1. For parts (a), (b), (c) and (d) below, you must choose four DIFFERENT languages from the six given here and are required to find a regular expression, a context-free grammar, a DFA, and a PDA for them respectively. Choose carefully to minimize your effort.

The six languages to choose from:

$L_1 = \{a^p : p$ is prime $\}$.

$L_2 = \{w \in \{a, b\}^* :$ the number of a's in w is equal to the number of b's in $w\}$.

$L_3 = \{w \in \{a, b\}^* :$ the number of a's in w is congruent to the number of b's in w modulo 2$\}$.

$L_4 = \{w \in \{a, b, c\}^* : w = a^n b^n c^n, n \geq 0\}$.

$L_5 = \{w \in \{a, b\}^* : w$ contains $aaba$ and $ababb\}$.

$L_6 = \{u \in \{a, b\}^* : w = w^R\}$.

Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Context-free Grammar	
(c)	Deterministic Finite Automaton	
(d)	Pushdown Automaton	

(a) [10 marks] Give a regular expression for one of the languages.

(b) [10 marks] Give a context-free grammar for one of the languages.

(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments).

(d) [10 marks] Describe a PDA for one of the languages (include comments).

2.(a) [10 marks] State the pumping lemma for regular languages (as presented in class, the ‘‘beginning of the string’’ pumping lemma).

(b) [10 marks] Let $w = a^k b a^{k^2}$. Describe all possible ways of choosing x, y, z such that $w = xyz$, and $y \neq \varepsilon$.

(c) [10 marks] Apply the pumping lemma to $w = a^k b a^{k^2}$ to prove that $L = \{a^n b a^m : n^2 \leq m \leq n^3\}$ is not accepted by a DFA with $k^2 + k + 1$ states.

(d) [10 marks] A more judicious choice for w would have made the argument for (c) much simpler. Suggest a better choice for w. How does this simplify the argument you gave for (c)?

3. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

(a) [7 marks] $L = \{a\}^*$ is countable.

True False

(b) [7 marks] Every subset of a regular language is regular.

True False

(c) [7 marks] If $x \notin L_1$ and $y \notin L_2$ then $xy \notin L_1 \cdot L_2$.

True False