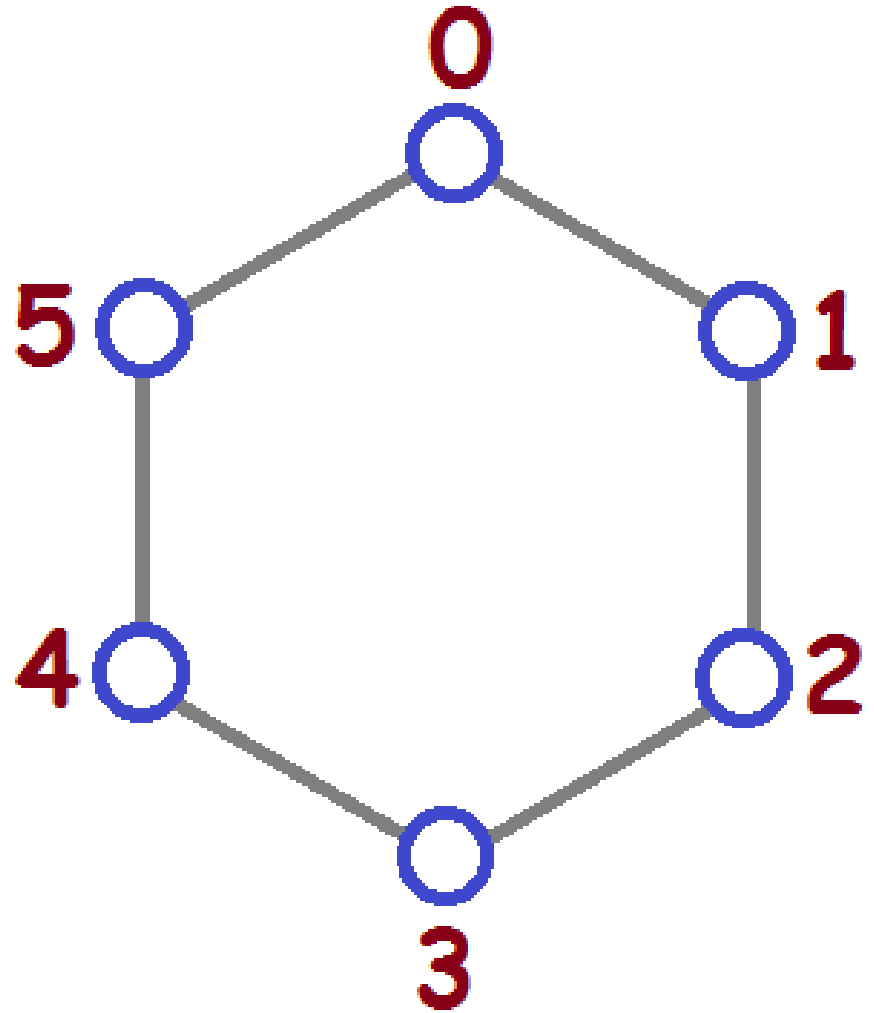


1. Write down the permutations that are the automorphisms of this graph.
2. Write the cycle structure notation for each of the automorphisms.
3. How many independent sets of order 2 does it have?

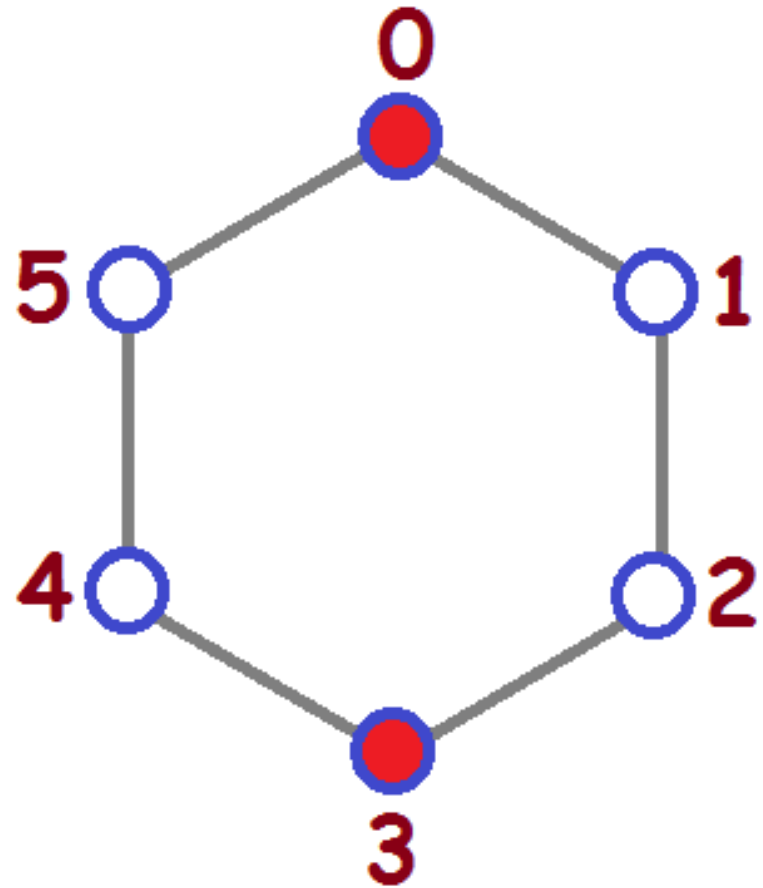


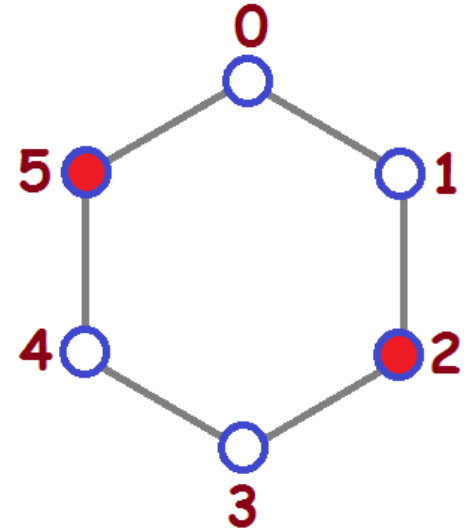
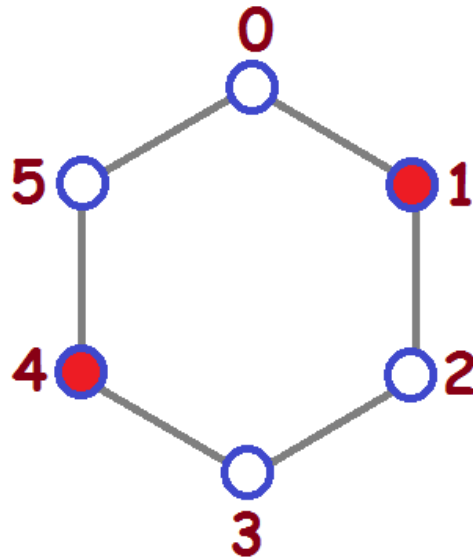
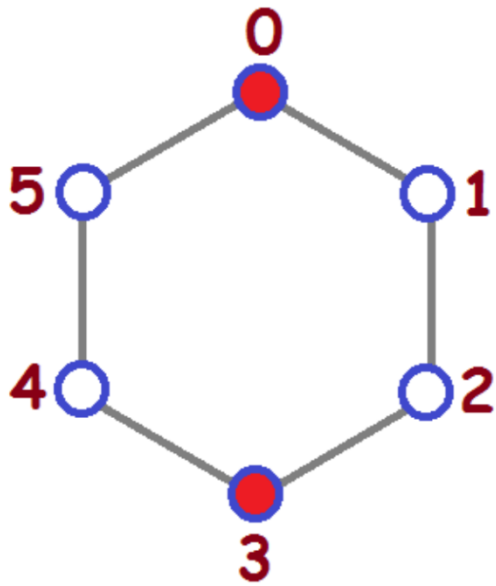
Two independent sets S and T are **equivalent (isomorphic)** if there is an automorphism of G mapping the vertices of S to those in T .

Given a graph G with automorphism group order g and an independent set S such that G has k automorphisms mapping the independent set S to itself, there will be g/k different independent sets of the graph that correspond to S .

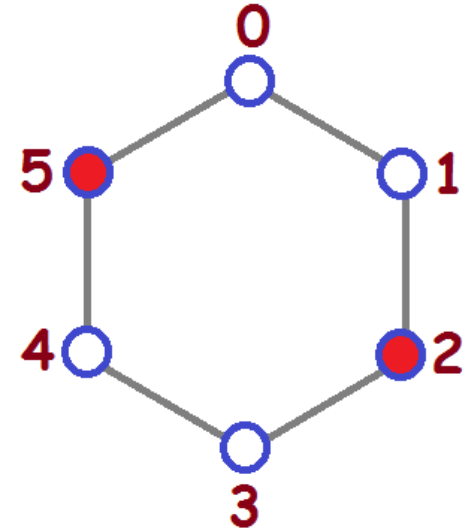
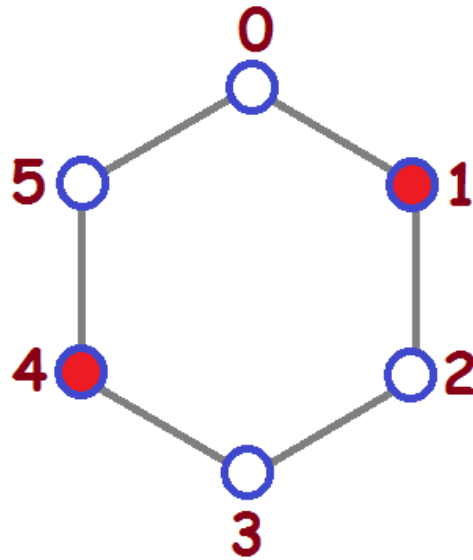
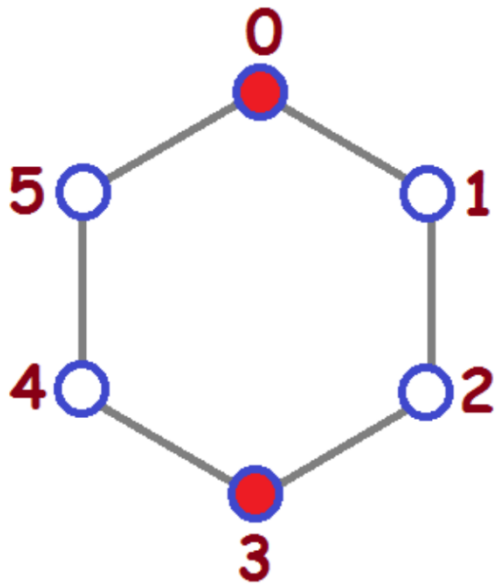
Usually the minimum of these is chosen to be the canonical representative of its equivalence class.

1. Which automorphisms map this independent set to itself?
2. Which other independent sets are equivalent to it?



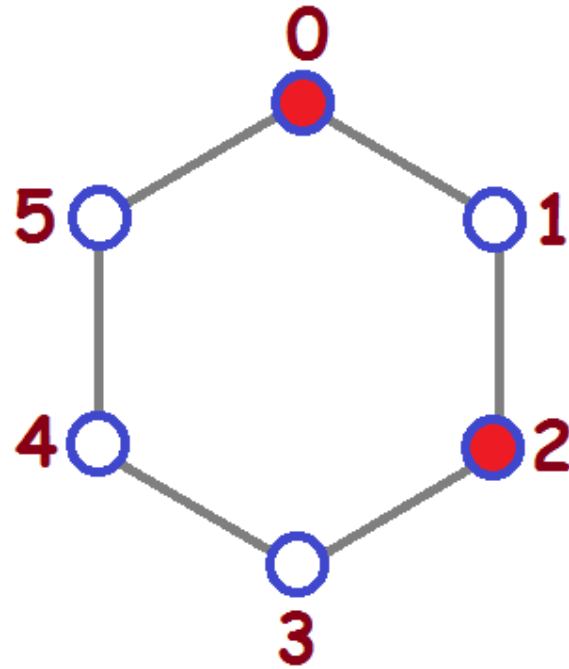


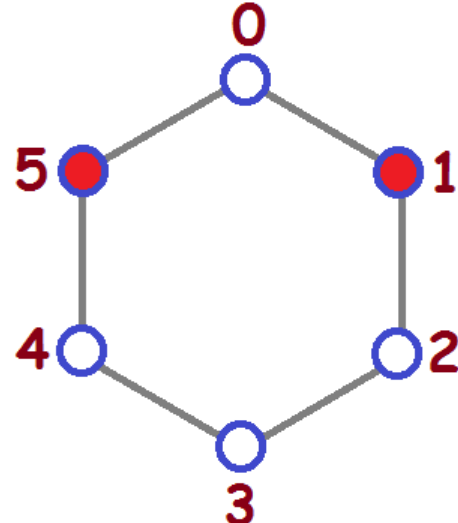
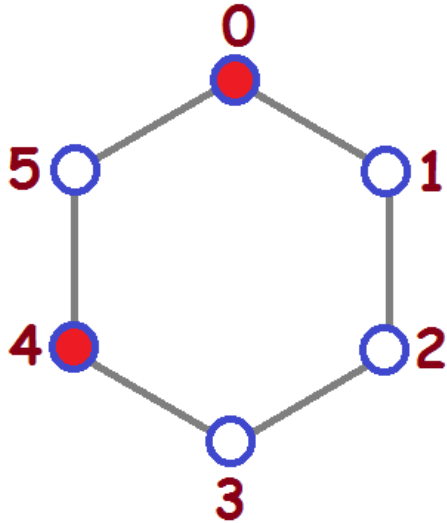
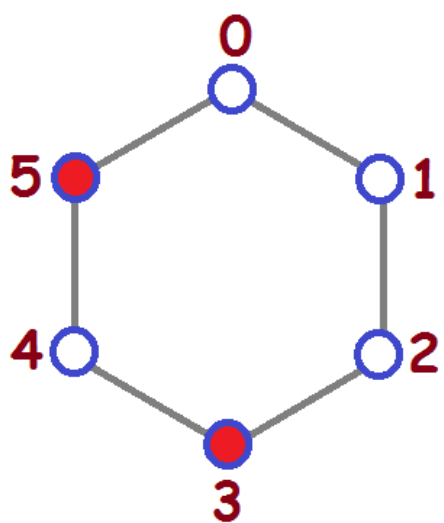
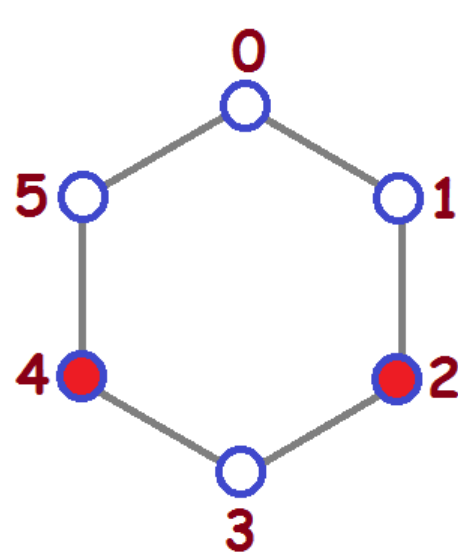
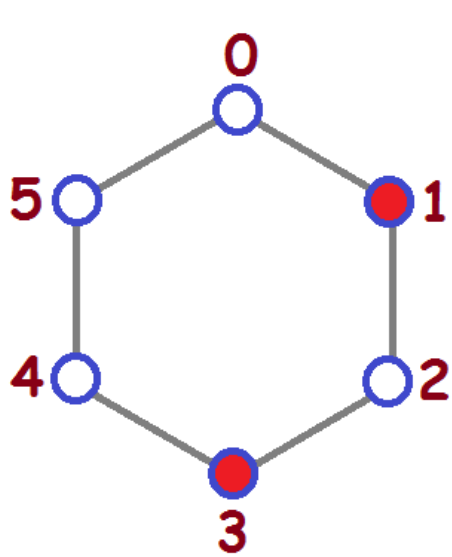
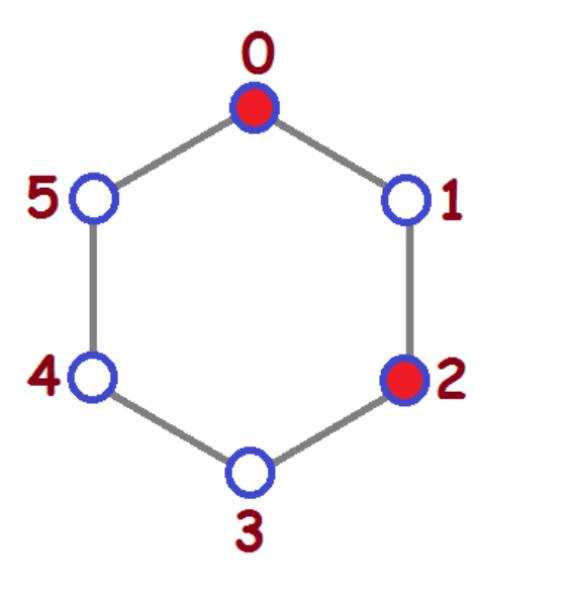
$$4 * 3 = 12$$



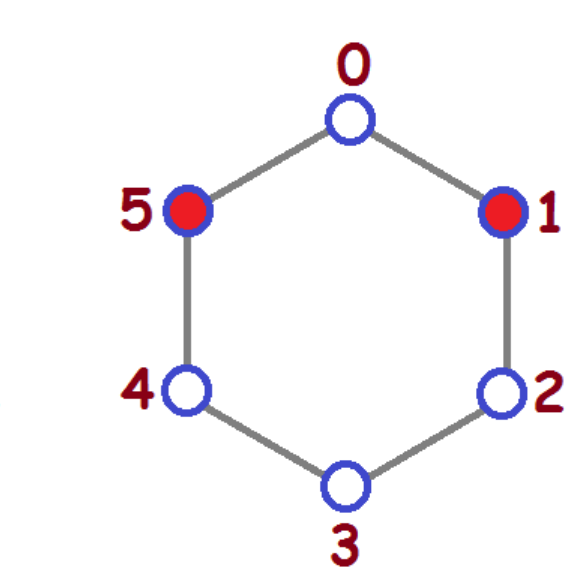
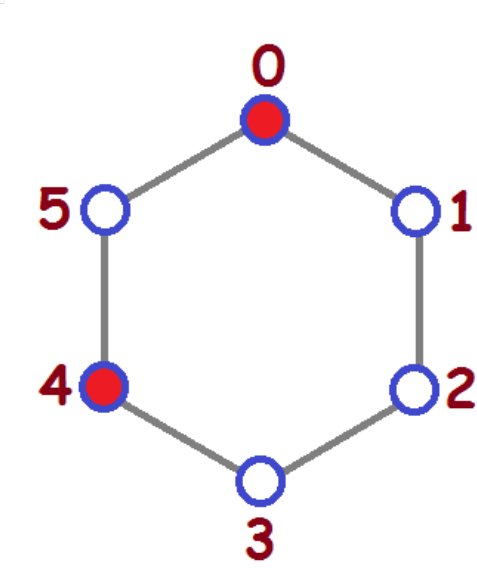
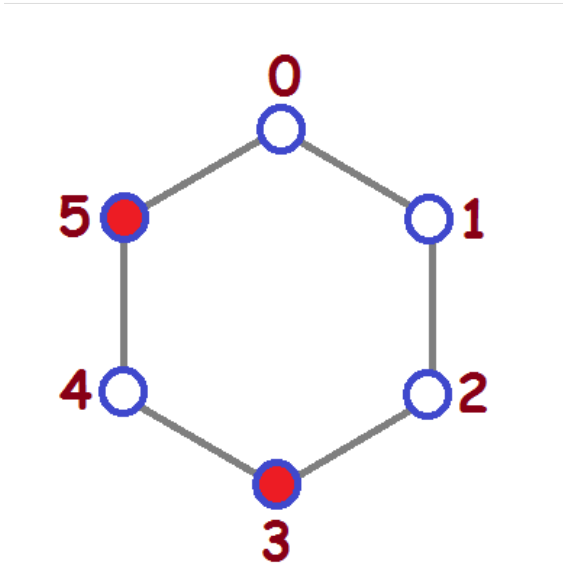
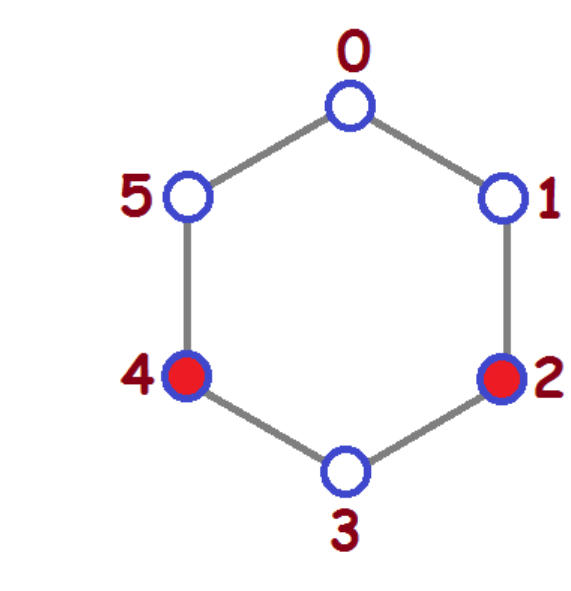
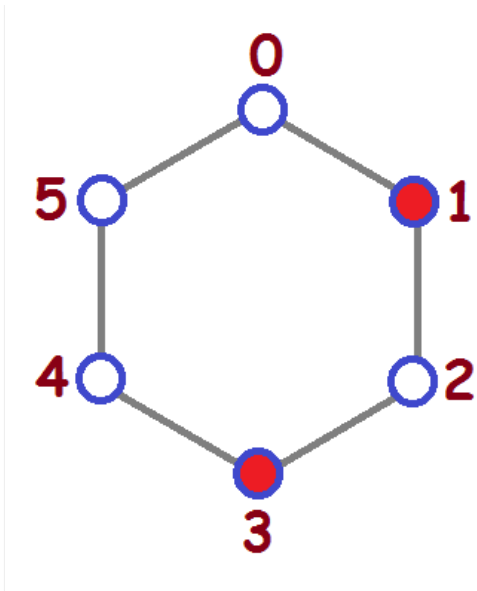
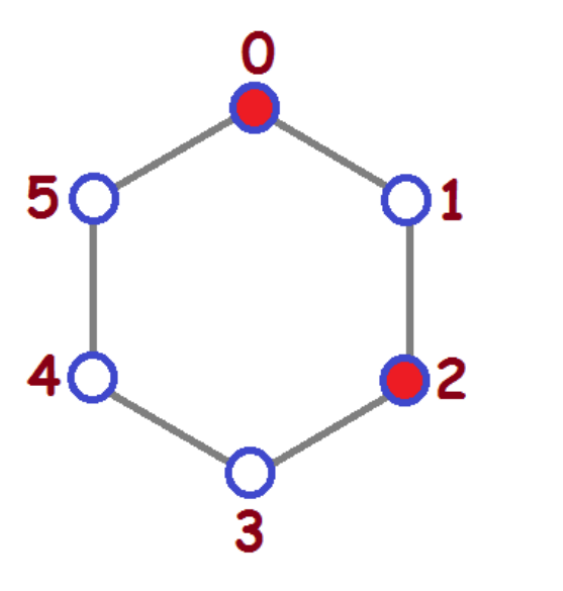
Sorting these lexicographically:
 $\{0, 3\} < \{1, 4\} < \{2, 5\}$

1. Which automorphisms map this independent set to itself?
2. Which other independent sets are equivalent to it?





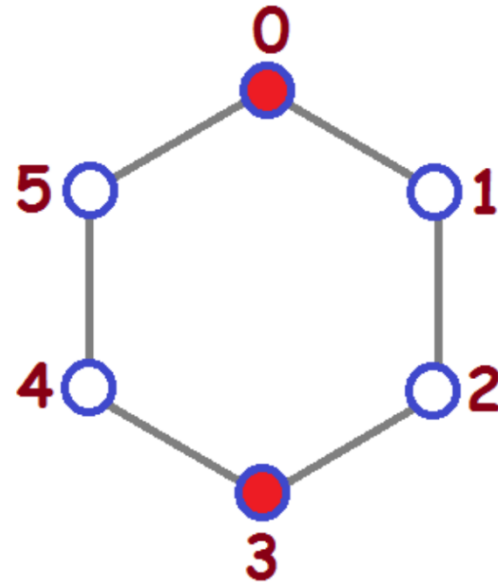
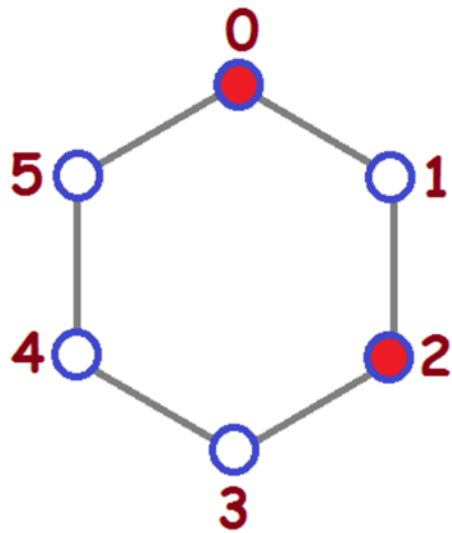
$$2 * 6 = 12$$



Sorting these lexicographically:

$\{0, 2\} < \{0, 4\} < \{1, 3\} < \{1, 5\} < \{2, 4\} < \{3, 5\}$

The independent sets of order two fall into two equivalence classes. We could choose the minimum one in each equivalence class as the representative for its class:



Let S and T be subsets of the vertices.

Examples: independent set, dominating set.

Subsets S and T are **isomorphic** for G if there is some automorphism of G that maps the vertices in S to the vertices in T .

g = group order for G .

d = number of ways to have a distinct subset of vertices so that it is isomorphic to S .

r = number of automorphisms mapping S to S .

Theorem: $r * d = g$