For each language, give a DFA that accepts it.

$L_1 = \{ w \in \{a, b\}^* :$ w at most 2 a's$\}$

$L_2 = \{ w \in \{0, 1\}^* :$ has both 001 and 11 as substrings$\}$

$L_3 = \{ w \in \{a, b\}^* :$ w ends with abab$\}$

$L_4 = \{ w \in \{a,b\}^* : w = a^{2r} b^s$ for some integers r and s where $r, s \geq 0\}$

$L_5 = \{ w \in \{0, 1\}^* :$ has both 010 and 101 as substrings$\}$

Assignment #2 has been posted.
Due Friday June 2 at the beginning of class.

Tutorial #3 has been posted.
No tutorial this week.
The next tutorial is Tuesday May 30.

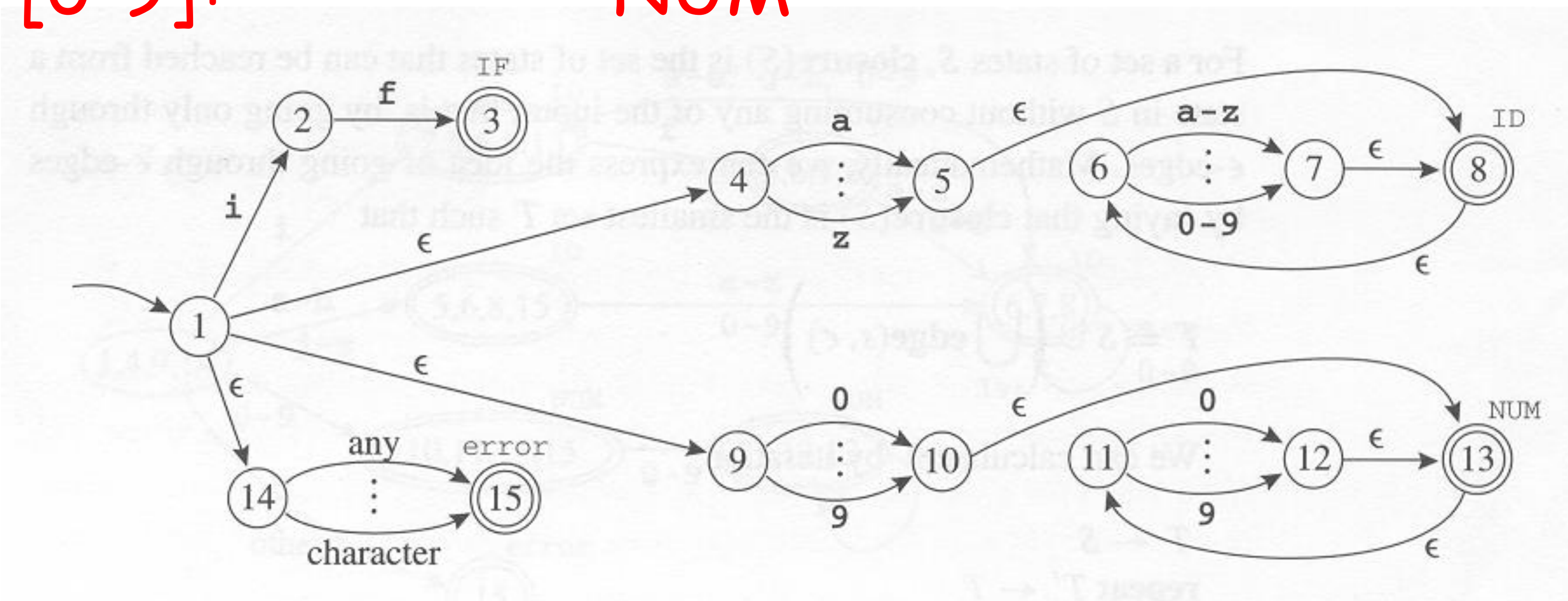
$L_5 = \{ w \in \{0, 1\}^*$: the number of 0's in w is even and the number of 1's is congruent to 1 modulo 3$\}$

$L_6 = \{ w \in \{a, b\}^*$: w contains abaab$\}$

Nondeterministic Finite Automata

if IF
[a-z][a-z0-9]* ID
[0-9]+ NUM

Picture from: Raymond Wisman, Notes from compiler design class



Outline: NDFA's are like DFA's but their operation is not deterministic. NDFA's are defined and several examples are given. We prove that for every NDFA there is an equivalent DFA by giving an algorithm which constructs one.

Context: By definition a language is regular if and only if there is a regular expression for it. We will soon prove that a language is regular if and only if there is a DFA which accepts it.

Ultimate Consequences:

You can prove that a language is regular by either

1. finding a regular expression which generates it,
2. finding a DFA which accepts it, or
3. finding a NDFA which accepts it.

A Nondeterministic Finite Automaton (NDFA) M is defined to be a quintuple

$(K, \Sigma, \Delta, s, F)$ where

K is a finite set of states,

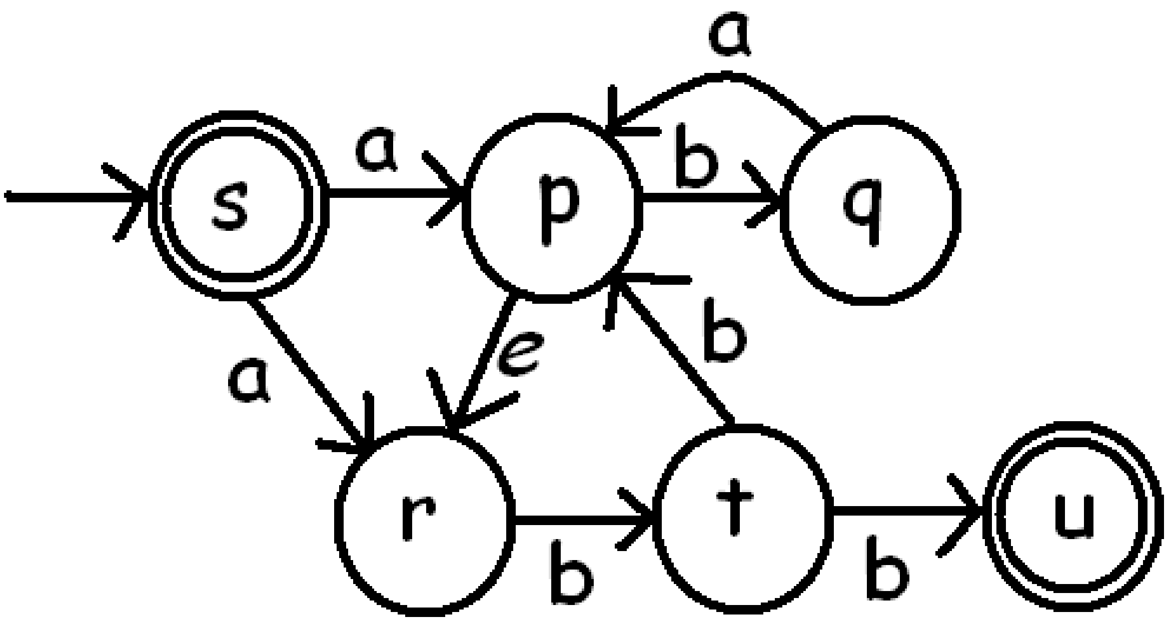
Σ is an alphabet,

Δ, the transition relation, is a subset of $K \times (\Sigma \cup \{\varepsilon\}) \times K$,

$s \in K$ is the start state, and

$F \subseteq K$ is the set of final states.

M



$M = (K, \Sigma, \Delta, s, F)$:

A configuration of a M is an element of $K \times \Sigma^*$.

For $\sigma \in (\Sigma \cup \{\varepsilon\})$, configuration

$(q, \sigma w) \vdash (r, w)$ if (q, σ, r) is in Δ.

The notation $\vdash^*$ means yields in zero or more steps.

$M = (K, \Sigma, \Delta, s, F)$:

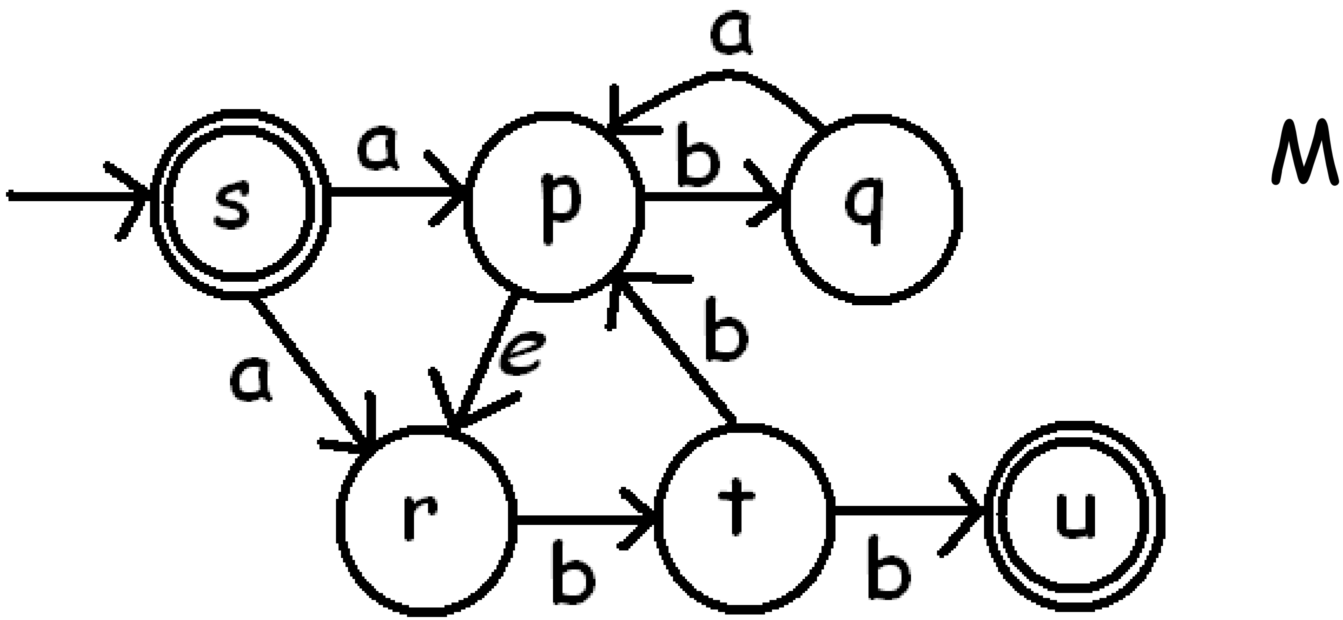
NDFA M accepts input w if and only if

there exists some computation such that

$(s, w) \vdash^* (f, \varepsilon)$ for some $f \in F$.

L(M), the language accepted by M is

$\{ w \in \Sigma^* : (s, w) \vdash^* (f, \varepsilon) \text{ for some } f \in F\}$.



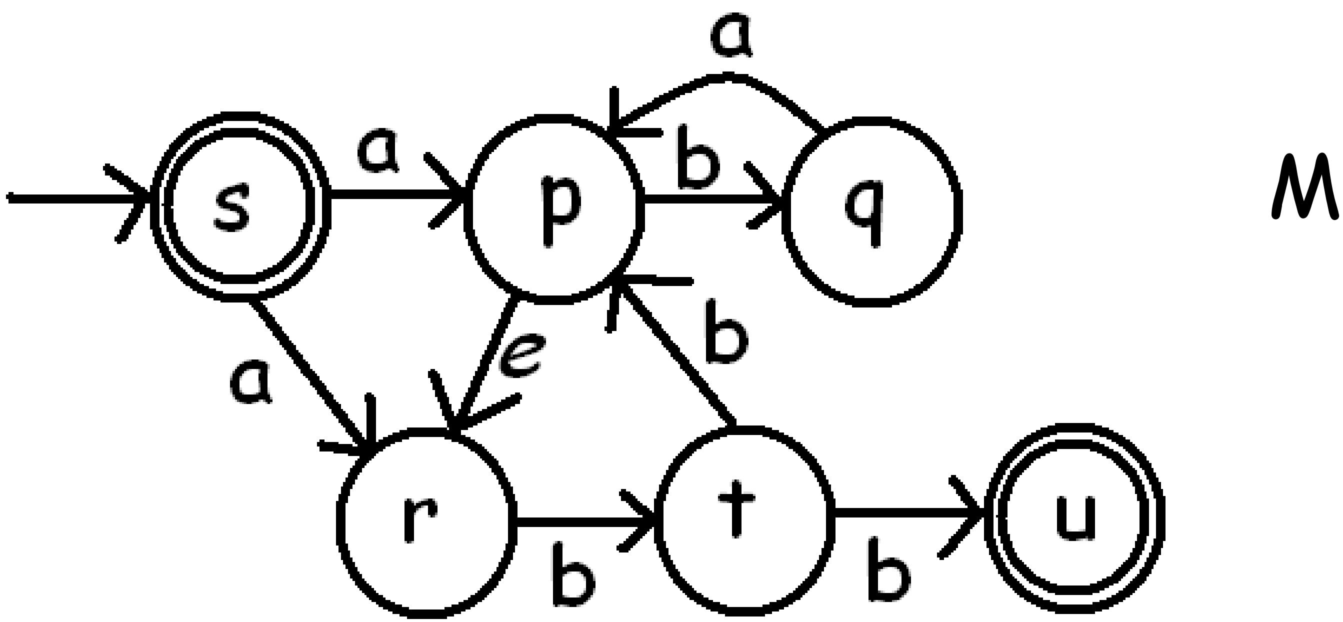
Some non-accepting computations:

(s, abb) $\vdash$ (p, bb) $\vdash$ (q, b)

STUCK- no way to finish consuming the input.

(s, abb) $\vdash$ (r, bb) $\vdash$ (t, b) $\vdash$ (p,*e*)

Ends in a non-final state p.



Some accepting computations:

(s, abb) $\vdash$ (r, bb) $\vdash$ (t, b) $\vdash$ (u, e)

(s, abb) $\vdash$ (p, bb) $\vdash$ (r, bb) $\vdash$ (t, b) $\vdash$ (u, e)

abb is in L(M) since it has at least one accepting computation.

Prove the following languages over $\Sigma = \{0, 1\}$ are regular by constructing NDFA's which accept them.

1. $L_1 = \{ w : w$ starts and ends with $0\}$.
2. $L_2 = (000 \cup 11 \cup 01)^*$
3. $L_1 \cup L_2$
4. $L_1 \cdot L_2$

A set S is closed with respect to a binary operation $\cdot$ if for all s and t in S, $s \cdot t$ is in S.

Examples:

N= {0, 1, 2, 3, ...}

Closed for addition and multiplication.

Not closed under subtraction or division.

The set $S = \{ L : L \text{ is } L(M) \text{ for some DFA } M\}$ is closed under union.

Let $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1

and $M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$ accept L_2 .

Proof 1: A construction for a new DFA $M = (K, \Sigma, \delta, s, F)$ which accepts $L_1 \cup L_2$.

Proof 2: A construction for a new NDFA $M = (K, \Sigma, \Delta, s, F)$ which accepts $L_1 \cup L_2$.

The set $S=\{ L : L$ is $L(M)$ for some DFA $M\}$ is closed under concatenation.

Let $M_1= (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1

and $M_2= (K_2, \Sigma, \delta_2, s_2, F_2)$ accept L_2.

Proof: A construction for a new NDFA $M= (K, \Sigma, \Delta, s, F)$ which accepts $L_1 \cdot L_2$.

The set S= { L : L is L(M) for some DFA M} is closed under Kleene star.

Let $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1.

Proof: A construction for a new NDFA $M = (K, \Sigma, \Delta, s, F)$ which accepts L_1*.