CSC 422/522 Fall 2014

Dr. Wendy Myrvold, ECS 552, wendym@cs.uvic.ca

http://xkcd.com/
Announcements

Powerpoint slides will be posted on the class web pages:

http://webhome.cs.uvic.ca/~wendym/422.html

Or on connex if they contain material that cannot be placed on the web.

Welcome to CSC 422/522!
Office hours:
Let me know either at the end of class or by e-mail what time you would like to come by so that I don't have to be there when nobody wants to see me.

TWF 11:30am - 12:20pm.

Tuesdays 1:30pm: when no dept. meeting.

WF: 1:30pm (until all questions are answered).

Or by appointment.
Outline

• Who is the instructor?
• My research interests.
• Logistics for CSC 422/522 - the critical points are included on the course outline and class web pages.
• Don’t worry about taking notes today
About me:

B.Sc.: Computer Science, McGill University, 1983
M.Math.: Combinatorics and Optimization, University of Waterloo, 1984
Ph.D. in Computer Science: Waterloo, 1988

University of Victoria: started in 1988, currently a full professor

Currently have 64 CPU’s that can be used for long computations (provided by NSERC).

From: Gurl Guide to programming.
Bring your parents to work day at Google.
My Research: Large Combinatorial Searches

Clique: Set of vertices which are pairwise adjacent
Keller graph with dimension $d$: vertices which are numbered with each of the $4^d$ $d$-digit numbers, digits are to 0, 1, 2, or 3. Two vertices are adjacent if their labels differ in at least two positions, and in at least one position the difference in the labels is 2 mod 4.

Examples (Dimension 5):

1 0 1 2 0
2 0 3 2 0

0 3 1 2 0
1 2 0 3 1

Adjacent

NOT adjacent

“In a sense, these cases require only patience- and maybe a high speed computer the size of a major galaxy.

No one in his right mind, and no mathematicians either, would set our to sort through \(2^{128}\) possible tilings to check the case of \(d=7\).”

p. 24, Barry Cipra and Paul Zorn
Finishing the Keller conjecture only requires the answer to whether the clique order for dimension 7 is 128 or less than 128.

Determining if it was 127 or less than 127 took only 3 days on 64 CPU's.

Determining that the maximum clique order is 124 took 109 days on 64 CPU's.

Funding for CPU’s was provided by:

Double checking:

8192 CPU’s
Applications of Graph theory to chemistry

Working with Patrick Fowler (chemist)

Graphite

Diamond

Benzenoids
The graph for naphthalene has 3 perfect matchings:
Randić current model: Consider ordered pairs of perfect matchings.
Conjugated Circuits:

- Occurs 2 times. (BC, CB).
- Occurs 2 times. (AB, BA)
- Occurs 2 times. (AC, CA)
Current flow: counterclockwise in $4n+2$ cycles and clockwise in $4n$ cycles.

Sum the currents for each pair of matchings to get current estimate.
Fullerenes are all-carbon molecules that correspond to 3-regular planar graphs with all face sizes equal to 5 or 6.
Pentagons cause grief for some hexagons.

Yellow hexagons – only 2 independent set vertices.

Linear time algorithm for maximum independent set of a fullerene: joint work with Sean Daugherty.
Topological Graph Theory: Algorithms and Obstructions
Torus Obstructions Found So Far:

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Our research interests include:
Graph Theory and Graph Algorithms
Combinatorics
Combinatorial Algorithms
Computational Geometry
Randomized Algorithms
Computational Complexity
Network Reliability
Topological Graph Theory
Computational Biology
Cryptography
Design Theory

Join our listserv to get information about conferences and research talks.

Undergrads are welcome to all events.
**Course description:**

This course provides an introduction to graph theory and graph algorithms. We will start with basic definitions in order to make the class accessible to all. The algorithms studied range from classical polynomial time algorithms for problems such as network flows to those geared towards dealing with intractible problems such as finding a maximum independent set in a graph. The material also includes cutting edge research tactics for solving real world problems. The class is especially valuable for students requiring graph theory and combinatorics as a tool for research in areas such as networks, database, computer graphics, and software engineering.
Course objectives:

• To provide a solid foundation in graph theory and algorithms.
• To teach some useful algorithms and algorithm design tactics.
• To develop research skills which include:
  • Background literature search,
  • Formal writing for graph theory topics (as required for theses, conference or journal papers), and
  • Programming graph algorithms.
• To intrigue and excite students about graph theory research topics.
• To take students to the leading edge of graph theory research.

Other references will be provided if we deviate from the text.
Students with a disability

Please let me know as soon as possible how I can accommodate your disability.

It’s sometimes possible to go beyond what is first offered by the disability center.

Parents of young children

Let me know if you require accommodation because of the school strike.
Grading scheme:

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<th>Component</th>
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<tr>
<td>Programming Project</td>
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<td>Dec. 9</td>
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New Grading System (effective Summer 2014)

• Instructor will submit grades in percentages.
• The University will use the following Senate approved standardized grading scale to assign letter grades.
• Both the percentage mark and the letter grade will be recorded on the academic record and transcripts.

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For graduate students:
B- or lower is unacceptable work revealing some deficiencies in knowledge, understanding or techniques.
Assignments (30%):

• 8-10 equally weighted small assignments, No late assignments.

• Your lowest assignment grade will be dropped when computing your assignment average.

If necessary due to limitations of TA resources, a random selection of question might be chosen for marking. Students are strongly advised to submit answers to all of the assignment questions.
Exams (20%)

• 8-10 equally weighted small pop quizzes that are during a class slot.
• Your lowest quiz mark will be dropped.
• Start: week of Sept. 23.
• No quizzes during reading break week (Nov. 10-14).
• Students who arrive to class late and miss the start of a quiz will not be provided with any extra time to finish the quiz. *Come to class on time!*
Literature Review Project (20%)
Due: Tues. Oct. 14, beginning of class.

- Choose a pre-approved subdomain of graph algorithms.
- Write a paper that defines the problem considered and summarizes some papers in the area. More substantial for graduate students (CSC 522).
- Students who exceed expectations can get bonus marks.
- Late submissions: Tues. Oct. 21 with a 10% late penalty.
Programming Project (30%)
Due: Tues. Dec. 9 by 11pm.

• Design and implement an algorithm (or for CSC 522 students, 2 algorithms) for a hard problem in graph algorithms.
• Students who exceed expectations can get bonus marks.
• Final submission: a paper that describes the algorithm, and the program(s).
• Late submissions: Tues. Dec. 16 at 11pm with a 10% late penalty.
A *dominating set* of a graph $G$ is a subset $D$ of the vertices of $G$ such that every vertex $v$ of $G$ is either in the set $D$ or $v$ has at least one neighbour that is in $D$. 
A Map of the Town of Iceberg
Dominating Set Challenge

In spite of more than 2,000 papers on dominating sets, not much has been done algorithmically to search for and evaluate practical algorithms.

In CSC 425/520 in the spring, students collected a large selection of interesting test graphs for this problem: http://webhome.cs.uvic.ca/~wendym/425.html
## Open problems?:

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Fullerenes

• Correspond to 3-regular planar graphs.
• All faces are size 5 or 6.
• Euler’s formula: exactly 12 pentagons.
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Some conjectures for fullerenes:

If $n$ is divisible by 4 then the minimum dominating set order is either $n/4$, $n/4 + 1$, or $n/4 + 2$. Can we characterize the cases that are $n/4$?

If $n$ is not divisible by 4 ($n$ is congruent to 2 mod 4) then the minimum dominating set order is $\left\lfloor \frac{n}{4} \right\rfloor$ or $\left\lfloor \frac{n}{4} \right\rfloor + 1$.

There is a linear time (or maybe $O(n^2)$ time) algorithm for finding a minimum size dominating set of a fullerene.
The **Cartesian product**, $G \square H$, of graphs $G$ and $H$ is a graph $F$ such that

1. $V(F) = V(G) \times V(H)$; and
2. any two vertices $(u,u')$ and $(v,v')$ are adjacent in $F$ if and only if either:
   - $u = v$ and $u'$ is adjacent with $v'$ in $H$, or
   - $u' = v'$ and $u$ is adjacent with $v$ in $G$.

http://mathworld.wolfram.com/GraphCartesianProduct.html
Vizing's conjecture concerns a relation between the domination number and the cartesian product of graphs. This conjecture was first stated by Vadim G. Vizing (1968), and states that, if $\gamma(G)$ denotes the minimum number of vertices in a dominating set for $G$, then $\gamma(G \square H) \geq \gamma(G) \gamma(H)$.

Conjecture predicts $\geq 1$ for this graph so it is not tight.

http://en.wikipedia.org/wiki/Vizing's_conjecture
A recent survey paper:

Brešar, Boštjan; Dorbec, Paul; Goddard, Wayne; Hartnell, Bert L.; Henning, Michael A.; Klavžar, Sandi; Rall, Douglas F. Vizing's conjecture: a survey and recent results. J. Graph Theory 69 (2012), no. 1, 46-76.