Auspicious tatami mat arrangements

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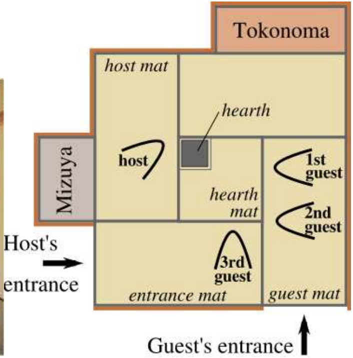
Lucky Tea House Floors

Tatami mats

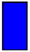
Traditional Japanese floor mats made of soft woven straw.

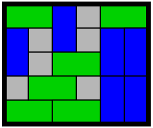


Certain floors, like tea houses, required that no four mats touch at any point.

Monomer-dimer tiling

Tile a subset of the grid with

monomers , and dimers  and .



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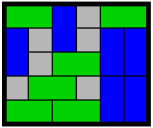
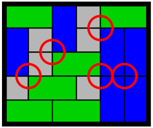
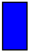


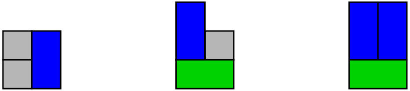
Tatami condition

Forbid configurations
with four tiles at one point.

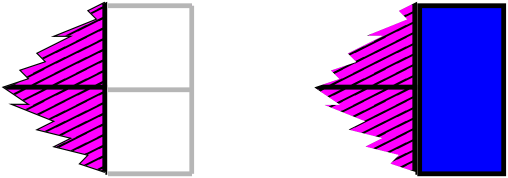


Every interior grid intersection is on a long edge of at least one dimer , .

For example

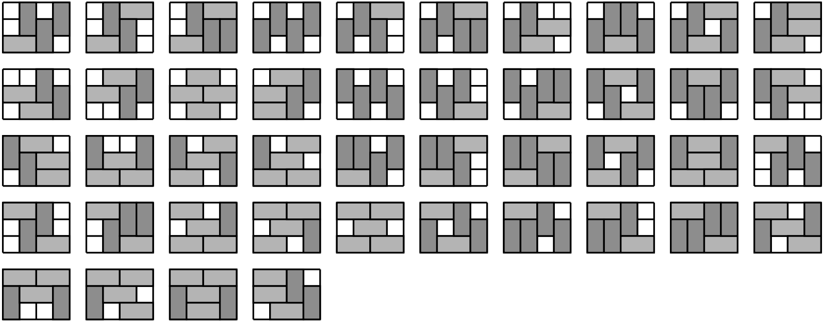


Adjacent tiles force the placement of a dimer:



Small tilings

All tatami tilings of the 3×4 grid.



Previous work on tatami dimer-tilings:

Dean Hickerson, 2002

OEIS a068920

`http://www.research.att.com/~njas/sequences/`

Donald Knuth, 2009

The Art of Computer Programming, volume 4, fascicle 1B

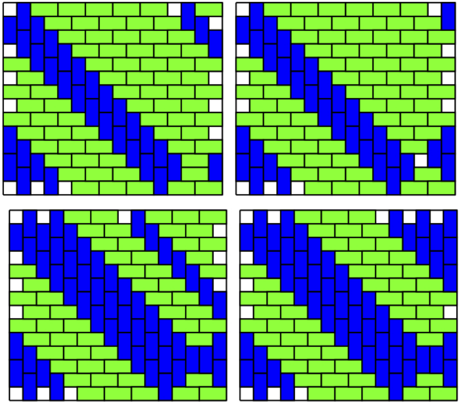
Frank Ruskey and Jenni Woodcock, 2009

Counting Fixed-Height Tatami Tilings, Electronic Journal of Combinatorics.

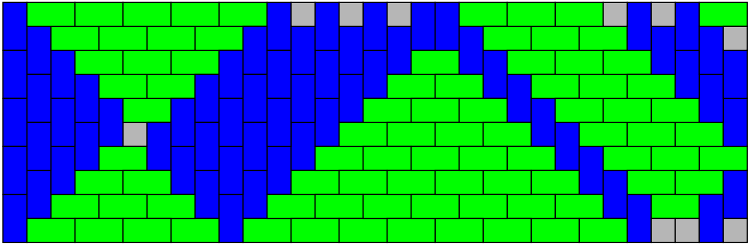
Alhazov, Morita, Iwamoto, 2009

A note on [monomer-dimer] tatami tilings, Proceedings of 2009 LA Winter Symposium.

Larger tilings expose patterns

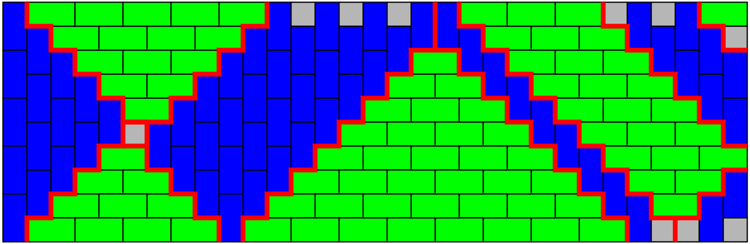


A closer look

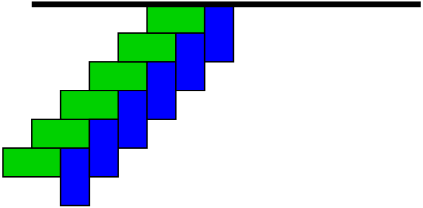


A closer look

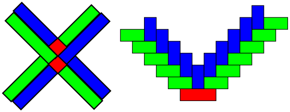
T-diagram



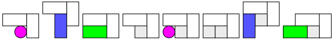
These boundaries form *rays*. Rays force the structure.



1. Rays propagate to the boundary.
2. Rays cannot cross.



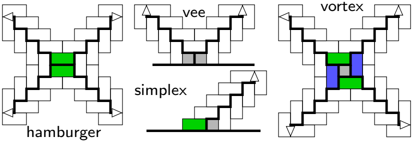
Where do rays begin?



...

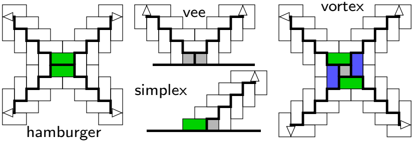
The “beginning” of a ray: 

- Not the beginning .



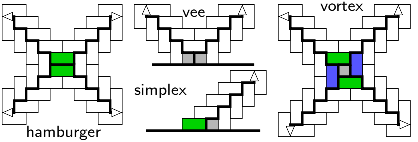
The "beginning" of a ray: 

- Not the beginning .
- Case 1, *hamburger*: . Occurs anywhere.



The “beginning” of a ray: 

- Not the beginning .
- Case 1, *hamburger*: . Occurs anywhere.
- Case 2: monomer at beginning .
  - Case 2(a), *simplex*: . Only on boundary.
  - Case 2(b), *vortex*: . Not on boundary.
  - Case 2(c), *vee*: . Only on boundary.



Results from our paper

Theorem

Let m be the number of monomers in an $r \times c$ tatami tiling. Then m has the same parity as rc and $m \leq \max(r + 1, c + 1)$.

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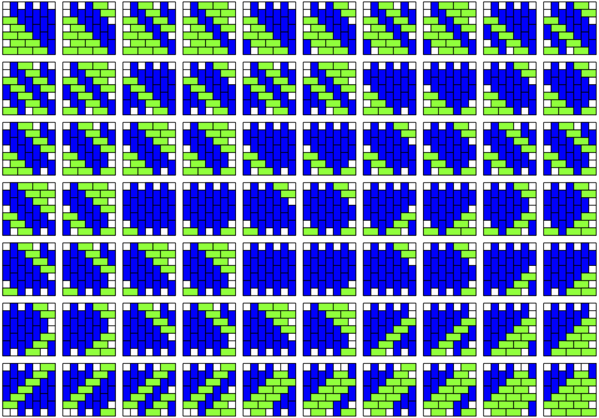
Lemma

An $n \times n$ square tiling has n monomers  *if and only if it has no sources of type*  *or* .

Theorem

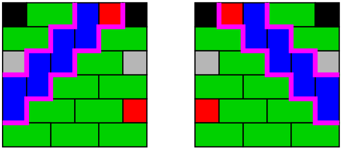
In an $n \times n$ grid there are $t(n) = n2^{n-1}$ tilings with n monomers.

Some 7×7 tilings with 7 monomers



A piece of the proof

“Diagonal slices” can be flipped to give a new tiling.



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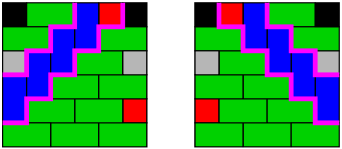
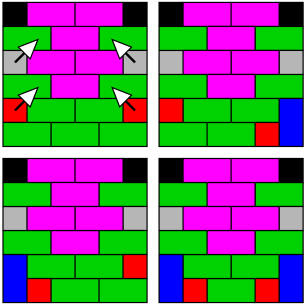
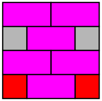


Fix the black corner-monomers. Use recursion.



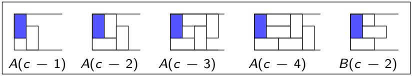


Recurrence for fixed black monomers is
$s(n) = 4s(n-2) + 2^{n-2}$.

Generating functions

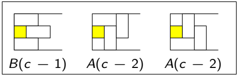
For height 3

Let $A(c)$ be the number of $3 \times c$ tilings which start with the blue tile shown on the right.

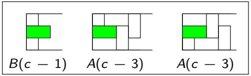


Recurrences for $A(c)$

Similarly for $B(c)$ and $C(c)$

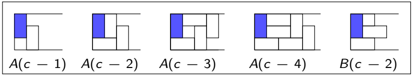


Recurrences for $B(c)$

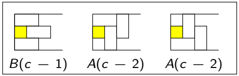


Recurrences for $C(c)$

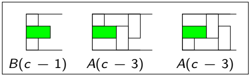
This is a linear recurrence relation in A, B, C so we have rational generating functions.



Recurrences for $A(c)$



Recurrences for $B(c)$



Recurrences for $C(c)$

The number of tilings with r rows and c columns is the coefficient of z^c in the generating function $T_r(z)$.

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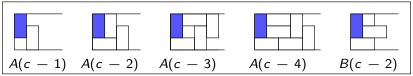
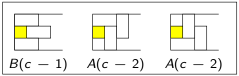
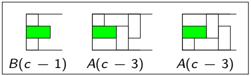


Recurrences for $A(c)$



Recurrences for $B(c)$



Recurrences for $C(c)$

The number of tilings with r rows and c columns is the coefficient of z^c in the generating function $T_r(z)$.

Theorem

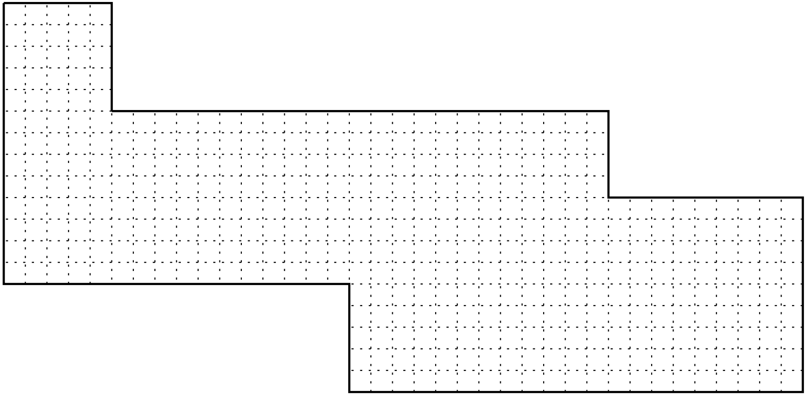
For height $r = 1, 2, 3$ the generating functions $T_r(z)$ are

$$T_1(z) = \frac{1+z}{1-z-z^2}, \qquad T_2(z) = \frac{1+2z^2-z^3}{1-2z-2z^3+z^4}, \quad \textit{and}$$

$$T_3(z) = \frac{1+2z+8z^2+3z^3-6z^4-3z^5-4z^6+2z^7+z^8}{1-z-2z^2-2z^4+z^5+z^6}.$$

More tatami problems

Given an arbitrary shaped grid, what is the minimum number of monomers in a tatami tiling?



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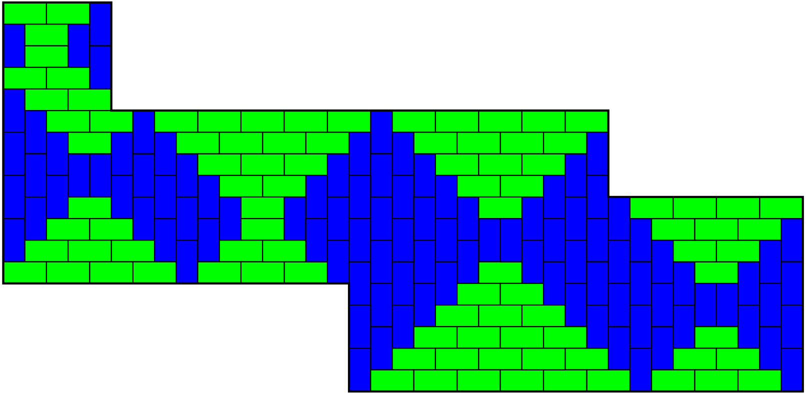
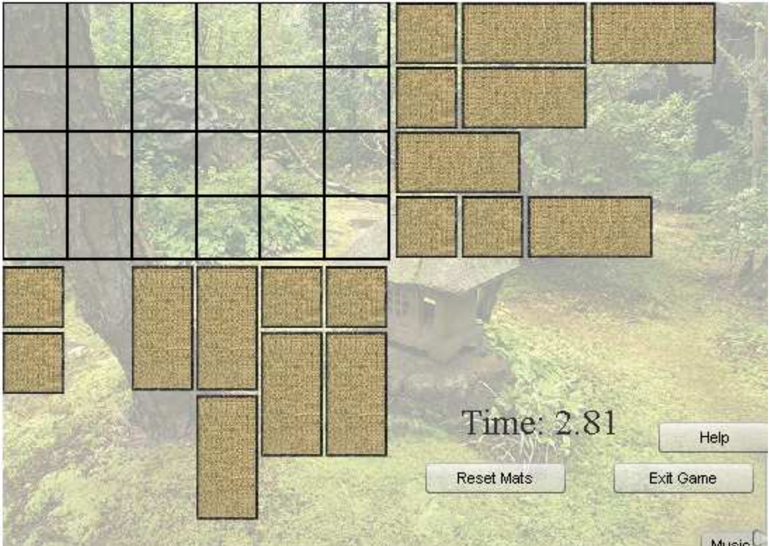
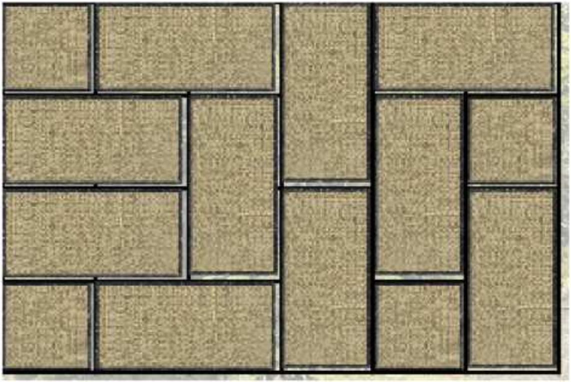


Tomography

Is it possible to tile a grid with these row and column projections? What is the complexity of this?



Solution



Play this flash game at `http://miniurl.org/tomoku` (or `http://tiny.cc/tomoku`). Use "CC" in your high score's name to identify yourself as a COCOON attendee.