Name: ______________________________

ID Number: ____________________________

UNIVERSITY OF VICTORIA
EXAMINATIONS- DECEMBER 2011
CSC 320 - Foundations of Computer Science
Section A01/A02, CRN 15735/10898
Instructor: Wendy Myrvold
Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

This question paper has eleven pages (the last page is blank in case you need extra space) plus the header page.

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

Question	Value	Mark
1	20	
2	10	
3	10	
4	10	
5	10	
6	10	
7	15	
8	15	
Total	**100**	

1. [20 marks] For each of the following languages, indicate the most restrictive of the classes below into which it falls

   (a) finite

   (b) regular

   (c) context-free

   (d) Turing-decidable

   (e) Turing-acceptable

   (f) None of the above.

Example:

$L = \{ a^n b^n : n \geq 0\}$ The correct answer is (c) since L is context-free, but is not regular.

_____ i) $\{u u^R v : u, v \in \{0, 1\}^+\}$

_____ ii) $(a \cup b)^* \phi (a \cup b)^*$

_____ iii) $\{ w w : w \in \{0\}^* \}$

_____ iv) $\{ w w : w \in \{0, 1\}^* \}$

_____ v) $\{ w \in \{0,1\}^* : w$ *has* 0 1 0 0 *and* 1 0 1 1 0 as substrings $\}$

_____vi) The complement of $(a \cup b)^* (a \cup b)^*$

_____vii) The complement of $\{ a^n b^n c^n : n \geq 0 \}$

_____viii) $\{ a^n b^{n^3} : n \geq 0\}$

_____ix) $\{ a^n b^m c^p : 2n \leq m \leq 3n$ or $2n \leq p \leq 3n\}$

_____x) $\{ a^n b^m c^p : 2n \leq m \leq 3n$ and $2n \leq p \leq 3n\}$

For each of the following languages, indicate the most restrictive of the classes below into which it falls

(a) finite

(b) regular

(c) context-free

(d) Turing-decidable

(e) Turing-acceptable

(f) None of the above.

_____xi) $\{w : w$ represents a correspondence system with alphabet $\{a\}$ that has a match $\}$

_____xii) $\{w : w$ represents a correspondence system with alphabet $\{a, b\}$ that has a match $\}$

_____xiii) { (M, w) *: TM M halts on input w* }

_____xiv) { $''M''$ *: there is some string on which M halts* }

_____ xv) { (M, c) : TM M does not print the symbol c when started on a blank tape }

_____ xvi) { $''M''$ *: M writes a nonblank symbol when started on a blank tape* }

_____ xvii) { $''M''$ *: there are no strings on which M halts* }

_____ xviii) { $''M''$: *M is a TM* }

_____ xiv) { $''M''$ *: TM M accepts a regular language* }

_____ xx) { (M, w) *: TM M moves its head to the right on input w* }

2.(a) [4 marks] Design a DFA that accepts the language $L = \{w \in \{0, 1\}^* : w$ has 011 or 110 as a suffix $\}$.

(b) [4 marks] Design a construction that starts with a DFA $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ and creates a new DFA $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$ such that $L(M_2) = \{w \in L(M_1) : w$ starts with $101\}$. Give precise and formal mathematical descriptions for K_2, δ_2, s_2 and F_2 for your construction.

(c) [2 marks] Annotate your picture from (a) to show what happens from your construction from (b) writing NEW beside the new states you add.

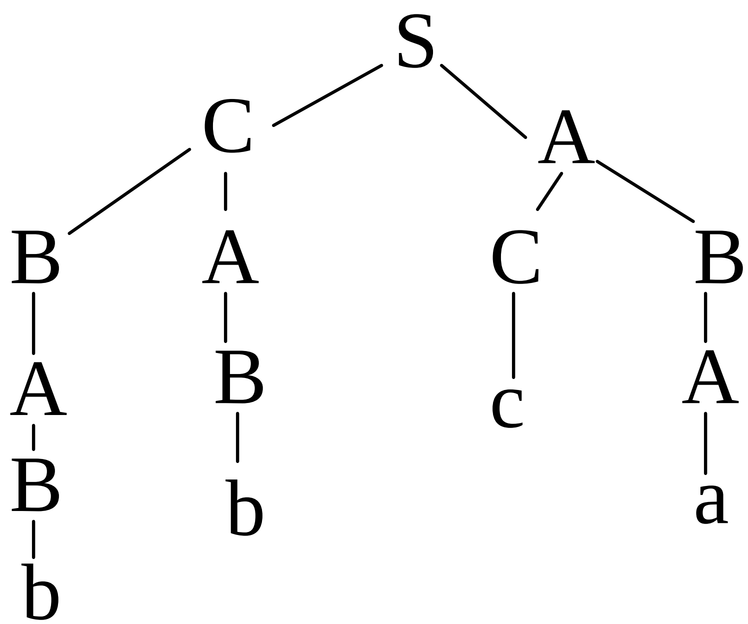
3.(a) [5 marks] State our definition of a regular expression over the alphabet $\{0, 1\}$.

(b) [5 marks] Use the definition from (a) to construct a context-free grammar that generates $L = \{w \in \{(,), 0, 1, \cup, *, \phi\}^* : w \text{ is a regular expression } \}$.

4. [10 marks] Let $L = \{w \in \{a, b\}^* : w = a^p\ b\ a^q\ b\ a^r \textit{ where } q \geq p \textit{ and } r \leq p\}$.

A CSC 320 student is trying to prove that this language is not context-free, and one of the cases is this one: $w = a^i\ a^j\ a^{k-i-j}\ b\ a^s\ a^t\ a^{k-s-t}\ b\ a^k$ where $v = a^j$ and $y = a^t$. The student writes: ''Pump zero times and the string is not in L because there is less a's in the first block than in the second one.'' The TA, being draconian with respect to part marks for incorrect or incomplete solutions, gives the student zero for this answer. Give a correct proof for this case.

5. Consider the following parse tree for a context-free grammar:



(a) [5 marks] Which strings correspond to u, v, x, y and z of the pumping theorem ?

(b) [5 marks] Use the ideas from the pumping theorem to find an infinite language which is contained in the language generated from the grammar for this question.

6. Suppose that a professor of CSC 320 using a newly invented quantum computer has written a program which takes as input a TM M_1 and answers
Question 1: "Does M_1 halt on input 101010?".

For your CSC 320 homework, you have created a TM M_2 and you want to know the answer to
Question 2: "Does M_2 halt on input 42?".

(a) [7 marks] Prove that it is possible to decide the answer to Question 2 using the algorithm that the professor has developed for Question 1. If you create a new TM in your proof, give its machine schema.

(b) [3 marks] Does your answer from (a) prove OUTCOME 1 or 2:

1. If Question 1 is not Turing-decidable then Question 2 is not Turing-decidable.
2. If Question 2 is not Turing-decidable then Question 1 is not Turing-decidable.

7(a) [4 marks] State the definition of what it means for a TM $M = (K, \Sigma, \delta, s)$ to decide a language L defined over an alphabet Σ_1.

(b) [7 marks] Suppose you are given a TM C which makes a copy of an input string w. Starting with # w [#] on the input tape, the final result is # w # w [#]. You are also given a TM M_1 that decides a language L_1 and a TM M_2 that decides a language L_2. Give the machine schema for a machine that decides $L_1 - L_2$.

(c) [4 marks] What does part (b) tell you about closure under difference?

8. A path that contains every vertex of a graph is a *Hamilton path*. A cycle that contains every vertex of a graph is a *Hamilton cycle*.

HAMILTON PATH PROBLEM

INSTANCE: Graph $G = (V, E)$.

QUESTION: Does G have a Hamilton path?

HAMILTON CYCLE PROBLEM

INSTANCE: Graph $G = (V, E)$.

QUESTION: Does G have a Hamilton cycle?

(a) [5 marks] Prove that if there is polynomial time algorithm for HAMILTON PATH then there is a polynomial time algorithm for HAMILTON CYCLE.

(b) [3 marks] What is the time complexity of your approach from part (a) assuming that the algorithm for HAMILTON PATH takes time $O(n^3 + m^2)$, where n is the number of vertices of the graph and m is the number of edges?

(c) [3 marks] Assuming that both HAMILTON PATH and HAMILTON CYCLE have been proved to be in NP, which of these statements have you proved with your answer to (a):
S1: If HAMILTON PATH is NP-complete then HAMILTON CYCLE is NP-complete.
S2: If HAMILTON CYCLE is NP-complete then HAMILTON PATH is NP-complete.

(d) [4 marks] Prove that HAMILTON PATH is in NP. Give pseudocode and analyse the time complexity of any algorithms that you develop.

Use this page if you need extra space. Clearly indicate the question you are answering.

The End.