If SAT is solvable in polynomial time, then so is HAMILTON CYCLE .

This is on p. 303 in text but there are numerous typos. Use my notes not text.

Graph G has n vertices 0, 1, 2, ..., n-1.

Variables: $x_{i,j}$: $0 \leq i, j \leq n-1$

Meaning: Node i is in position j in Hamilton cycle.

Variables: $x_{i,j}$: $0 \leq i, j \leq n-1$ Meaning- Node i is in position j in Ham. cycle.

Conditions to ensure a Hamilton cycle:

1. Exactly one node appears in position j.

(a) At least one node appears in position j.

For each j, add a clause:

($x_{0,j}$ OR $x_{1,j}$ OR $x_{2,j}$ OR ... $x_{n-1,j}$)

(b) At most one node appears in position j.

For each pair of vertices i,k add a clause

(not $x_{i,j}$ OR not $x_{k,j}$)

Variables: $x_{i,j}$: $0 \le i, j \le n-1$ Meaning- Node i is in position j in Ham. cycle.

2. Vertex i occurs exactly once on the cycle.

(a) Vertex i occurs at least once.

For each i, add a clause:

$(x_{i,0}$ OR $x_{i,1}$ OR $x_{i,2}$ OR ... $x_{i,n-1})$

(b) Vertex i occurs at most once.

For each vertex i and pair of positions j,k add a clause (not $x_{i,j}$ OR not $x_{i,k}$)

Variables: $x_{i, j}$: $0 \le i, j \le n-1$ Meaning- Node i is in position j in Ham. cycle.

3. Consecutive vertices of the cycle are connected by an edge of the graph.

For each edge (i, k) which is missing from the graph and for each j add a clause

(not $x_{i, j}$ OR not $x_{k, j+1 \bmod n}$)

Known: SAT is NP-complete.

We just proved that if SAT is solvable in polynomial time, then so is HAMILTON CYCLE.

Is this a proof that HAMILTON CYCLE is NP-complete?

Known: SAT is NP-complete.

We showed that if SAT is solvable in polynomial time, then so is HAMILTON CYCLE.

Is this a proof that HAMILTON CYCLE is NP-complete?

NO. Wrong direction. We would have to show that you can solve SAT in polynomial time using HAMILTON CYCLE instead.

Another example: proving you can solve 2-SAT using a SAT solver does not mean 2-SAT is hard.

A problem Q in NP is NP-complete if the existence of a polynomial time algorithm for Q implies the existence of a polynomial time algorithm for all problems in NP.

How do we prove SAT is NP-complete?

Cook's theorem: SAT is NP-complete.