uniform control point distributions, as well as create distributions with specific conditions, it does not use any data from real-world porous materials. Generating the mesoscopic details of quasi-homogeneous materials can be time-consuming for the end user. Integrating a simple and fast texture synthesis procedure or generating various textures and applying them to the surface will reduce the pressure on the user, who can benefit from the combination of our methods and texture synthesis and rendering techniques to attain beautiful and intricate images.

The Voronoi diagram has been used because of the natural and varied structures it produces. In the future, we hope to investigate alternative tessellation methods so that we can further increase the variety of shapes.

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