

CSC 425/520 Fall 2016: Assignment #2: Written Questions
Due at beginning of class, Friday Oct. 7, 2016

1. Put your name on your assignment. Questions should be **in order**.
2. The written questions should be submitted on paper at the beginning of class on the due date. Submissions can be handed in up to 4 days late with a 10% penalty for each day past the deadline. If you want to submit an assignment on a day that we do not have class, please write the day and time submitted on your first page and slip it under my office door (ECS 552).
3. Show your work unless otherwise stated.
4. Draw BIG boxes for your marks on the top of the first page of your submission. Place a 0 in the corresponding box for any questions you omit. For this assignment:

| | | | | | |
|-----------|---|---|---|---|---|
| Question: | 1 | 2 | 3 | 4 | 5 |
| Marks | 0 | 0 | 0 | 0 | 0 |

1. For the following questions, consider the set of Stable Matching Problems that have four undergraduates labelled 1, 2, 3, and 4, and four graduate students labelled A, B, C, D. The aim is to find a perfect matching representing project partners for a class project. The undergraduates will make the proposals to the graduate students. At each stage, the undergraduate chosen to propose is the unmatched student that has the smallest number. For the following questions, if your answer is *yes*, give the preference tables for both the undergraduates and graduates and show the steps of the algorithm on an example that shows the situation can occur. If your answer is *no*, give a proof to justify that.
 - (a) [5] Is it possible to have a problem such that in the solution, nobody has their first choice of a project partner?
 - (b) [5] Is it possible to have a situation where there is some undergraduate student u ranking graduate student g first on their preference list and also g ranks u first on their preference list, but the stable matching selected does not match u and g together?
 - (c) [5] Suppose that graduate student A has preference list 1, 2, 3, 4. Can graduate student A ever end up with a higher ranked partner by lying about the preference list and using 1, 2, 4, 3 instead?

2. [5] Problem 3, pages 22-23 of the text.
3. [10] Problem 4, p. 23 of the text.
4. Consider Problem 6 on p. 191 of the text. Suppose the students have these swimming, biking and running times (in minutes):

| Student | Swim | Bike | Run |
|---------|------|------|-----|
| Alice | 5 | 10 | 10 |
| Bob | 10 | 15 | 10 |
| Carl | 15 | 20 | 10 |
| Dawn | 5 | 20 | 20 |

- (a) [2] If the race starts at 9am, what time does each student finish the race if the order used is Alice, Bob, Carl then Dawn. At what time is everybody finished?
 - (b) [2] If the race starts at 9am, what time does each student finish the race if the order used is Dawn, Carl, Bob then Alice. At what time is everybody finished?
 - (c) [6] Given a schedule S , denote the swim, bike and run times for the k th student by s_k , b_k and r_k respectively. Prove that if two students appear consecutively in a schedule S in positions k and $k + 1$, and $b_k + r_k < b_{k+1} + r_{k+1}$ then the schedule T that is the same as S except that the students in positions k and $k + 1$ are swapped takes the same time or less than schedule S .
5. [10] Problem 6. p. 191 of the text.