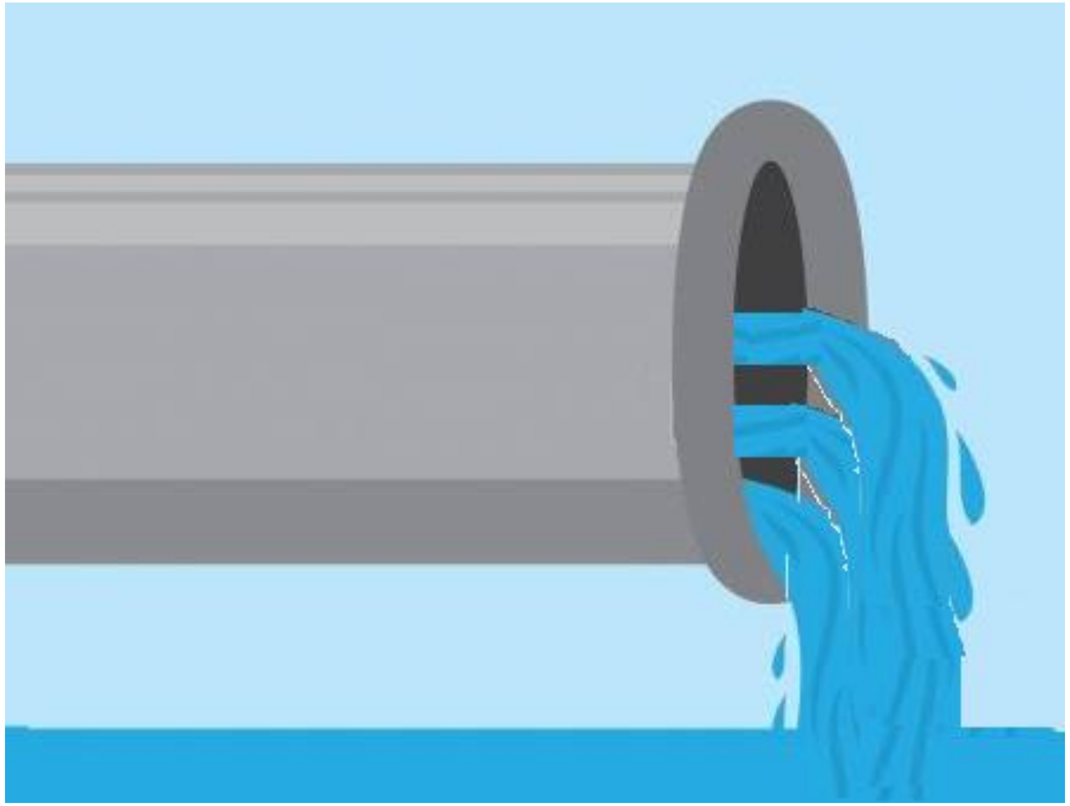
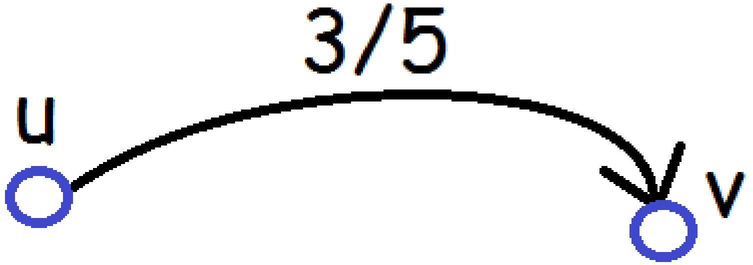
Some pipes are designed so that flow can only move in one direction.

http://www.flowcontrolnetwork.com/flowmeter-piping-requirements/

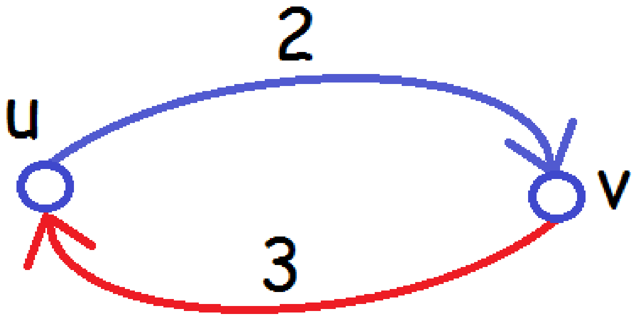
Suppose we have a pipe with capacity of 5 units and it currently has 3 units of flow:



In the graph, flow/capacity:

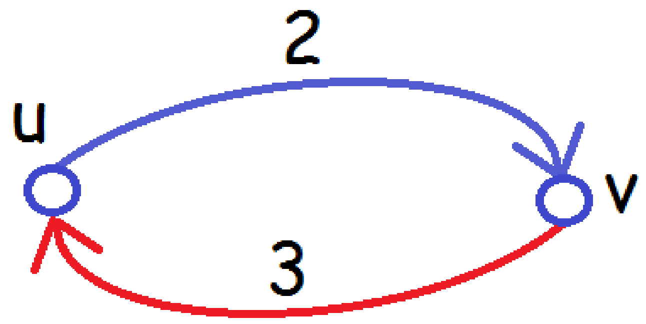


In the auxillary graph:

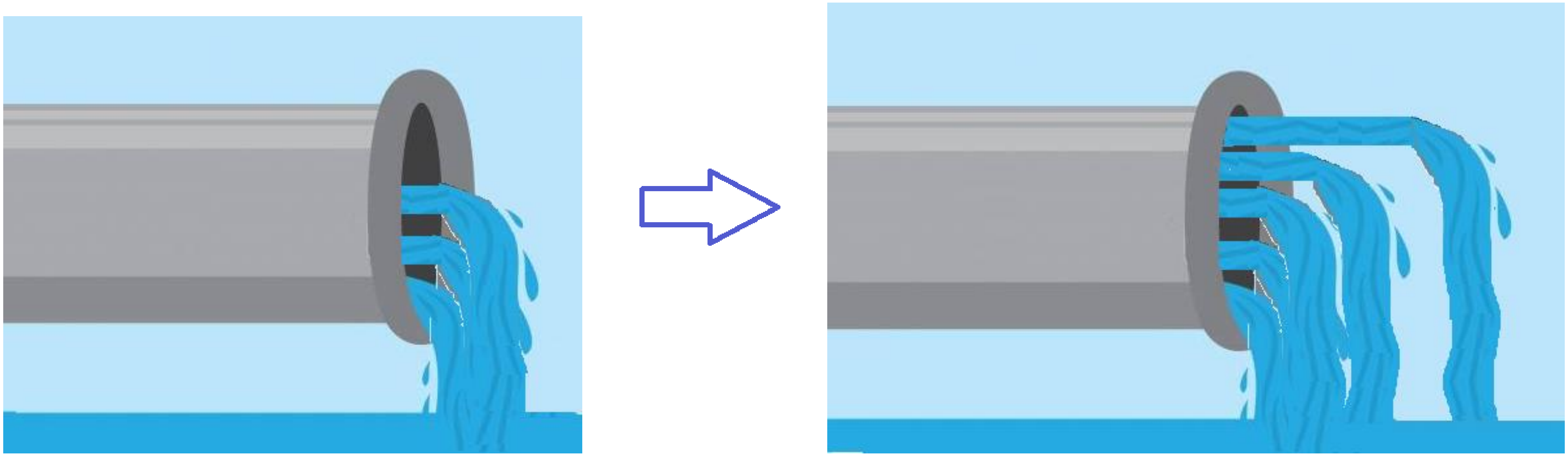


The blue arc is indicating that we can send up to 2 more units from u to v.
The red arc is there because we can send up to 3 less units from u to v.

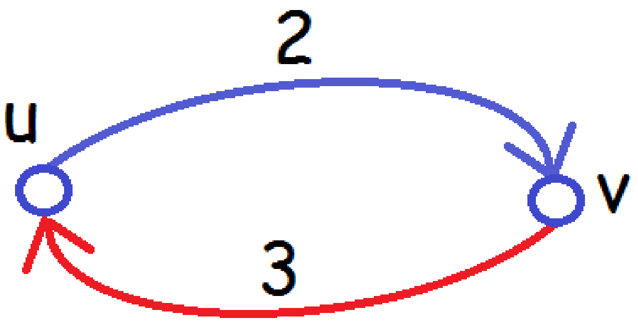
If we send 2 units of flow on the blue arc:



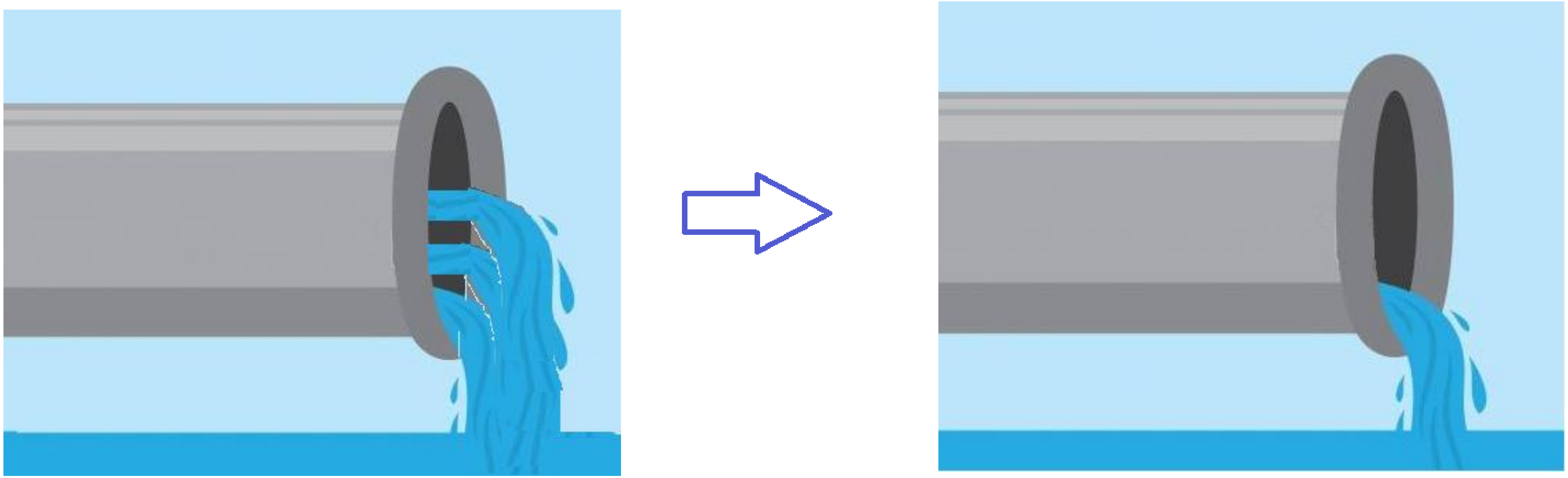
New flow from u to v is 5 units.

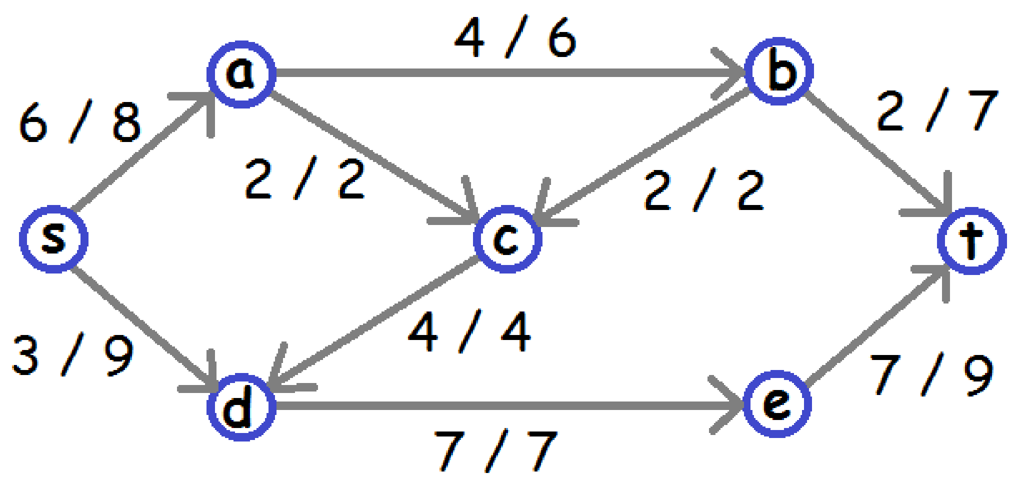


If we send 2 units of flow on the red arc:

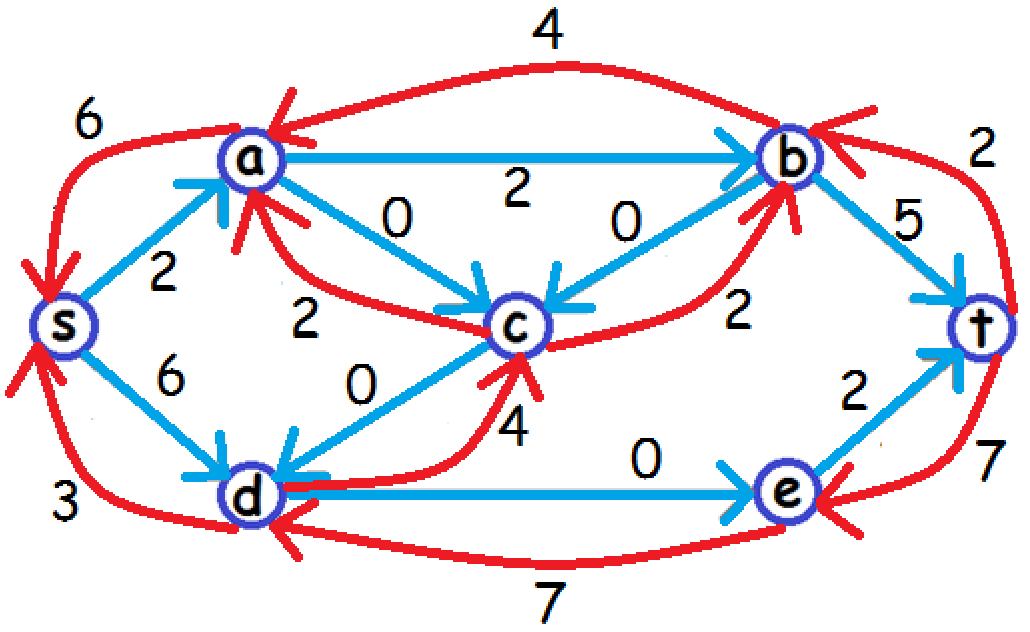


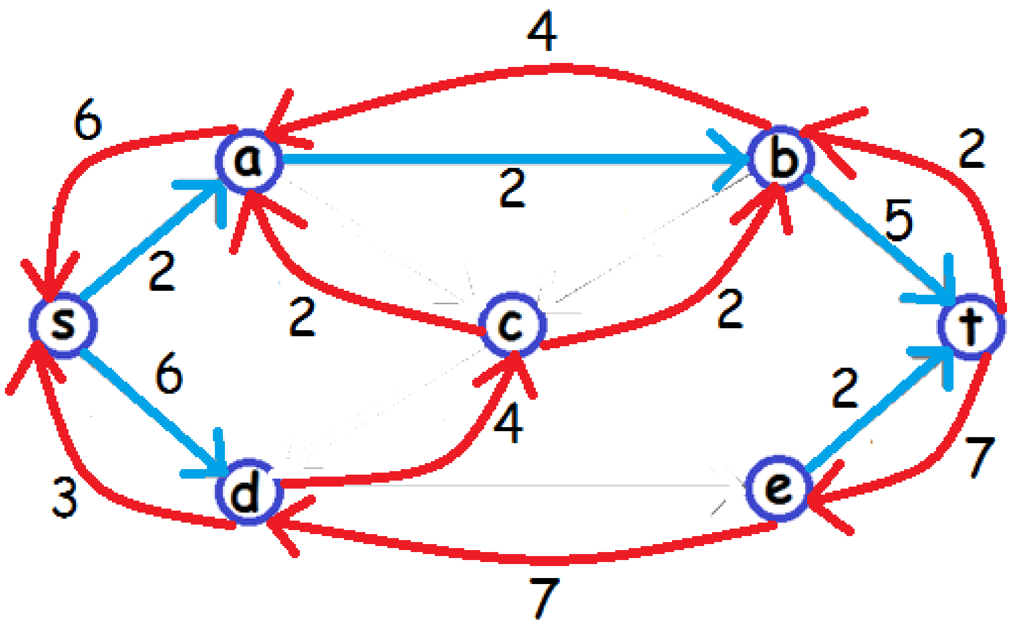
New flow from u to v is 1 unit.

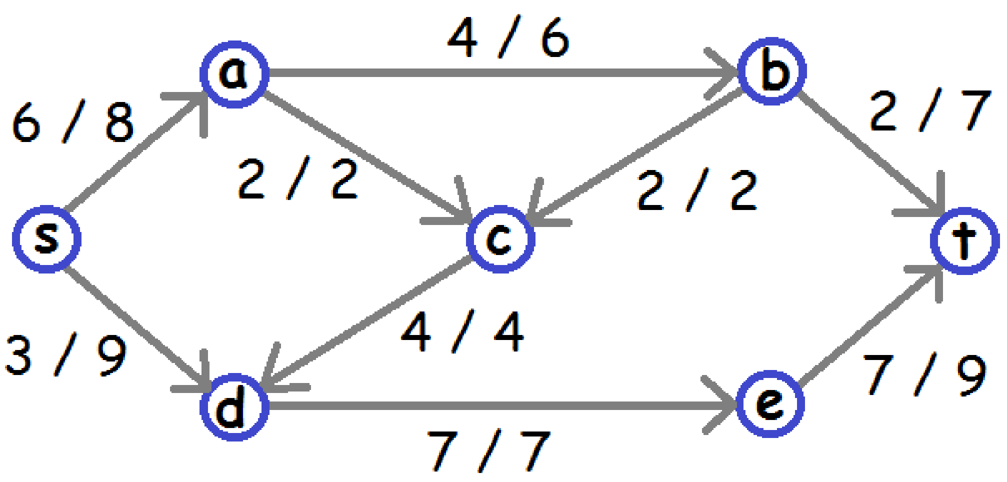


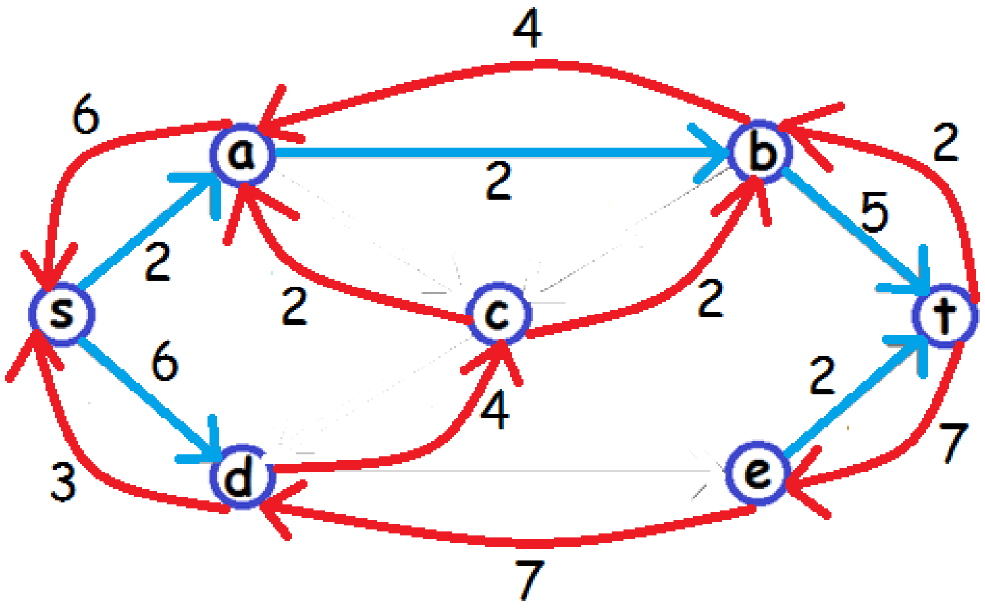


We don’t usually include the arcs with 0 capacity since they cannot be used to increase the flow:

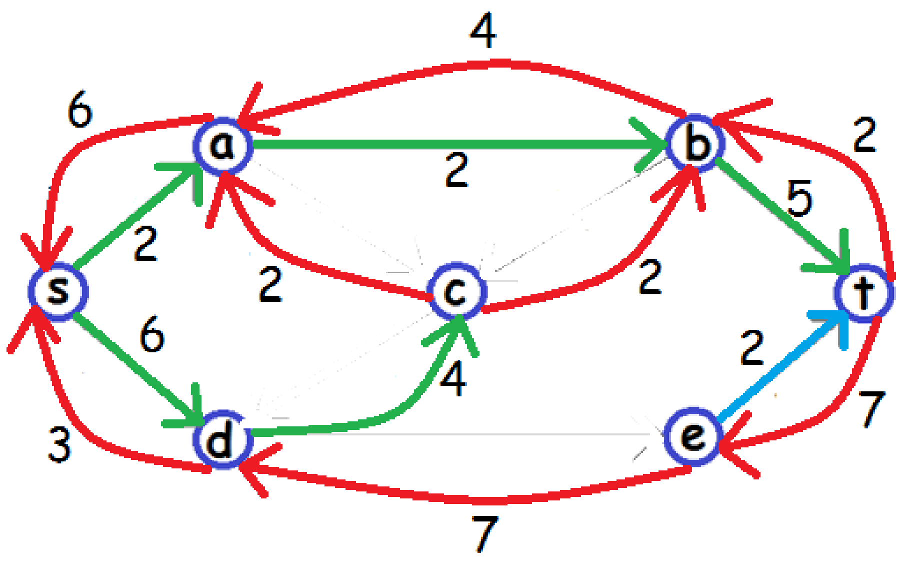




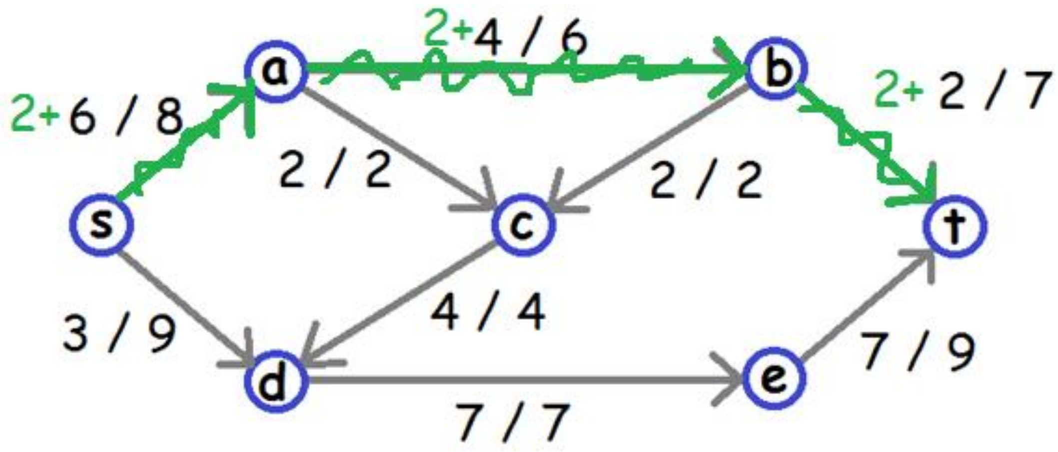


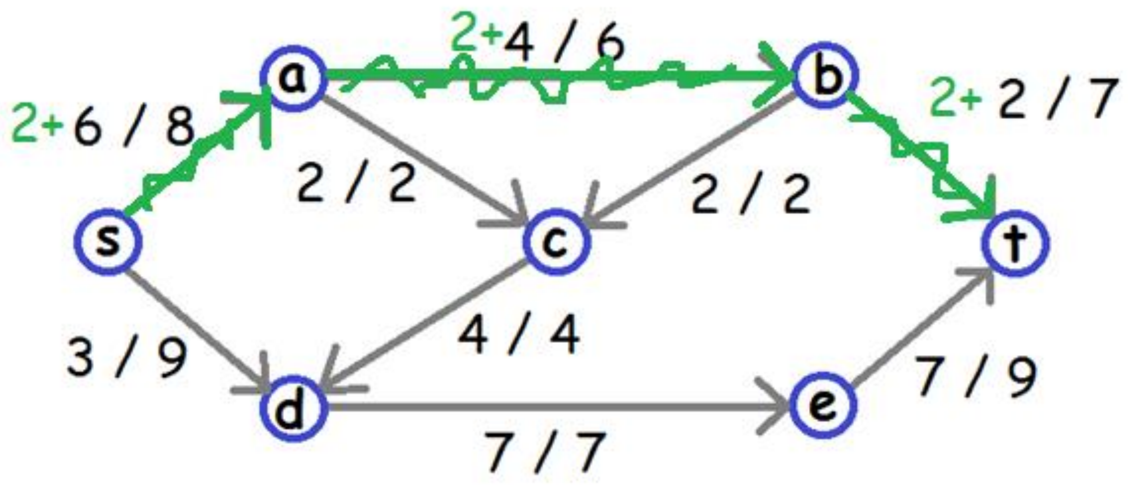


1. Find an s,t path by doing BFS:

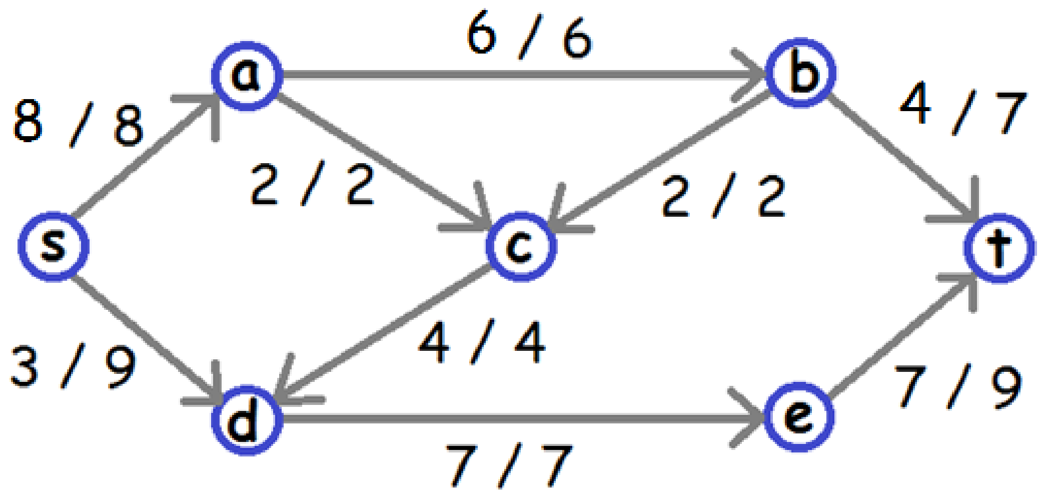


2. The min capacity arc on the s,t-path has capacity two. Send two more units of flow from s to t:

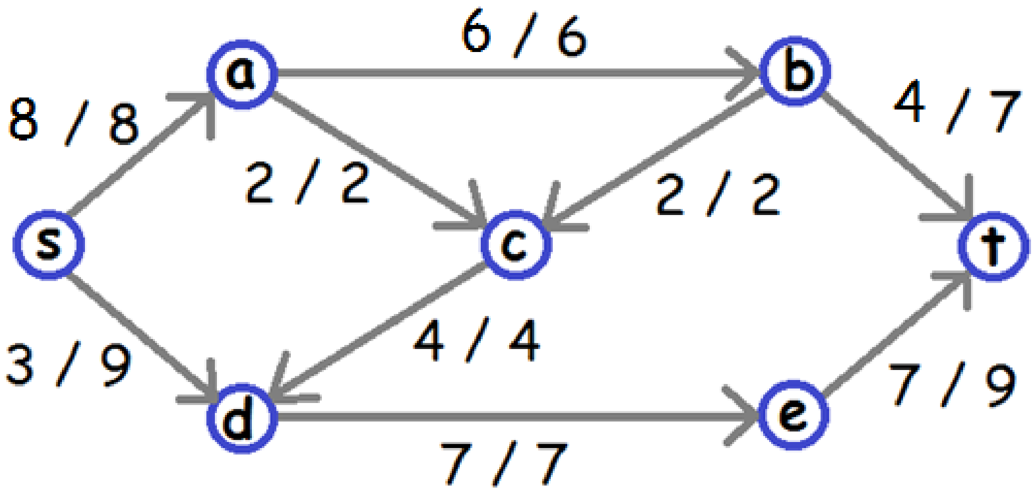




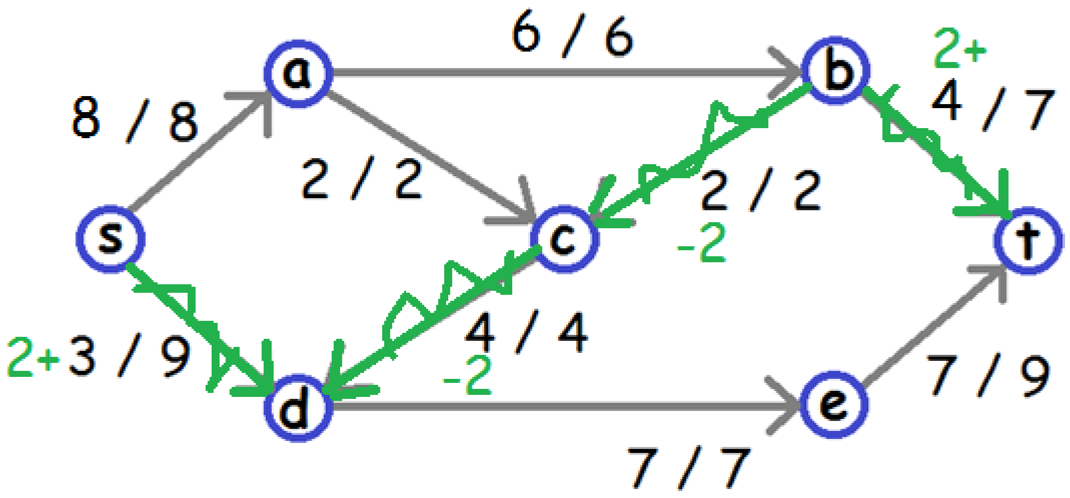
Flows on other arcs stay the same.
The new flow is:

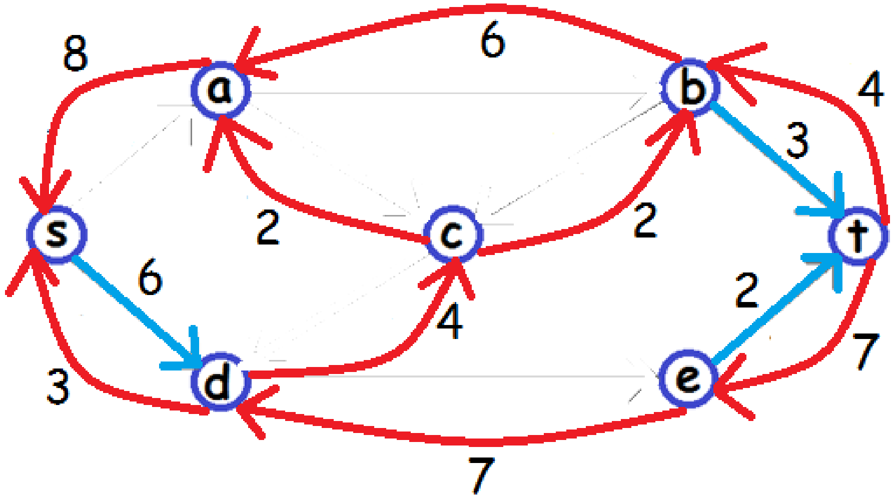


After you update the flow it is wise to check that for each vertex not equal to the source or the sink, that you have conservation of flow. If you do not, then you have made a mistake somewhere.

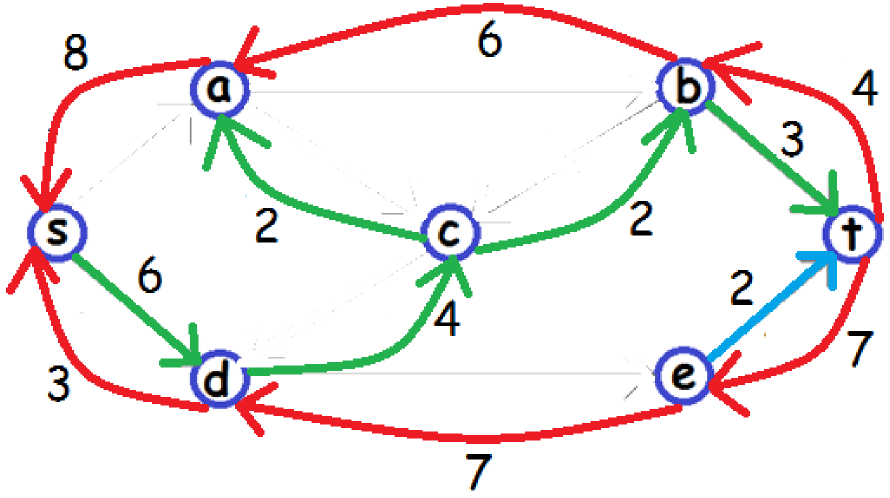


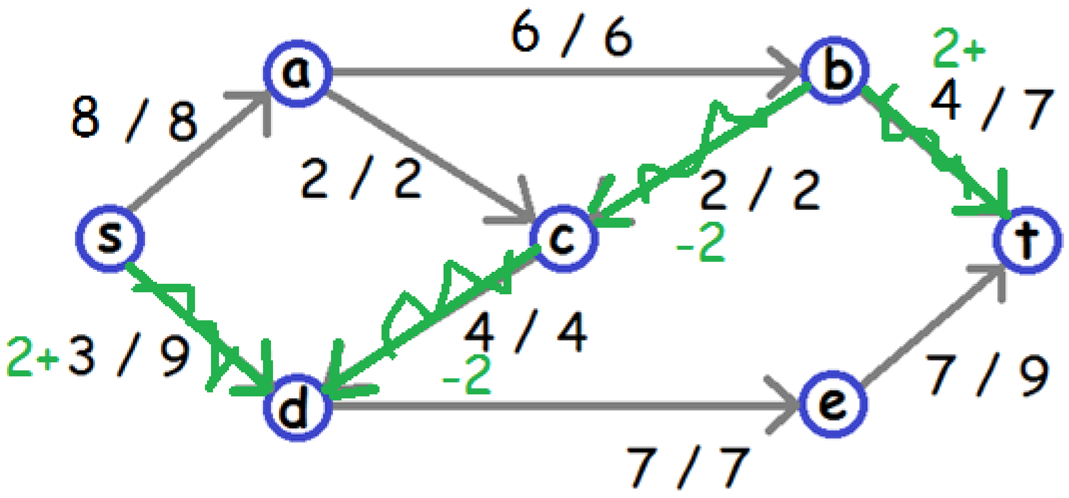
2. Send two more units of flow from s to t. Flow is added if blue in aux. graph and subtracted if blue.



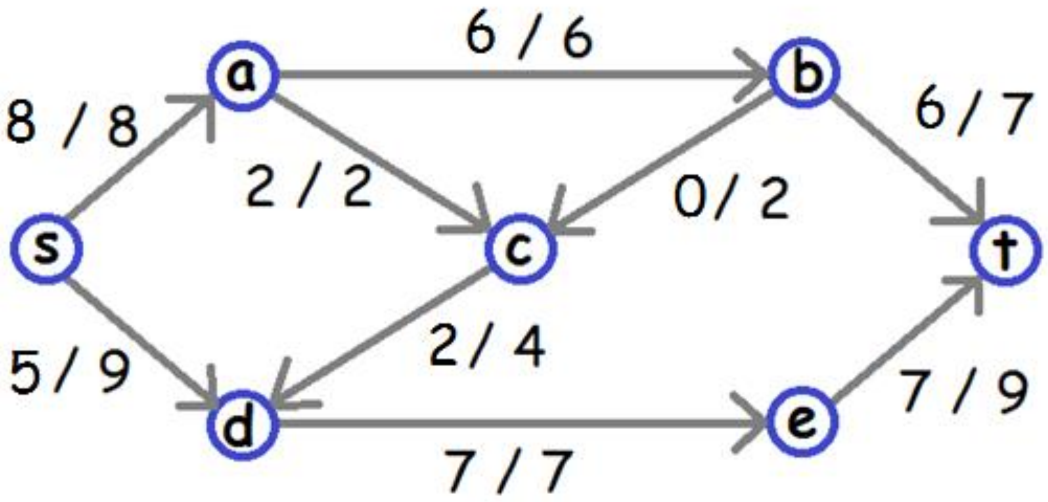


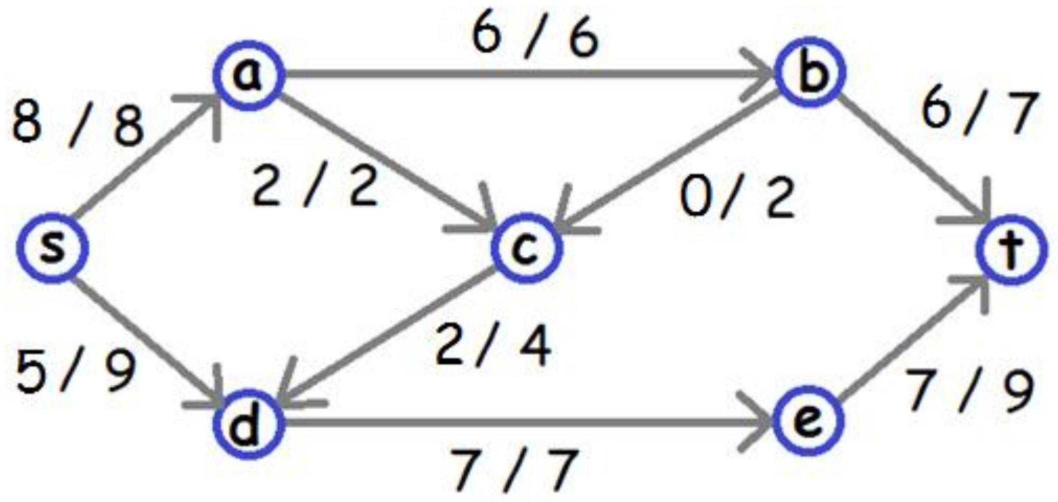
1. Find an s,t path by doing BFS:

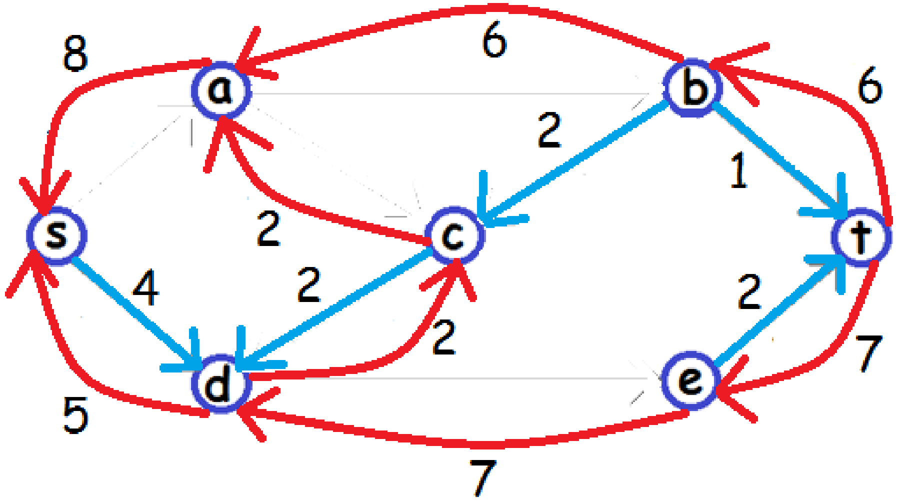




Flows on other arcs stay the same.
The new flow is:

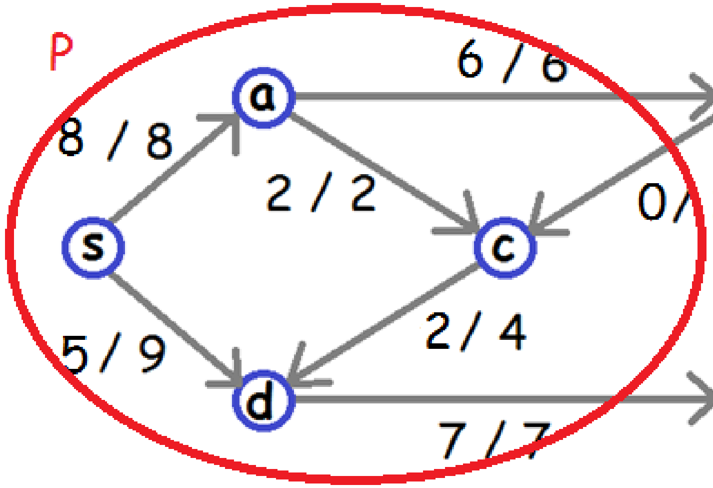


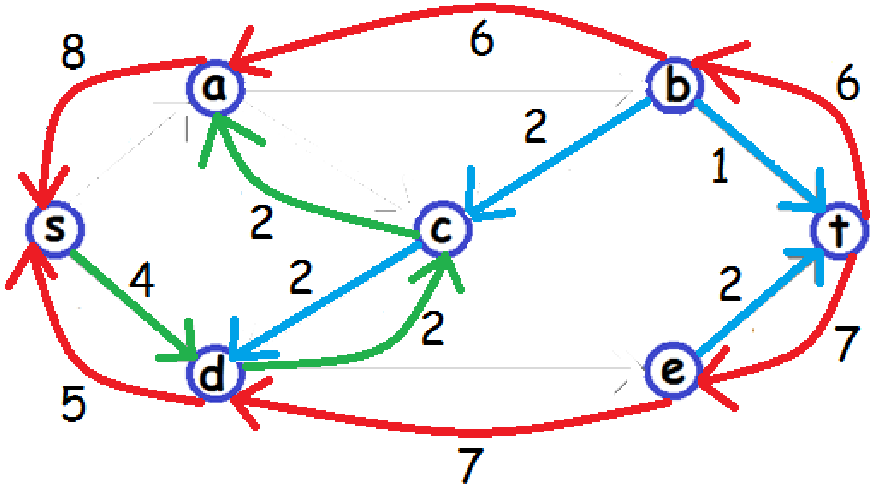




2. Set P to be the vertices reachable from s from the BFS.

1. Find an s,t path by doing BFS, Vertices reachable from s: s, a, c, d:





The proof of the famous maximum flow, minimum cut theorem says that the flow in the network should equal the capacity of this cut.

The flow is 13= 8+5 + 6+7. The cut arcs are (a, b) and (d,e) and the cut has capacity 6+7 = 13. Arc (b,c) is NOT an edge of this cut since it goes from $\overline{P}$ to P and not from P to $\overline{P}$. If you include it then the capacities of the edges sum to 15 which is greater than the flow of 13.