CSC 482/582 Summer 2012 Quiz #3 NAME: I.M. Solution

Instructions: Closed book and notes. Answer all questions.

1. [5 marks] Below is the Farey series $\mathcal{F}_6$. Show were to insert the missing numbers to get $\mathcal{F}_7$.

$$\underbrace{\frac{0}{1}\ \ \frac{1}{6}}_{\frac{1}{7}}\ \ \frac{1}{5}\ \ \underbrace{\frac{1}{4}\ \ \frac{1}{3}}_{\frac{2}{7}}\ \ \underbrace{\frac{2}{5}\ \ \frac{1}{2}\ \ \frac{3}{5}}_{\frac{3}{7}\ \ \frac{4}{7}}\ \ \underbrace{\frac{2}{3}\ \ \frac{3}{4}}_{\frac{5}{7}}\ \ \frac{4}{5}\ \ \underbrace{\frac{5}{6}\ \ \frac{1}{1}}_{\frac{6}{7}}$$

2. [5 marks] Give a simplified expression for the binomial coefficient

$$\binom{-2}{k} = \frac{(-2)(-3)\cdots(-(k+1))}{(1)(2)\cdots(k)} = (-1)^k(k+1)$$

3. [6 marks] In the set $\{66j \bmod 100 : j = 0, 1, \ldots, 99\}$ how many times does the number 14 occur? What about the number 15?

ANSWER: Note that $d = 2 = \gcd(66, 100)$. Since $14 = 7d$, it occurs 2 times. On the other hand the number 15 is not divisible by d and thus it does not occur.

4. [9 marks] Let $\sigma(n)$ be the sum of the divisors of n. For example, $\sigma(6) = 1+2+3+6 = 12$. Simplify (and explain why your simplification works)

$$\sum_{d\backslash m} \mu(d)\sigma(m/d).$$

ANSWER: By the definition of σ,

$$\sigma(m) = \sum_{d\backslash m} d.$$

Now apply Möbius inversion to get

$$m = \sum_{d\backslash m} \mu(d)\sigma(m/d).$$