Name: _____

ID Number:_____

CSC 320 Midterm Exam

Wed. Oct. 26, 2011

Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. Read through the entire exam before beginning. You should have 9 pages including this header page.
4. If you need more space you can write on the backs of the pages.

Question	Value	Mark
1	40	
2	20	
3	25	
4	15	
Total	**100**	

1. For parts (a), (b), (c) and (d) below, you must choose four DIFFERENT languages from the five given here and are required to find a regular expression, a context-free grammar, a DFA, and a PDA for them respectively. Choose carefully to minimize your effort.

$L_1 = \{ w\,w \ : \ w \in \{ a, b \}^* \}$
$L_2 = \{ w \in \{ 0, 1 \}^* \ : \ w$ contains 01001 $\}$
$L_3 = \{ a^p \ b^q \ c^r \ d^s \ : \ (p + q) = (r + s), \quad p, q, r, s \geq 0 \ \}$
$L_4 = \{ u\,u^R \ v\,v^R \ : \ u \in \{ 0, 1 \}^*, \ v \in \{ 0, 1 \}^+ \}$
$L_5 = \{ w \in \{ a, b \}^* \ : \ w$ has both *abba and baab* as substrings $\}$
Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Context-free Grammar	
(c)	Deterministic Finite Automaton	
(d)	Pushdown Automata	

(a) [10 marks] Give a regular expression for one of the languages.

(b) [10 marks] Give a context-free grammar for one of the languages.

[Question #1, continued]

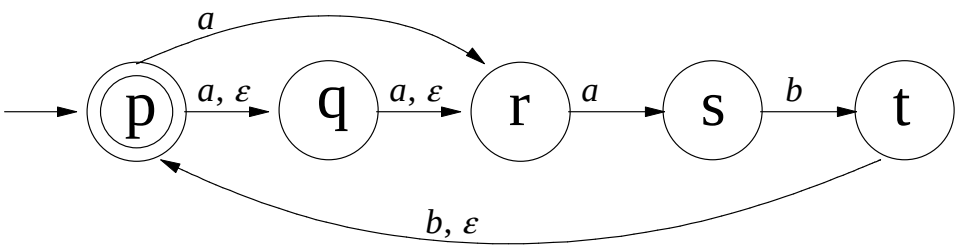
(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments).

(d) [10 marks] Design a PDA which accepts one of the languages:

Start state: **Final states:**

State	Read	Pop	Next State	Push

2. [20 marks] Use the construction described in class (which is the same as the one in the text) to convert this NDFA to an equivalent DFA:



State	Symbol	Q	Next state

Start state: _____

Final states: _____

A picture of your final DFA:

3.(a) [5 marks] State the pumping lemma for regular languages.

(b) [5 marks] Let $w = a^n b c^{2n}$. Describe all possible ways of choosing x, y, z such that $w = x\,y\,z$, and $y \neq \varepsilon$.

(c) [10 marks] Apply the pumping lemma to $w = a^n b c^{2n}$ to prove that $L = \{a^r b c^s : r \leq s \leq 2r\}$ is not accepted by a DFA with $3n + 1$ states.

(d) [5 marks] A more judicious choice for w would have made the argument for (c) much simpler. Suggest a better choice for w. How does this simplify the argument you gave for (c)?

4. Suppose you are given a boolean function *IsEmpty(M):*

   Input: A DFA M

   Returns: *true* if $L(M) = \phi$ and *false* otherwise.

(a) [9 marks] Describe a construction which given a DFA $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ yields a DFA $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$ so that if you call isEmpty(M_2) it returns the answer to the question: ‘‘Does M_1 accept any strings which start with 101?’’

[Question 4, continued]

The Question from part (a) is: ‘‘Does M_1 accept any strings which start with 101?’’

(b) [6 marks] Show how to apply your construction from part (a) to this DFA M_1 and draw a picure of the resulting DFA M_2.

Start state: s Final states: { s, u }

State	Symbol	Next State
s	0	t
s	1	s
s	2	u
t	0	u
t	1	t
t	2	s
u	0	s
u	1	u
u	2	t

Use this page if you need more space.
Clearly indicate the question you are answering.