For each of these two embeddings, apply clockwise BFS starting at the red vertex with first child the green vertex and direction clockwise.







Lexicographically smaller.

$$\begin{array}{c}3\\ \\ 5\\ \\ 4\\ \end{array}$$



Lexicographically smaller.



Lexicographically smaller

0 (2): 1 2 1 (3): 0 3 2 2 (3): 0 1 4 3 (3): 154 4 (3): 2 3 5 5 (2): 3 4



Automorphism since the same.

Automorphism: Isomorphism from an object to itself. How many automorphisms does this embedding have?











Flip over a horizontal axis.





Rotate 180°



Flip over the vertical axis.





Then flip over a horizonal axis.



The original embedding.

Identity automorphism: Two line notation: 0 1 2 3 4 5 0 1 2 3 4 5

Cycle structure notation:

(0) (1)(2)(3)(4)(5)



Two line notation?

Cycle structure notation?



Two line notation: 0 1 2 3 4 5 0 1 5 4 3 2

Cycle structure notation:

(0)(1) (25)(34)



Two line notation?

Cycle structure notation?



Two line notation: 0 1 2 3 4 5 1 0 4 5 2 3

Cycle structure notation:

(01) (24)(35)



Two line notation?

Cycle structure notation?



Two line notation: 0 1 2 3 4 5 1 0 3 2 4 4

Cycle structure notation:

(01) (23)(45)

Permutations that are automorphisms:



The automorphism form a group: 1. The identity is always included. 2. If p is an automorphism, then so is p⁻¹.

3. If p and q are automorphisms, then so is p * q.

What is: rotate 180° horizonal flip (01) (24)(35) * (0)(1)(25)(34)

The automorphism form a group: 1. The identity is always included. 2. If p is an automorphism, then so is p^{-1} .

3. If p and q are automorphisms, then so is p * q.

What is:

rotation horizonal flip (01)(24)(35) * (0)(1)(25)(34)

= (01)(23)(45) vertical flip





Then flip over a horizonal axis.

Vertices u and v are in the same orbit if there is an automorphism that maps u to v. Each orbit is a subset of the vertices due to the properties of a group.

What are the orbits for this embedding?



Vertices u and v are in the same orbit if there is an automorphism that maps u to v.

What it means to be in the same orbit: An ant sitting at u and looking around at the unlabelled object will see exactly the same thing as an ant sitting at v.

Properties that do not depend on how vertices are labelled (e.g. the clockwise-BFS rotation systems that start at u) will be the same with respect to vertices in the same orbit.

Ant: http://www.1decision1day.com/2013/05/21/what-i-learned-from-an-ant/







If an embedding has an automorphism to its flip then the embedding is not chiral.







If an embedding has no automorphisms to its flip then the embedding is chiral.

Chiral embeddings have a sense of clockwise.