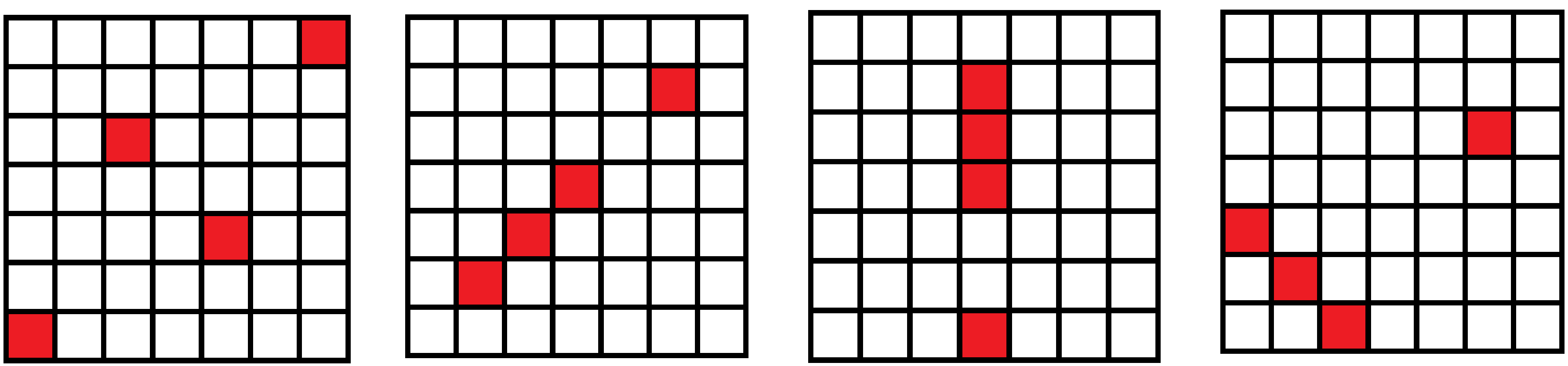
The n-Queen graph: Vertices- squares on an n by n chessboard. Two vertices are adjacent if a queen could move from one square to another. That is, if the squares are on the same row, column, diagonal or back diagonal.

Red vertices: dominating set vertices.

1. How many automorphisms of the graph map each dominating set to itself?
2. How many different dominating sets does each one correspond to?

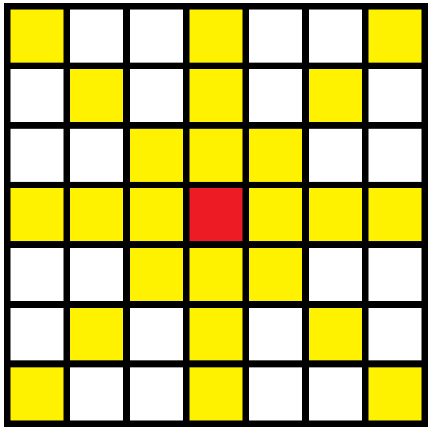


Annoucements

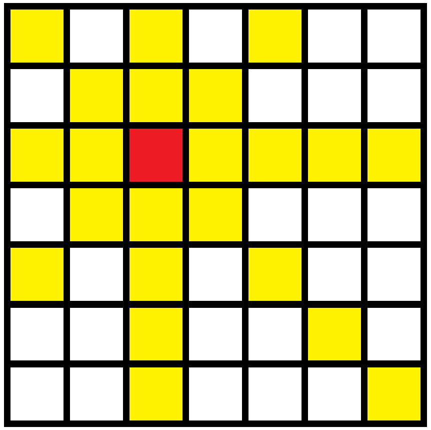
Some graph classes that may have polynomial time algorithms for hard problems:

planar, perfect, k-trees, chordal, permutation, interval, circular arc, intersection, bounded tree width, bounded facewidth, claw-free, fullerenes, benzenoids, ….

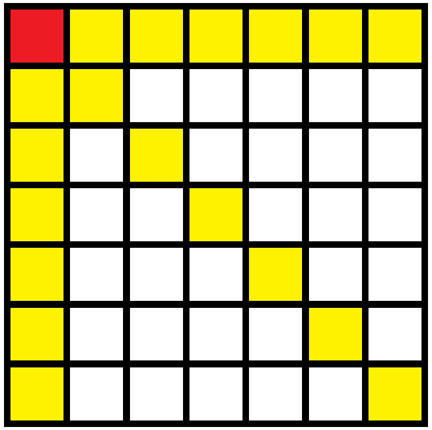
The queen's graph is not a regular graph.



Red cell dominates 25 vertices.



Red cell dominates 23 vertices.



Red cell dominates 19 vertices.

Number of cells each cell dominates:

19	19	19	19	19	19	19
19	21	21	21	21	21	19
19	21	23	23	23	21	19
19	21	23	25	23	21	19
19	21	23	23	23	21	19
19	21	21	21	21	21	19
19	19	19	19	19	19	19

Our naïve approach suggests that two cells can dominate 25 + 25= 50 cells.

But if we sort these numbers in reverse order:
25, 23, 23, …

We see that 2 cells could dominate at most 25 + 23 = 48 vertices/cells.

Choose center cell to be in dominating set:

19	19	19	19	19	19	19
19	21	21	21	21	21	19
19	21	23	23	23	21	19
19	21	23	25	23	21	19
19	21	23	23	23	21	19
19	21	21	21	21	21	19
19	19	19	19	19	19	19

8	11	9	8	9	11	8
11	10	11	8	11	10	11
9	11	10	12	10	11	9
8	8	12		12	8	8
9	11	10	12	10	11	9
11	10	11	8	11	10	11
8	11	9	8	9	11	8

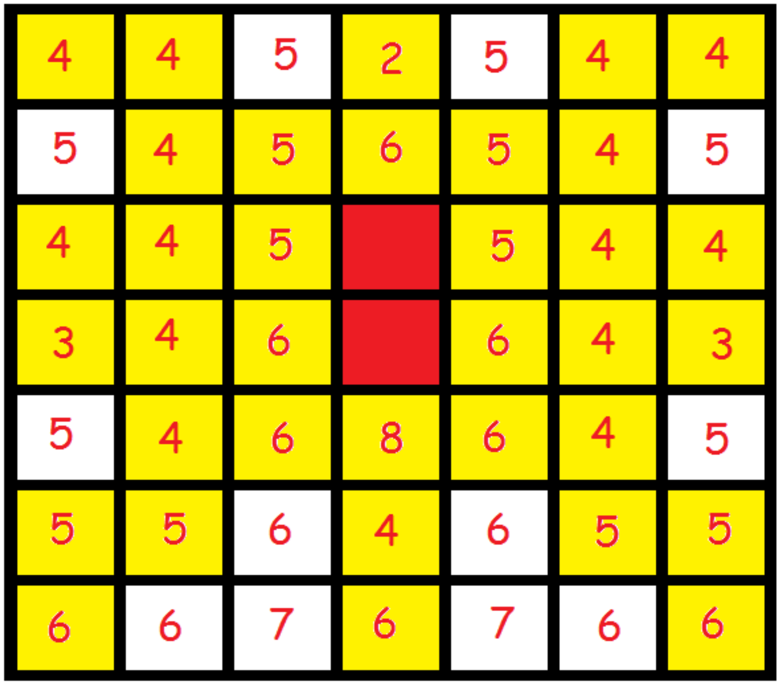
Yellow squares are dominated.

The number of undominated cells each cell would dominate decreases dramatically.

New sorted order: 12, 12, 12, 12, 10, 10 …

24 cells are not dominated: need at least 2 more ($12 + 12 \geq 24$).

Greedy approach: choose a cell dominating a maximum number of undominated vertices:



New sorted order: 8, 7, 7, 6, 6, ...

12 cells are not dominated:

need at least 2 more (8+7 ≥ 12).

If we backtrack and color center cell blue:

19	19	19	19	19	19	19
19	21	21	21	21	21	19
19	21	23	23	23	21	19
19	21	23	25	23	21	19
19	21	23	23	23	21	19
19	21	21	21	21	21	19
19	19	19	19	19	19	19

New sorted order: 23, 23, 23, ...

49 cells are not dominated:

need at least 3 more (23 + 23 + 23 ≥ 49).

Our simple formula for an upper bound:
n_extra ≥ num_not_dominated / (Δ + 1)
is very easy to compute: $O(1)$ time.

It takes more time to maintain a better bound.
One tactic: maintain for k= 0, 1, 2, … , n the number of white vertices dominating k undominated vertices (num[0…n]).

x= maximum number of undominated vertices some white vertex dominates.

Go from x downwards adding on for one vertex at a time the number it might newly dominate until the total is at least the number of undominated vertices.

```
n_extra=0; sum=0;
for (k= x; k ≥ 1; k--)
    for (j=0; j < num[k]; j++)
    {
        n_extra++;
        sum+= k;
        if (sum ≥ num_not_dominated)
            goto check_bound;
    }
}
// white vertices can't dominate all the rest.
return(0);

check_bound:;
if (size + n_extra >= min_size) return(0);
```