CSC 320 Midterm Exam #2 Summer 2002

1. For parts (a), (b), and (c) below, you must choose three DIFFERENT languages from the five given here and are required to find a regular expression, a context-free grammar, and a DFA for them respectively. Choose carefully to minimize your effort.

The five languages to choose from:

$L_1 = \{a^p : p$ is prime $\}$.
$L_2 = \{b\, a^n\, b\, a^{3n} b\}$
$L_3 = \{w \in \{a, b\}^* :$ the number of a's in w is even and the number of b's is even $\}$.
$L_4 = \{w \in \{a, b\}^* :$ the number of a's in w is equal to the number of b's in w $\}$.
$L_5 = \{w \in \{a, b\}^* : w$ contains *baba* or *abaab*$\}$
Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Context-free Grammar	
(c)	Deterministic Finite Automaton	

(a) [10 marks] Give a regular expression for one of the languages.

(b) [10 marks] Give a context-free grammar for one of the languages.

(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments).

2. [20] The *exclusive or* of two languages L_1 and L_2, denoted $L_1 \oplus L_2$, is defined to be $\{w : (w \in L_1$ *or* $w \in L_2)$ *and w is not in* $L_1 \cap L_2\}$.
Prove that regular languages are closed under **exclusive or** by describing a construction for a DFA $M = (K, \Sigma, \delta, s, F)$ for $L_1 \oplus L_2$ given DFA's $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$ for L_1 and L_2 respectively.
Hint: a construction similar to the ones derived for union and intersection on assignment #2 works.

3. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

(a) [5 marks] ϕ^* is a regular expression for a language containing no strings.
True False

(b) [5 marks] Every subset of a regular language is regular.
True False

(c) [5 marks] If $x \notin L_1$ and $y \notin L_2$ then $x\,y \notin L_1 \cdot L_2$.
True False

(d) [5 marks] The set containing all DFA's over the alphabet $\{a, b\}$ is countable.
True False

4.(a) [10 marks] State precisely the pumping lemma for regular languages.

(b) [10 marks] Describe all ways of factoring $w = a^r\, b^s\, c^{r+s}$ as $x\,y\,z$ where y is not equal to the empty string.

(c) [10 marks] Apply the pumping lemma to $w = a^r\, b^s\, c^{r+s}$. to prove that $L = \{a^n\, b^m\, c^p : n + m \leq p\}$ is not accepted by a DFA with $k = 2(r + s)$ states.