CSC 482/582 Summer 2012 Quiz #2 NAME: I.M. Solution

Instructions: Closed book and notes. Answer all questions.

1. [8 marks]

   What is the definition of $\{x\}$?

   What is the definition of n mod m for integer n and integer $m > 0$?

   What is the mediant of $3/5$ and $4/3$?

   In Maple how would you compute the sum

   $$\sum_{d \backslash n} \phi(d) 2^{n/d}.$$

2. [3 marks] Simplify (you can use any equation from the book):

   $$\lfloor 2x \rfloor + \left\lfloor 2x + \frac{1}{3} \right\rfloor + \left\lfloor 2x + \frac{2}{3} \right\rfloor .$$

3. [9 marks] We know that if p is prime then $\phi(p) = p - 1$. Is the converse true? That is, does $\phi(n) = n - 1$ imply that n is prime?

The function $\phi(n)$ is *multiplicative*. What does that mean?

Is $\phi(n)$ ever prime for $n \geq 1$? Carefully explain your answer.

ANSWER: If n has a prime factor p that is larger than 3, then $n = pn'$ and $\phi(n) = \phi(p)\phi(n') = (p-1)\phi(n')$. Since $p - 1$ is even and larger than 2 it is not prime and neither is $\phi(n)$. Thus if $\phi(n)$ is prime its only factors are 2 and 3. Since $\phi(3^b) = 3^{b+1} - 3^b = 2 \cdot 3^b$ the exponent of 3 can be at most 1. Similarly, since $\phi(2^a) = 2^{a-1}$, the exponent of 2 can be at most 2. An examination of the small cases reveals the final answer: *The function* $\phi(n)$ *is prime if and only if* $n \in \{3, 4, 6\}$.