CSC 225 Laboratory 2 Part A: Review of Recurrences and Induction

Induction and how to solve recurrences should have been learned in Math 122 or its equivalent. If you still have not mastered these, it is wise to do that now as you will need these skills to do well in CSC 225. In the lab you will be given a problem to solve and then will be asked to try induction proofs for some proposed solutions. If you want extra problems to use as practice, here are some questions you can do.

1. For CSC 225, there are three sums that appear often when analyzing time and space complexities of algorithms. For each of these, prove that the given formula is correct by induction.

   (a) $\sum_{i=1}^{n} i = n(n+1)/2$

   (b) $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

   (c) $\sum_{i=0}^{n} \frac{1}{2^i} = 2 - 1/2^n$

2. Solve these recurrences by repeated substitution. Number your steps by letting Step 0 represent the original recurrence and Step i is the step where the RHS has been replaced *i* times as done in class.

   (a) $T(n) = n + T(n-1)$, $T(1) = 1$.

   (b) This recurrence is only defined for $n = 2^k$ for some integer $k \geq 0$. $T(n) = n + T(n/2)$, $T(1) = 1$.

   (c) This recurrence is only defined for $n = 2^k$ for some integer $k \geq 0$. $T(n) = 1 + T(n/2)$, $T(1) = 0$.

3. To ensure you have the mechanics for solving recurrences mastered, try these ones as well which are variations on the recurrences from question 2. Solve these recurrences by repeated substitution. Number your steps by letting Step 0 represent the original recurrence and Step i is the step where the RHS has been replaced *i* times as done in class.

   (a) This recurrence is defined only for $n \geq 7$: $T(n) = 3n + 2 + T(n-1)$, $T(7) = 23$.

   (b) This recurrence is defined only for $n = 2^k$ for some integer $k \geq 3$. $T(n) = n + T(n/2)$, $T(8) = 42$.

4. Prove your solutions from question 3 are correct by induction.