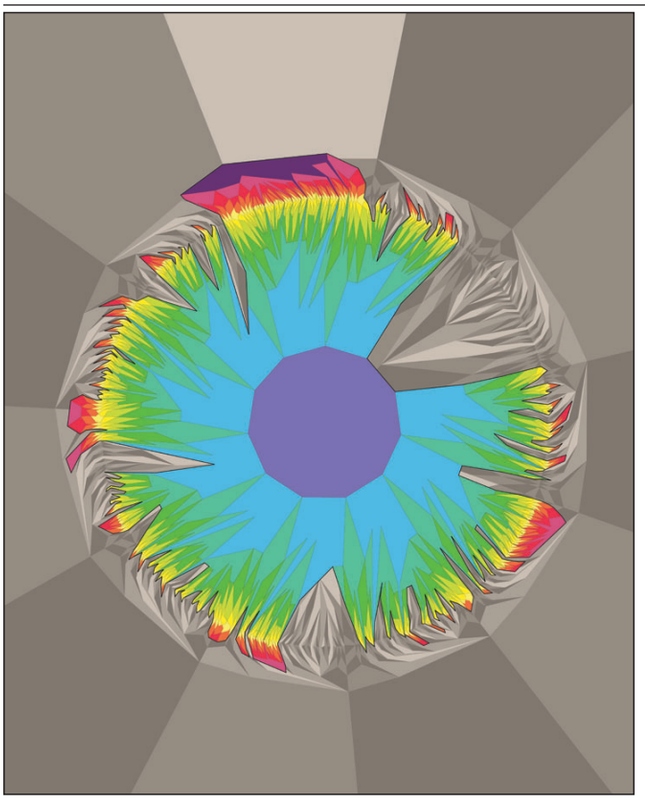

About the Cover

Symmetric Venn diagrams

The cover image shows a symmetric Venn diagram on 11 sets, but only half of the regions are shown. That is, the full diagram has 2048 regions, with 1024 lying inside each of the curves that represent sets. Here one such curve is shown in black (the boundary of the colored region), with the regions inside it colored according to their rank. The other curves can be obtained by rotating the one shown. The gray background shows the dual of the full Venn diagram, with its regions colored in four shades of gray.

These images were generated by computer, with some manual intervention. The main tool is a *Mathematica* package written by Stan Wagon that allows the manipulation and four-coloring of planar maps and graphs (contact him for a copy). The package has code to form the dual map of a planar map (by finding a capital of each country and using a triangulation of the country to get disjoint piecewise-linear edges to the middle of each boundary edge).

Inspired by the display of hand-drawn 11-Venn art by Peter Hamburger and Edit Hepp at the Banff Bridges Conference in 2005 [1], Wagon used his package to produce the set of all polygons in the Venn dual and Venn diagram. The package had the necessary tools, but the devil was in the details. First the data for the Venn dual (Figure 9) had to be entered, symmetry being used at every step to minimize manual labor. To then get a pleasing Venn diagram, attention had to be paid to vertex placement within each region of the Venn dual. Most regions of the dual are quadrilaterals that are convex or nearly so, and those cases can be handled by just choosing the centroid; the remaining cases were handled manually. And the edges connecting these vertices were also chosen for simplicity, always trying for piecewise linearity with as few pieces as possible.

In the full diagram it is hard to see the smaller regions. Here is a magnification of a small portion.

It was shown in [12] that a practical algorithm for 4-coloring planar maps could be devised based on ideas of Kempe and Kittell, "practical" meaning that it it could handle all examples without difficulty, even though it has not been proven to always work. In particular, this method can quickly 4-color the infamous hoax map that Martin Gardner published in the April 1, 1975, edition of *Scientific American*. The Venn dual is a map so the regions have to be colored somehow, and using a 4-coloring is a natural way to do it. While the Venn dual shown on the cover cannot be 3-colored, maps derived from "Siamese trees", such as those shown in figures 3(b) and 7(b), are always 3-colorable (Hutchinson [11]). The first examples of the use of Siamese trees in the construction of Venn diagrams may be found in [14]. The Venn diagram itself is a 2-colorable map by parity of rank, but it is more natural to use 12 colors, one for each rank.

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