

## What is wrong with my induction proof?

In a drunken haze I decided that the solution to the recurrence  $T(1)=1$ ,  $T(n)= 1 + T(n-1)$  is

$$1 + 2 + 3 + \dots + n.$$

Theorem: The solution to the recurrence is  $n(n+1)/2$ .

Proof. [Basis]  $T(1)=1$  and  $1 \cdot (1+1)/2 = 1$  as required.

[Induction step] Assume that  $1 + 2 + \dots + n-1 + n = n(n+1)/2$ .

We want to prove that  $1 + 2 + \dots + n-1 + n + (n+1) = (n+1)(n+2)/2 = (n^2 + 3n + 2)/2$ .

By induction,  $1 + 2 + \dots + n = n(n+1)/2$ .

So  $1 + 2 + \dots + n + (n+1) = n(n+1)/2 + (n+1)$ .

Simplifying:  $(n^2 + n + 2n + 2)/2 = (n^2 + 3n + 2)/2$  as required. <sub>1</sub>

# Announcements

- Assignment 1 is posted- due Fri. May 19.  
Submissions: on paper in class,  
code must be uploaded to connex.
- Tutorial **today** (material is posted on our web page for  
you to do before you go)  
1:30-2:20pm or 2:30-3:20pm in **Clearihue A 307**.  
Feel free to attend either section.
- Midterm study aid has a reading list, for now read Ch. 1.
- Powerpoint slides will be posted: click on the  
"Selected class notes" link on the course web page.

**Don't forget to vote today!**

# CSC 320:

## To Infinity and Beyond

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### BOOKS



**“Yes, we have *Chicken Soup for the Math Teacher’s Soul*.  
The price is  $\$475 \div 23 \times .018^2 - Y^3 + 4X \div \$73.99999 + 2.$ ”**

# Outline

- Definitions of equinumerous, cardinality, finite, infinite, countable, uncountable
- Tactics for proving that sets are countable
- Diagonalization for proving sets are uncountable

CSC 320 will challenge you to broaden how you think mathematically.

The idea that there is more than one "size" of infinity is a strange concept. Several new proof tactics are introduced.

On ongoing theme of this class will be that each of our language representation schemes represents a countable number of languages so some must be left out since the total number of possible languages is not countable.

# Definitions:

Two sets  $A$  and  $B$  are **equinumerous** if there is a bijection (pairing)  $f:A \rightarrow B$

A set  $S$  is **finite** (cardinality  $n$ ) if it is equinumerous with  $\{1, 2, \dots, n\}$  for some integer  $n$ .

A set is **infinite** if it is not finite.

A set is **countably infinite** if it is equinumerous with the set of natural numbers.

A set is **countable** if it is finite or countably infinite.

Theorem 1:

The set  $\{(p, q): p \text{ and } q \text{ are natural numbers}\}$  is countable

Proof tactic: Dovetailing

Note: this tactic does not provide an explicit formula for a bijection to the natural numbers but it does have enough information for someone to know how to compute it.

You should give this same level of information on the assignment.

Theorem 2:

The set  $\{ r : r \text{ is a real number, } 0 \leq r < 1 \}$   
is not countable.

Proof tactic: Diagonalization

Note: Use diagonalization on the assignment  
when you want to prove a set is not countable.  
Do not use other results.