CSC 482/582 Summer 2012 Quiz #1 NAME: _____

Instructions: Closed book and notes. Answer all questions.

1. [5 marks]

   (a) Simplify: $\lfloor n/2 \rfloor + \lceil n/2 \rceil =$ _____.

   (b) Simplify: $-\lfloor -x \rfloor =$ _____?

2. [5 marks] Recall that $J_2(n)$ is the Josephus function studied in Chapter 1.
   (a) What is $J_2(32+16+4+2)$?
   (b) Characterize those values of n for which $J_2(n) = n$.

3. [5 marks] Below we are interchanging the order of three sums.

$$\sum_{1\le j\le n}\ \sum_{1\le k\le j}\ \sum_{1\le l\le k} f(j,k,l) = \sum_{?_1\le l\le ?_2}\ \sum_{?_3\le k\le ?_4}\ \sum_{?_5\le j\le ?_6} f(j,k,l)$$

   What are $?_1, ?_2, ?_3, ?_4, ?_5, ?_6$. Justify your answer by writing out some equalities involving the $[\![P]\!]$ notation.

4. [10 marks]

   (a) What is $\Delta x^{\underline{3}}$? ANSWER: _____

   (b) What function v, if any, satisfies $\Delta v(x) = 5^x$? ANSWER: _____

   (c) Below is equation (2.55) from the book.

   $$\sum u \Delta v = uv - \sum Ev \Delta u.$$

   Use this once to get a "simpler" (it will still involve a summation) expression for

   $$\sum_{k=1}^{n} k^{\underline{3}} 5^k =$$

5. [5 marks] Solve the recurrence relation $D(0) = 1$ and for $n > 0$,

   $$D(n) = \frac{n+2}{n} D(n-1) + 1$$

   You can use whatever method you want. I suggest that you compute a few small values of $D(n)$ first.