Prove the following languages over $\Sigma=\{0,1\}$ are regular by giving regular expressions for them:

1. {w contains two or more 0's}
2. {|w| = 3k for some integer k}
3. {w has 001 or 1010 as a prefix}
4. {w contains both 00 and 11 as substrings}
5. {w contains 01001 and 0101 as substrings}
6. {w : w has at most two consecutive 1's}

Deterministic Finite Automata

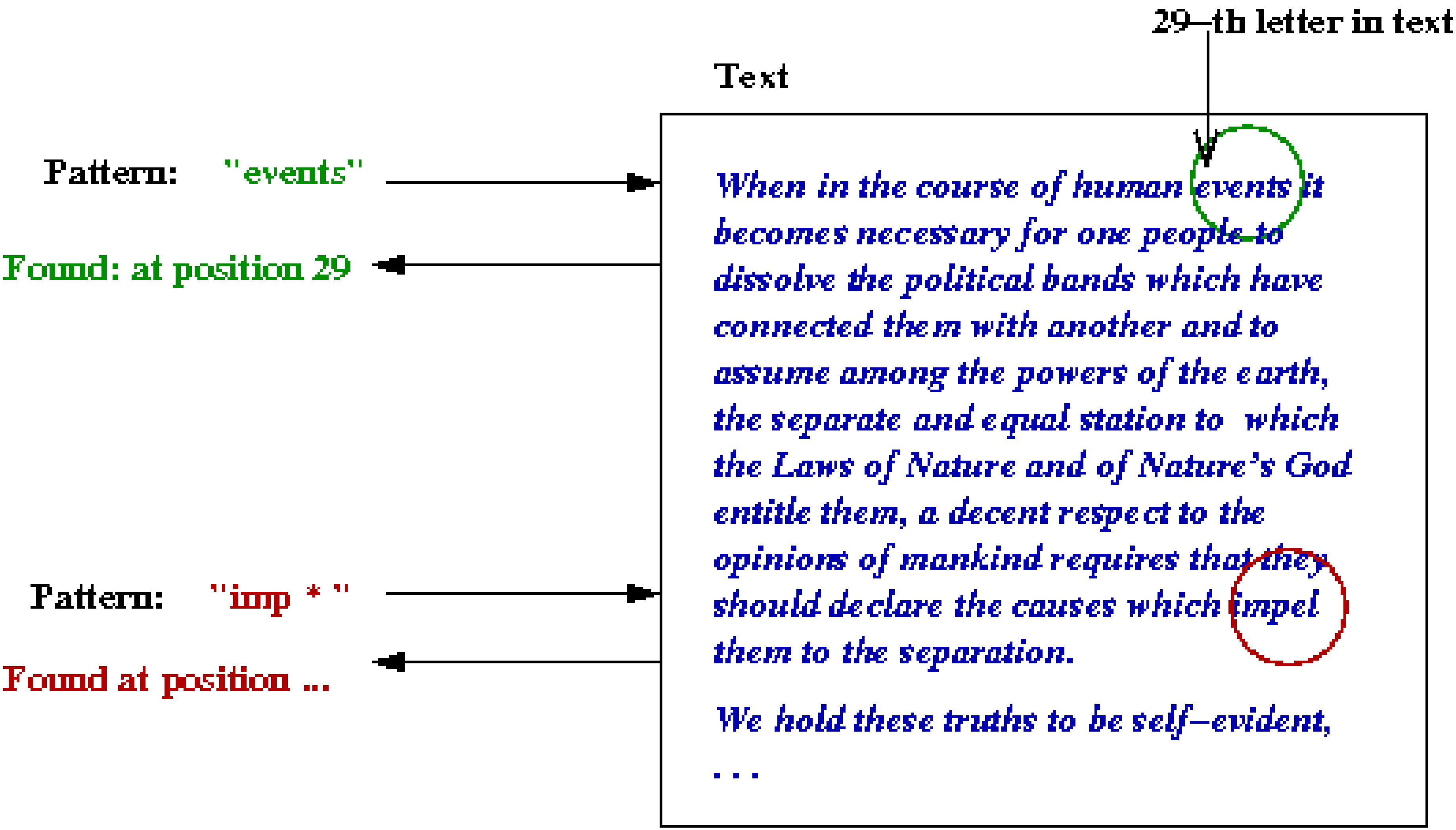
Given a language L, the language recognition problem is:

Given an input string w, is w in L?

Answer: yes or no.

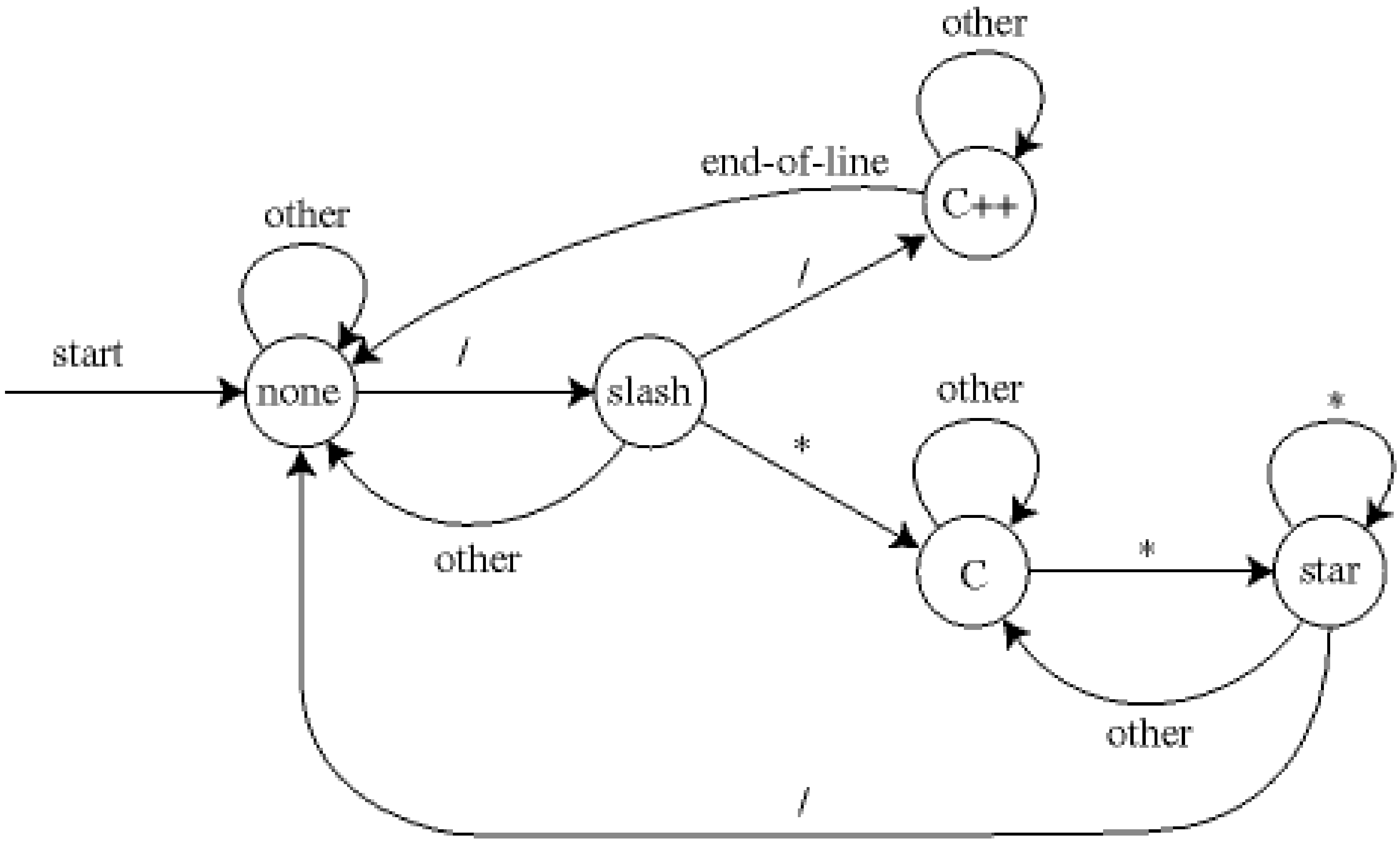
DFA's provide linear time language recognition algorithms for regular languages (languages which can be defined by a regular expression).

This class introduces DFA's, describes how they work, and defines the mathematical notation used to talk about them.



http://www.seas.gwu.edu/~simhaweb/cs151/lectures/module5/module5.html

To strip comments from a program:



Picture by David Eppstein.

Applications of DFA's or DFA-like code:

[*www.fbeedle.com/formallanguage/ch04.pdf*]

Compilers and other language-processing systems- to divide an input program into tokens like identifiers, constants, and keywords and to remove comments and white space.

Typed commands- the command language can be quite complex, almost like a little programming language.

Speech-processing and other signal-processing systems- to transform an incoming signal.

More Applications

[*www.fbeedle.com/formallanguage/ch04.pdf*]

Text processors- to search a text file for strings that match a given pattern. This includes most versions of Unix tools like awk, egrep, and Procmail, along with a number of platform-independent systems such as MySQL.

Controllers- to track the current state of a wide variety of finite-state systems, from industrial processes to video games, implemented in hardware or in software.

$L = \{w \in \{a,b\}^* :$ the number of a's in w is not divisible by 3$\}$.

1. Give a regular expression for L.

2. Design a DFA that accepts L.

Give regular expressions for:

$L_1 = \{w \in \{a,b\}^* : w = a^{2k}\, b^{2r+1}$ for some integers $k, r \geq 1\}$

$L_2 = \{w \in \{a,b\}^* : w$ has at least two a's and at least two b's$\}$

A Deterministic Finite Automaton (DFA) M is defined to be a quintuple

$(K, \Sigma, \delta, s, F)$ where

K is a finite set of states,

Σ is an alphabet,

δ is a function from $K \times \Sigma$ to K,

$s \in K$ is the start state, and

$F \subseteq K$ is the set of final states.

Some examples:

1. {w: w has odd length}
2. {w: w contains 0011}
3. {w: w does not contain 01}
4. {w: w starts and ends with the same symbol ($|w| \geq 1$)}

$M = (K, \Sigma, \delta, s, F)$:

A configuration of a M is an element of $K \times \Sigma^*$.

For $\sigma \in \Sigma$, configuration $(q, \sigma w) \vdash (r, w)$

[$(q, \sigma w)$ yields in one step (r, w)]

if $\delta(q, \sigma) = r$.

The notation $\vdash^*$ means yields in zero or more steps.

$M = (K, \Sigma, \delta, s, F)$:

DFA M accepts input w if and only if

$(s, w) \vdash^* (f, \varepsilon)$ for some $f \in F$.

L(M), the language accepted by M is

$\{ w \in \Sigma^* : (s, w) \vdash^* (f, \varepsilon)$ for some $f \in F\}$.

What language does this DFA accept?