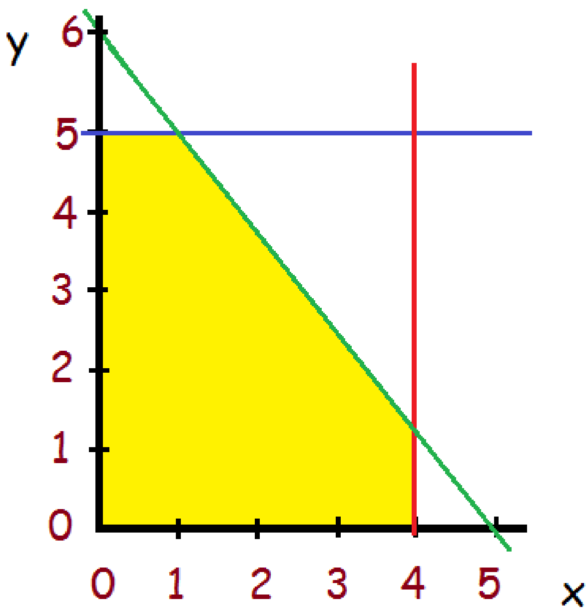
CSC 445/545: Linear Programming

Instructor: Wendy Myrvold

E-mail: wendym@csc.UVic.ca

Home page:

http://webhome.cs.uvic.ca/~wendym/445.html



Maximize $x + y$

subject to

$$x \leq 4$$

$$y \leq 5$$

$$6x + 5y \leq 30$$

$$x \geq 0,\ y \geq 0$$

Announcements

Powerpoint slides will be posted on the class web pages:

http://webhome.cs.uvic.ca/~wendym/445.html

Or on connex if they contain material that cannot be placed on the web.

Welcome to CSC 445/545!

Office hours:

Let me know either at the end of class or by e-mail what time you would like to come by so that I don't have to be there when nobody wants to see me.

TWF 11:30am - 12:20pm.

Tuesdays 1:30pm: when no dept. meeting.

WF: 1:30pm (until all questions are answered).

Or by appointment.

Outline

- Who is the instructor?
- My research interests.
- Logistics for CSC 445/545- the critical points are included on the course outline and class web pages.
- Don't worry about taking notes today

About me:

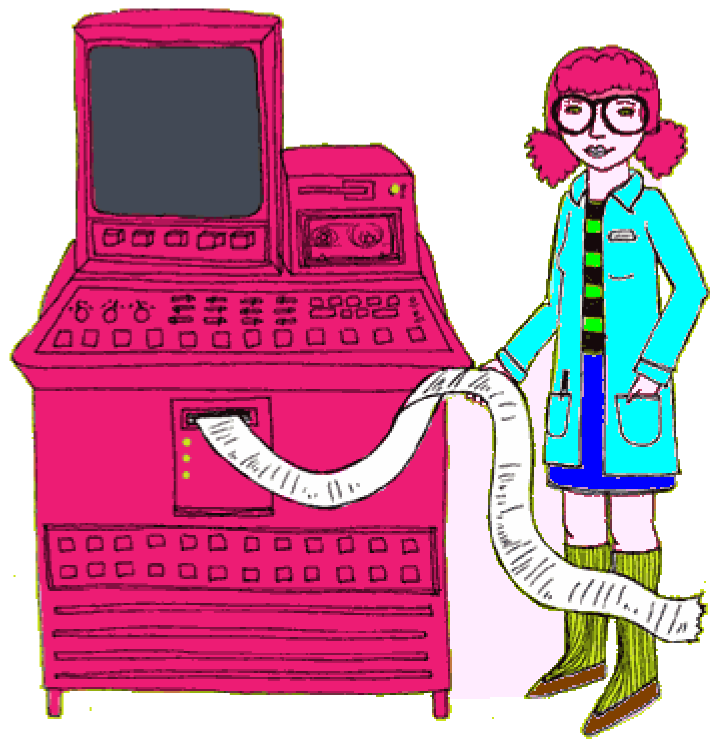
B.Sc. : Computer Science, McGill University, 1983

M.Math. : Combinatorics and Optimization, University of Waterloo, 1984

Ph.D. in Computer Science: Waterloo, 1988

University of Victoria: started in 1988, currently a full professor

by Mark A. Hicks, illustrator.



From: Gurl Guide to programming.

Bring your parents to work day at Google.

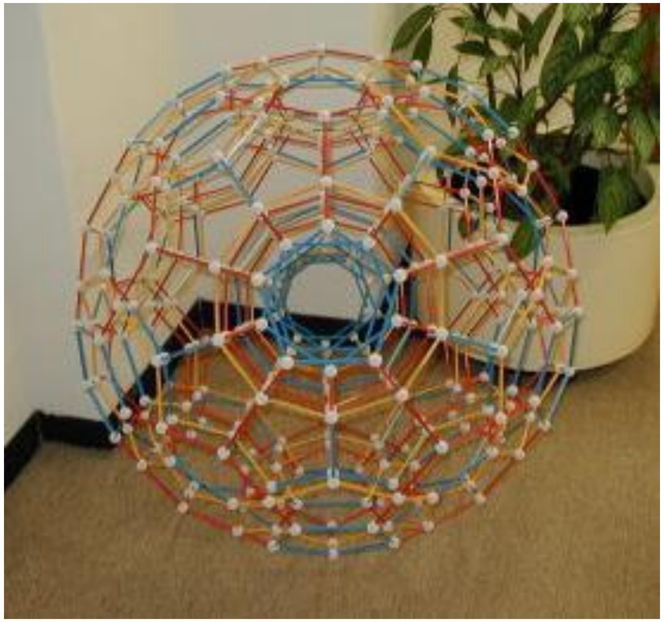

CCM

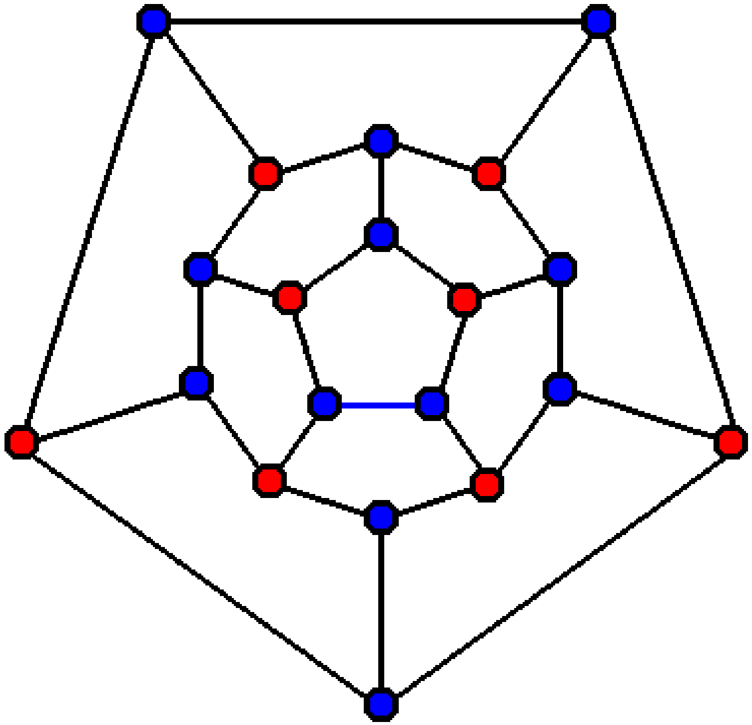

WHAT

My Research: Large Combinatorial Searches

Independent Set:

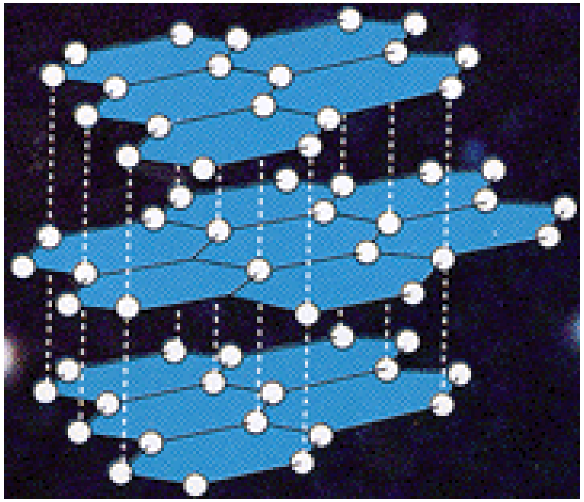
Set of vertices which are pairwise non-adjacent



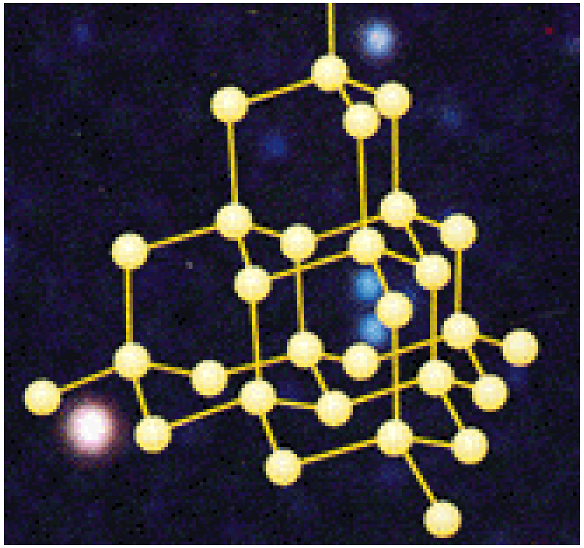


Fullerenes:

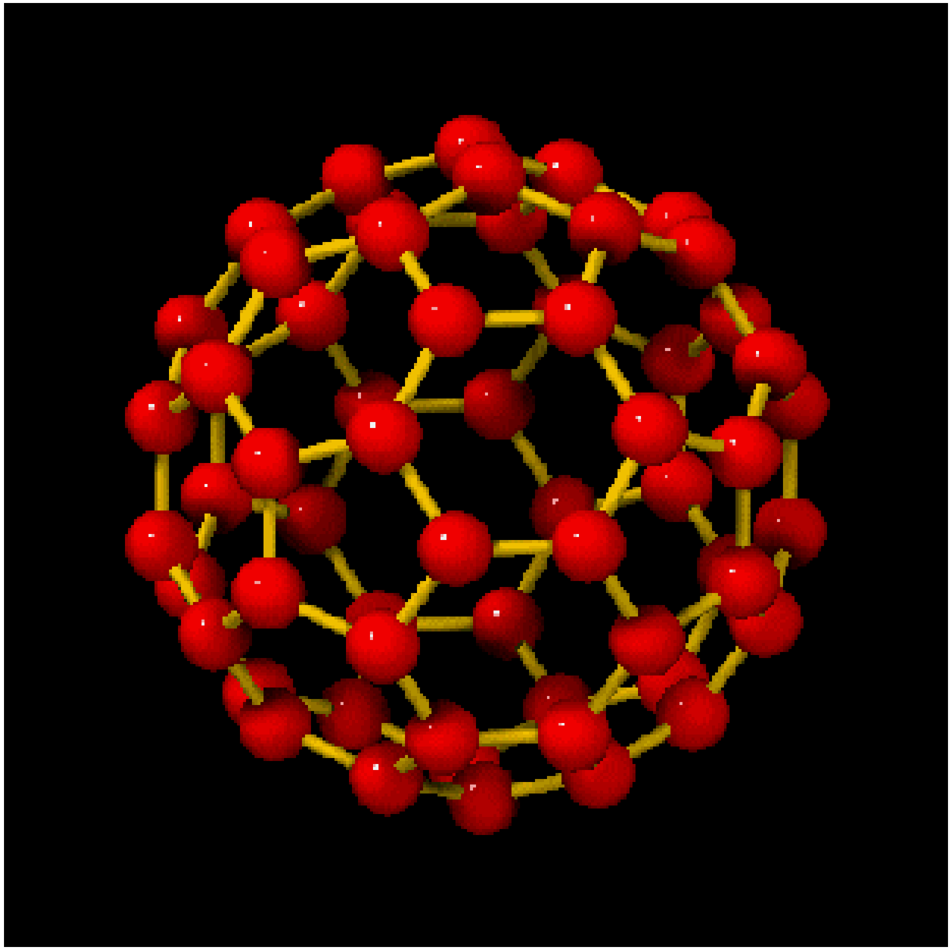
Working with Patrick Fowler (chemist)



Graphite

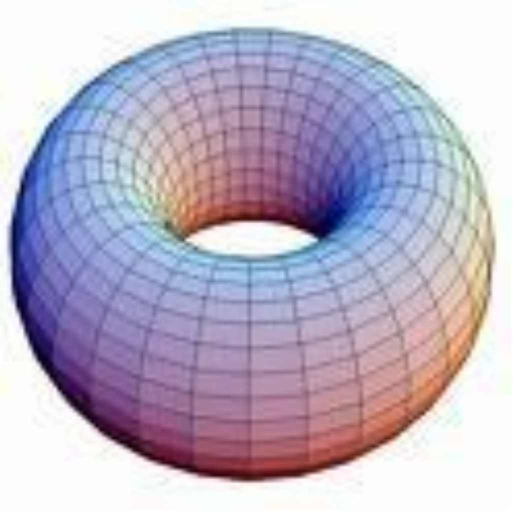


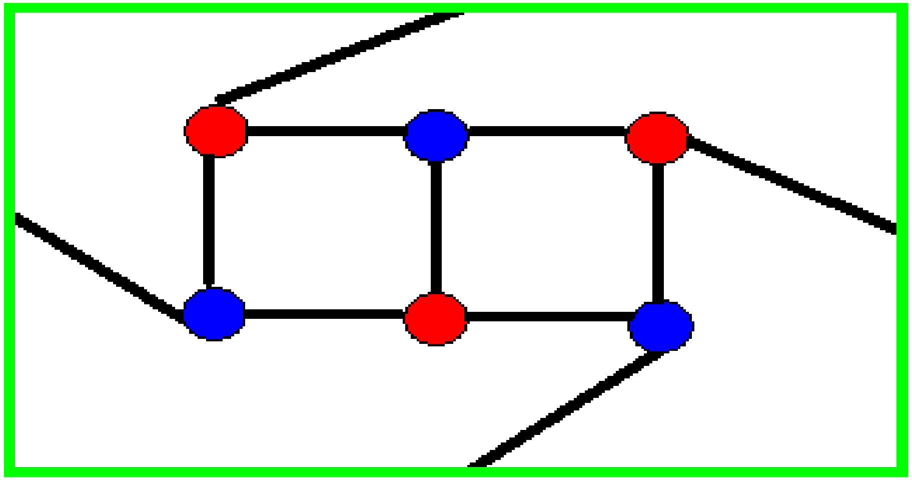
Diamond

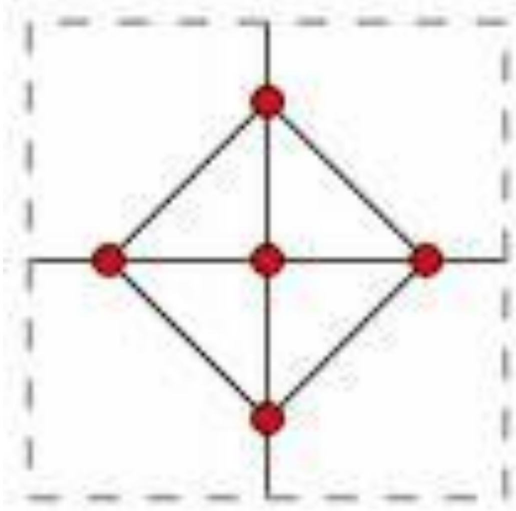


Topological Graph Theory:

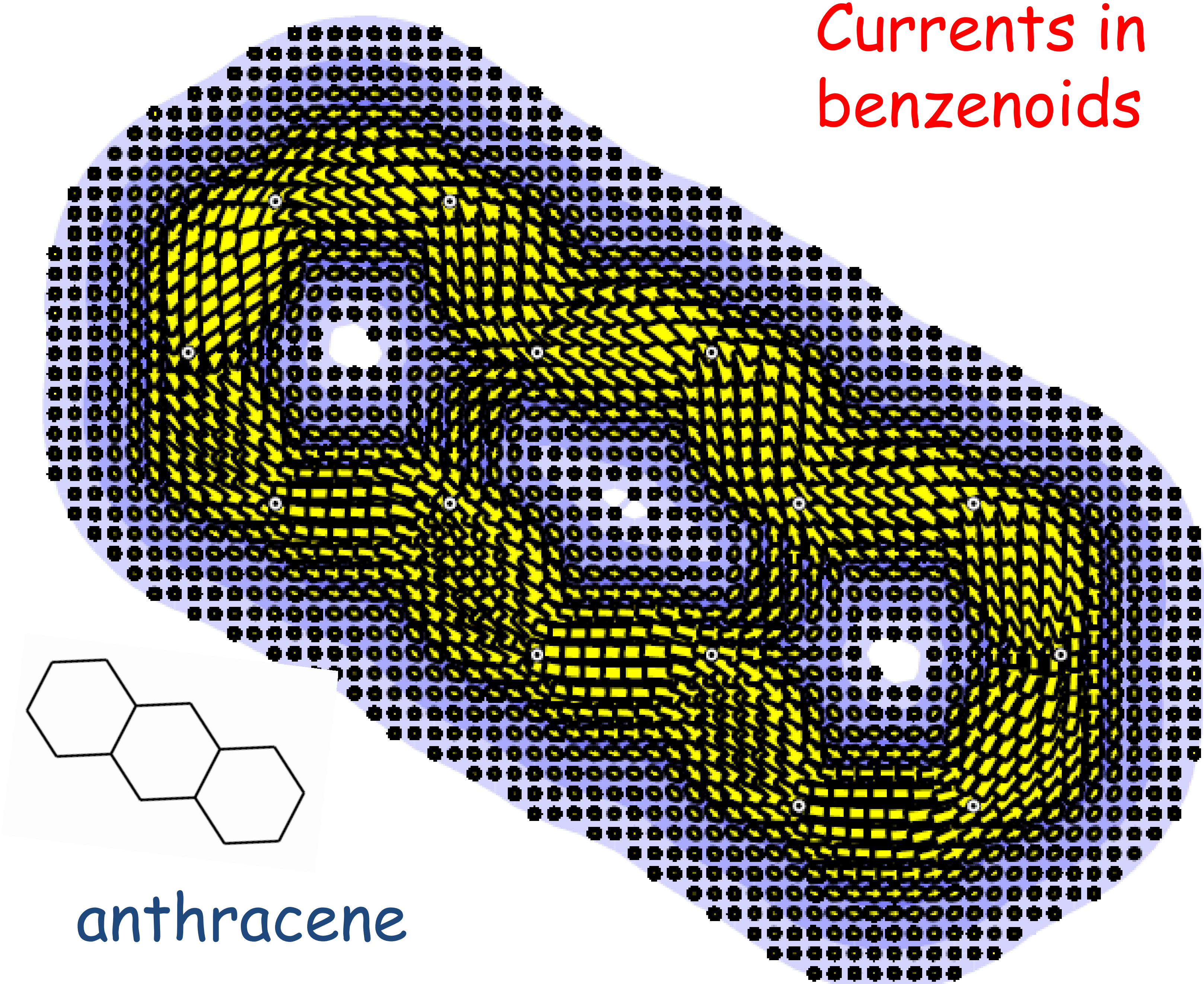
Algorithms and Obstructions







Currents in benzenoids

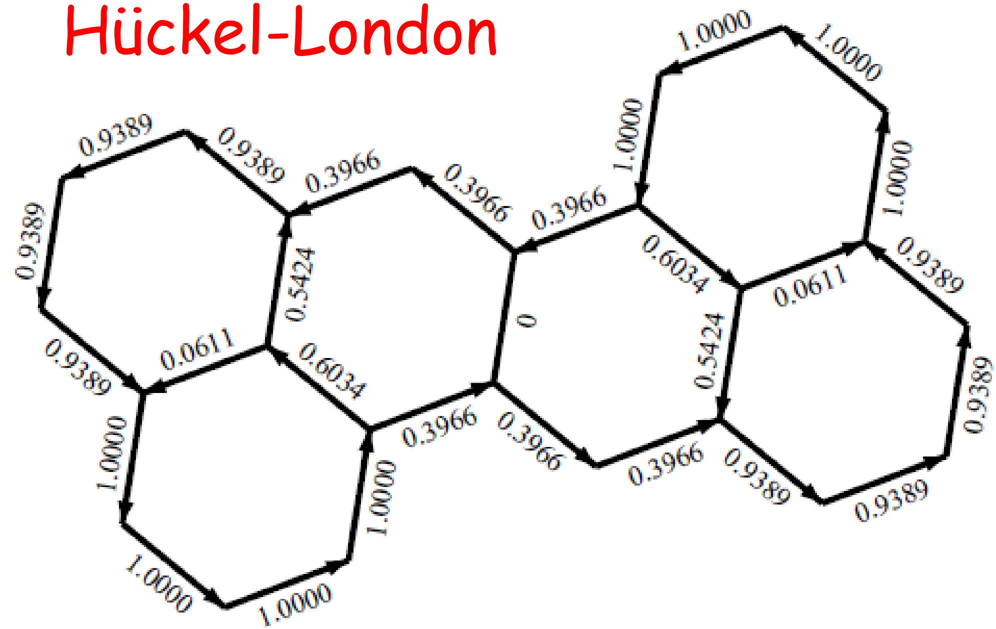


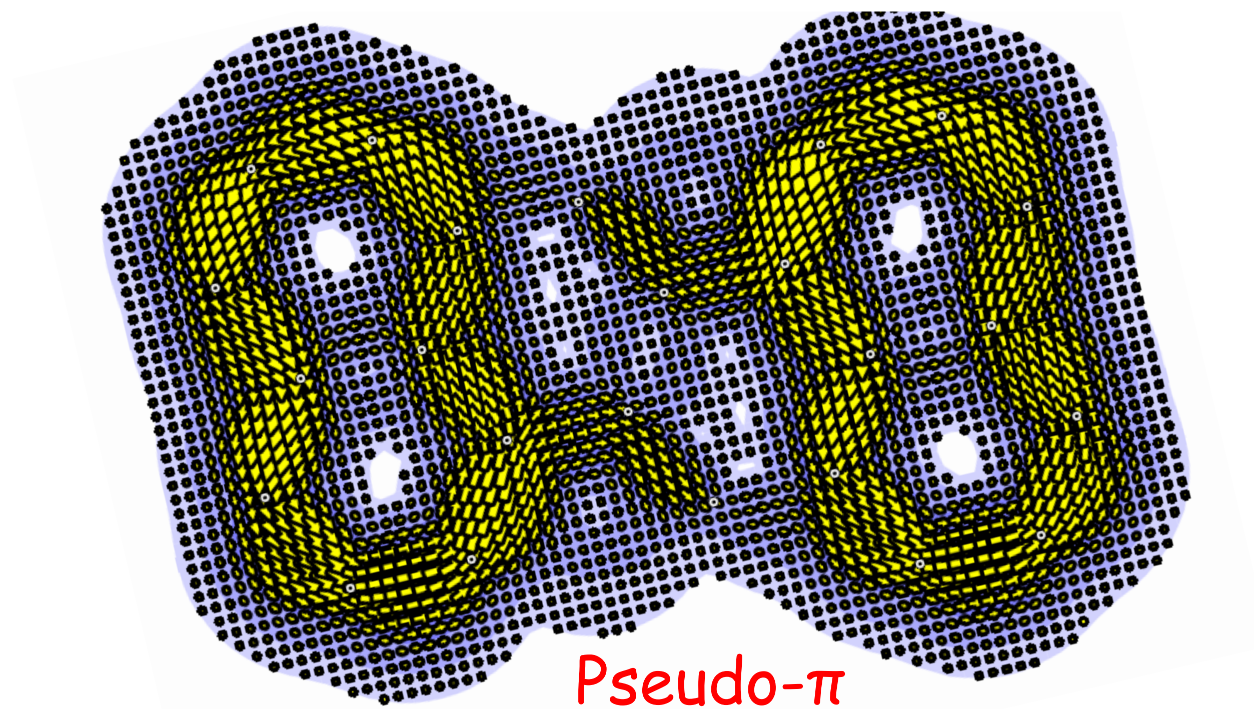
anthracene

Pseudo-π :
"Eye-brows" lacking conservation of flow.

One bad example of this is Zethrene:

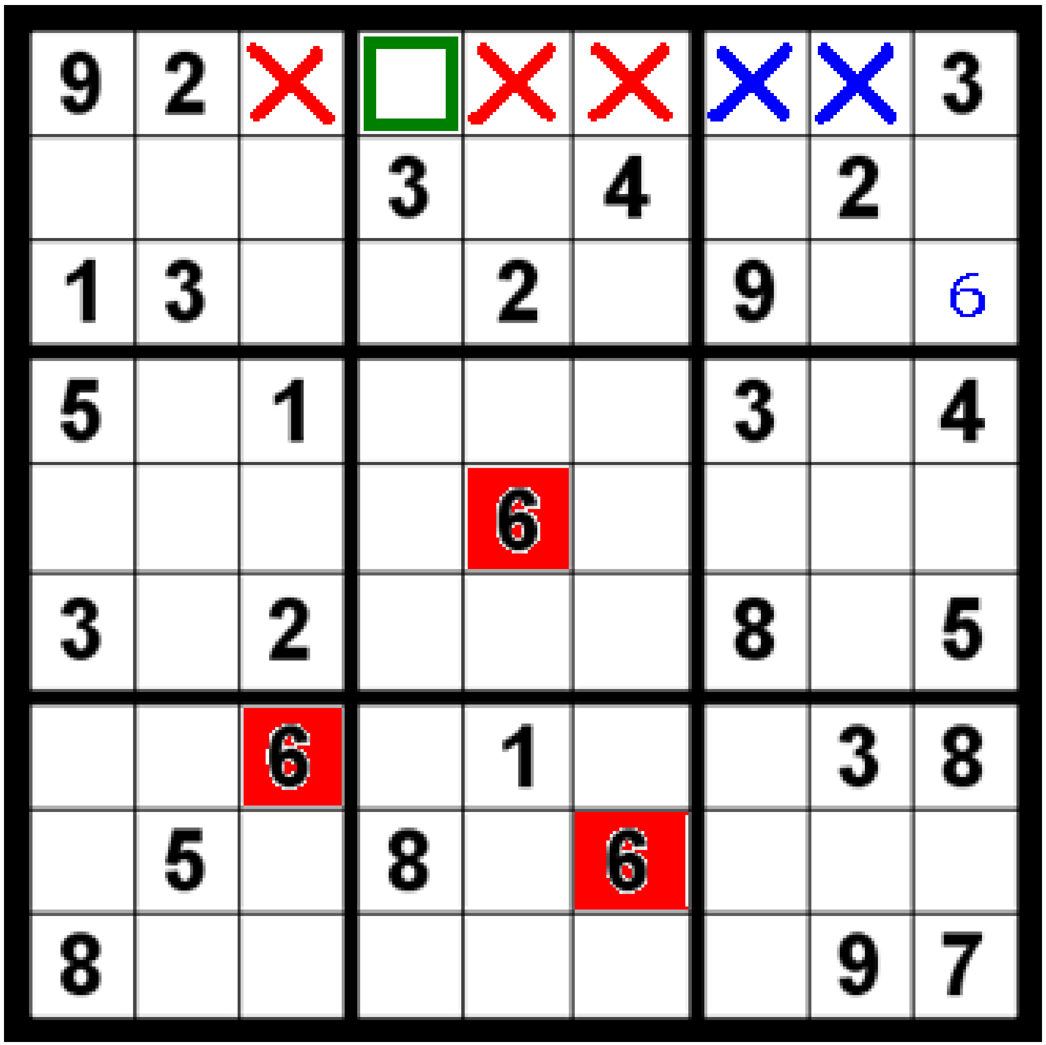
Hückel-London





Pseudo-π

Latin Squares



Please come talk to me if you are looking for Honours project research topics or for an NSERC undergraduate research project.

COMBINATORIAL ALGORITHMS GROUP
University of Victoria

http://www.cs.uvic.ca/~wendym/cag

Our research interests include:

Graph Theory and Graph Algorithms
Combinatorics
Combinatorial Algorithms
Computational Geometry
Randomized Algorithms
Computational Complexity
Network Reliability
Topological Graph Theory
Computational Biology
Cryptography
Design Theory

Join our listserv to get information about conferences and research talks.

Undergrads are welcome to all events.

Component	CSC 445	CSC 545
Assignments (4)	20	16
Programming Project (Sept. 26, Oct. 24).	20	16
Midterm: Oct. 10	20	18
Participation (Nov. 18-)	5	0
Final exam:	35	30
Survey paper (Oct. 17) [optimization technique]	0	10
Slides (Nov. 7) Presentation (Nov. 18-)	0	10

New Grading System (effective Summer 2014)

- Instructor will submit grades in percentages.
- The University will use the following Senate approved standardized grading scale to assign letter grades.
- Both the percentage mark and the letter grade will be recorded on the academic record and transcripts.

F	D	C	C+	B-	B	B+	A-	A	A+
0-49	50-59	60-64	65-69	70-72	73-76	77-79	80-84	85-89	90-100

For graduate students:

B- or lower is unacceptable work revealing some deficiencies in knowledge, understanding or techniques.

Late Assignments And Projects:

Assignments and projects will be due on Fridays at the beginning of class. They can be handed in late at 12:30pm on the following Tuesday with a 10% late penalty.

CSC 545 only:

1. Choose an optimization technique from a list provided (by **Wed. Sept. 24)**. Each topic: at most 3 students.
2. [10%] Survey paper: due on **Fri. Oct. 17**.
3. [10%] For each technique, the group of 2-3 students collaborate to create a 50 minute presentation. Slides: due **Fri. Nov. 7**. Talks scheduled in last 8 classes (starting **Tues. Nov. 18**). Each student in a group is expected to take an equal role in this presentation.

Some optimization tactics : simulated annealing, hill climbing, ant algorithms, tabu search…

Optional Programming Project:

An additional programming project is optional. The due date is Fri. Dec. 12 at 1:00 p.m. If students complete this project, the mark obtained will replace the contribution from one of

(a) the midterm, or (b) the two lowest assignment marks,

where the option chosen will be selected so that the final numerical score in the course is maximized.

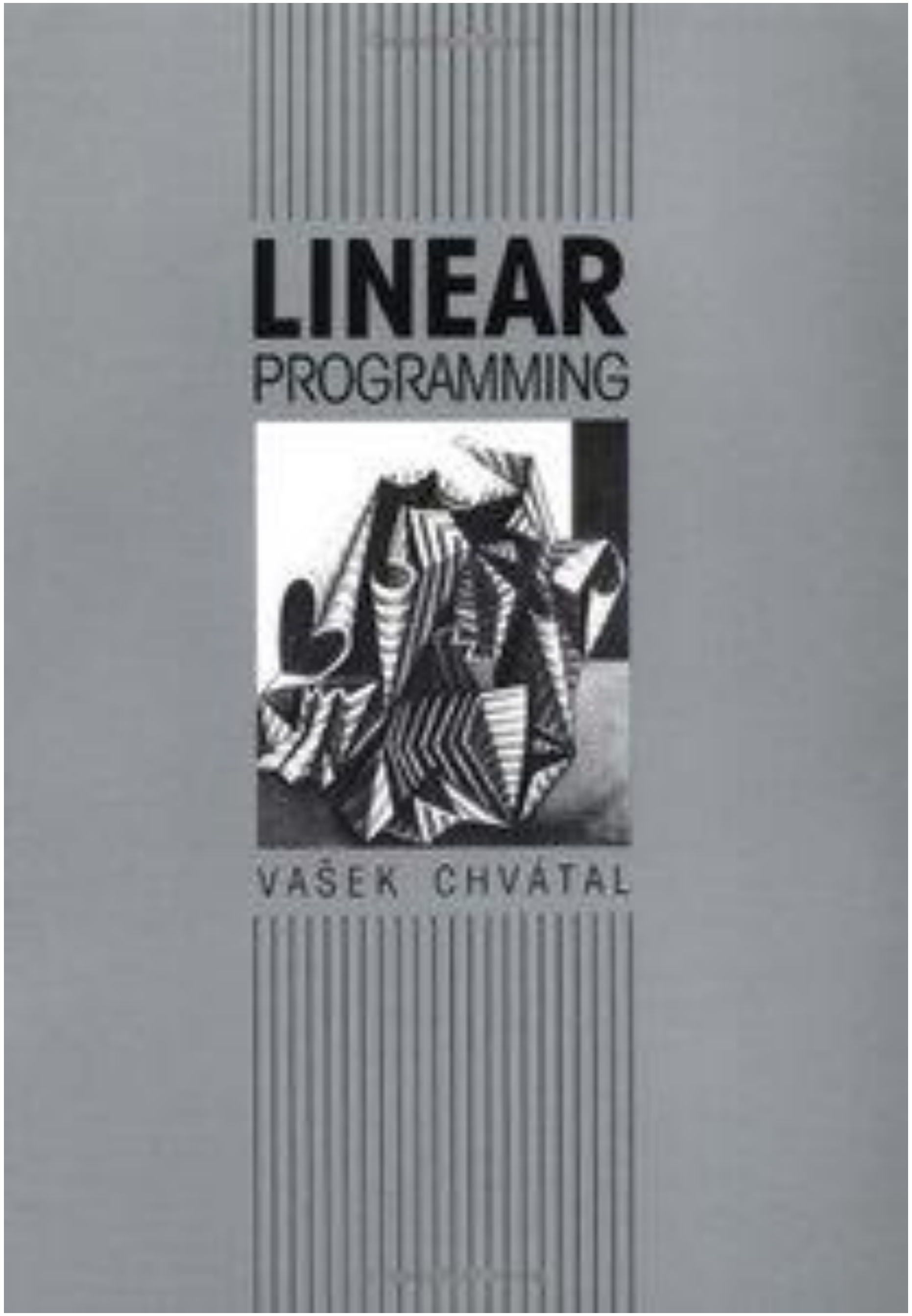
Students with a disability

Please let me know as soon as possible how I can accommodate your disability.

It's sometimes possible to go beyond what is first offered by the disability center.

Parents of young children

Let me know if you require accommodation because of the school strike.

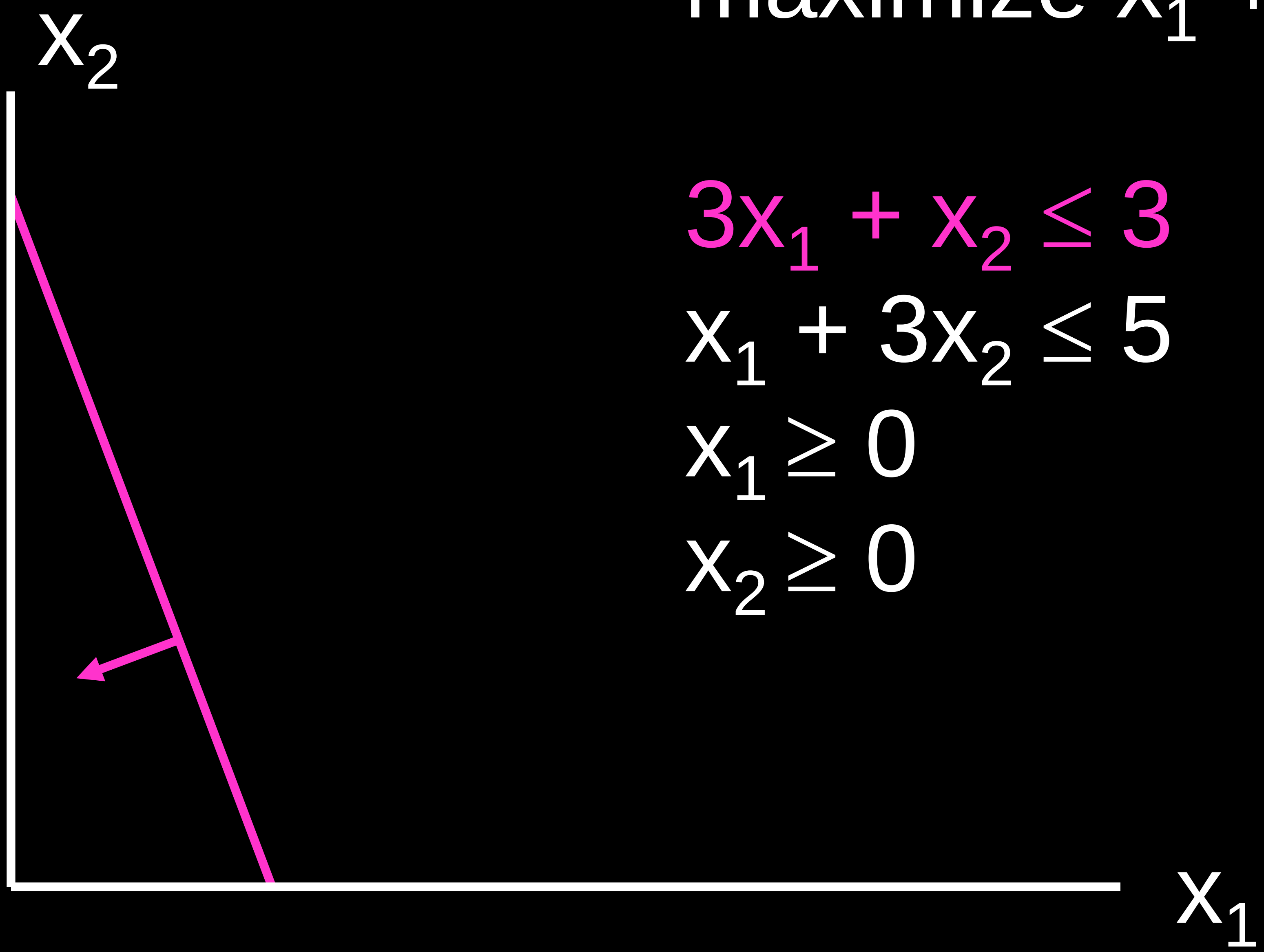


Who has a textbook?

The bookstore may have to order more due to the large class enrollment.

Let me know if you have problems getting a copy.

Image from: Daniel Stefankov,icwww.cs.rochester.edu/~stefanko/



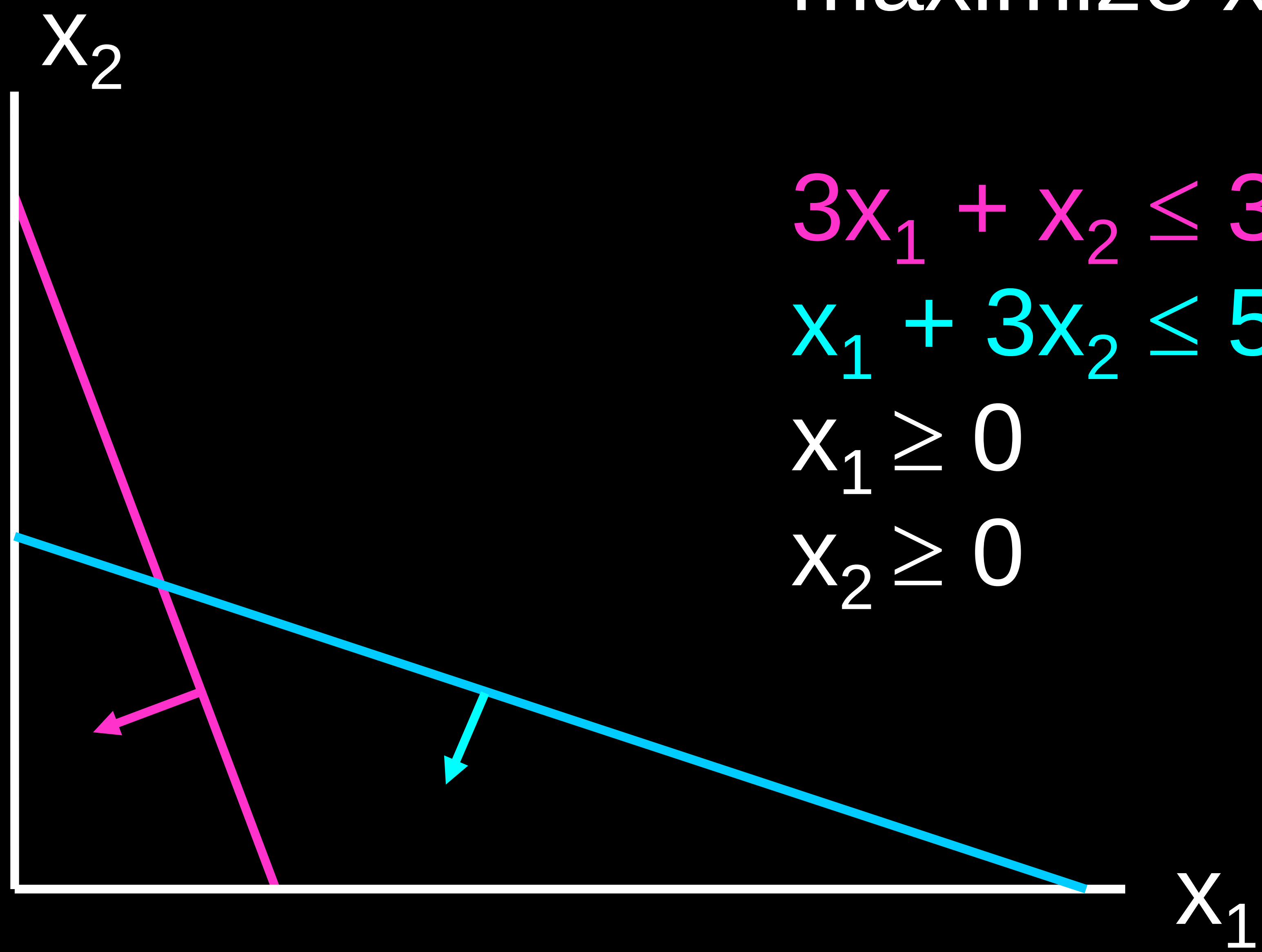
maximize $x_1 + x_2$

$3x_1 + x_2 \leq 3$

$x_1 + 3x_2 \leq 5$

$x_1 \geq 0$

$x_2 \geq 0$



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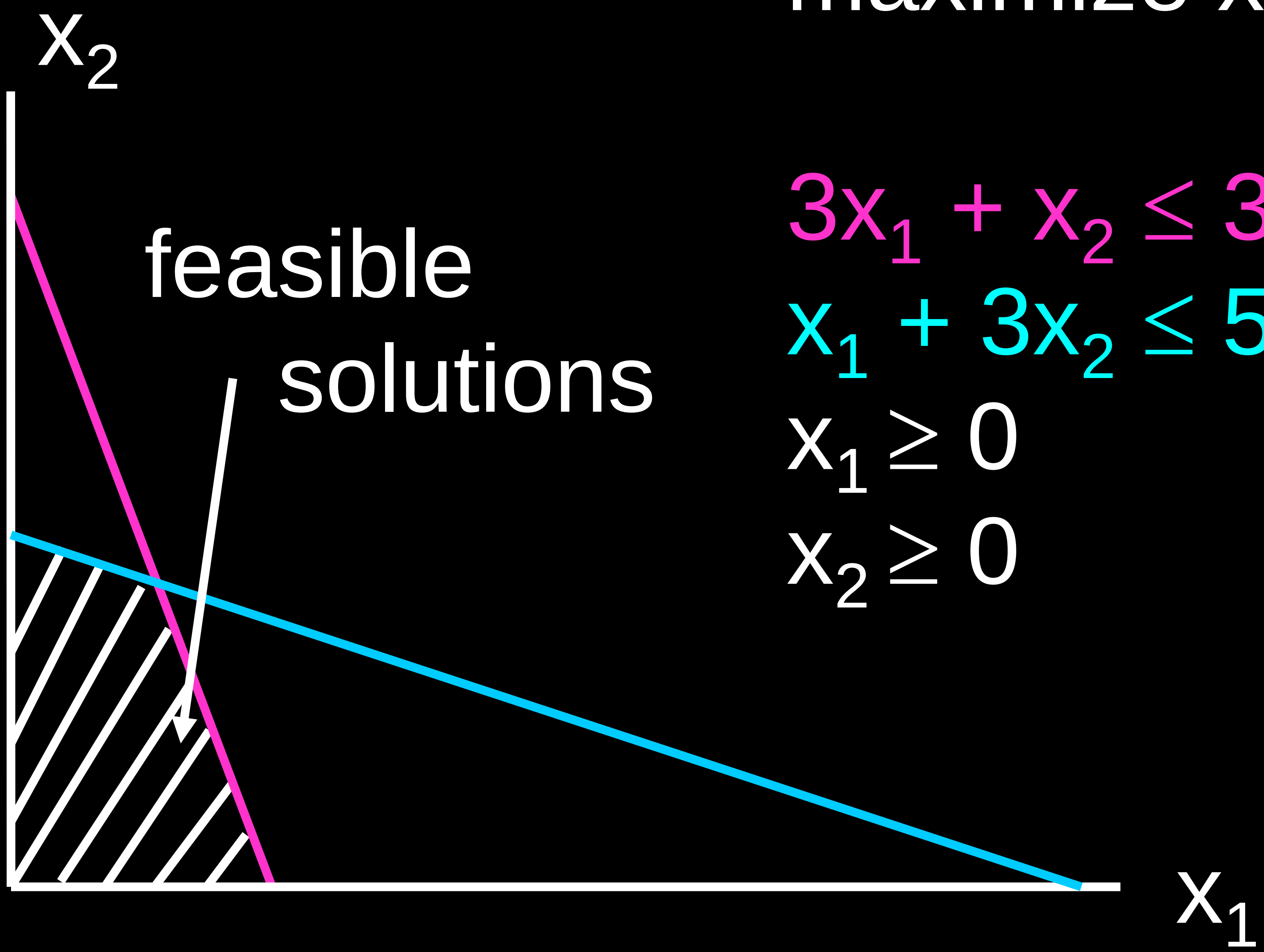
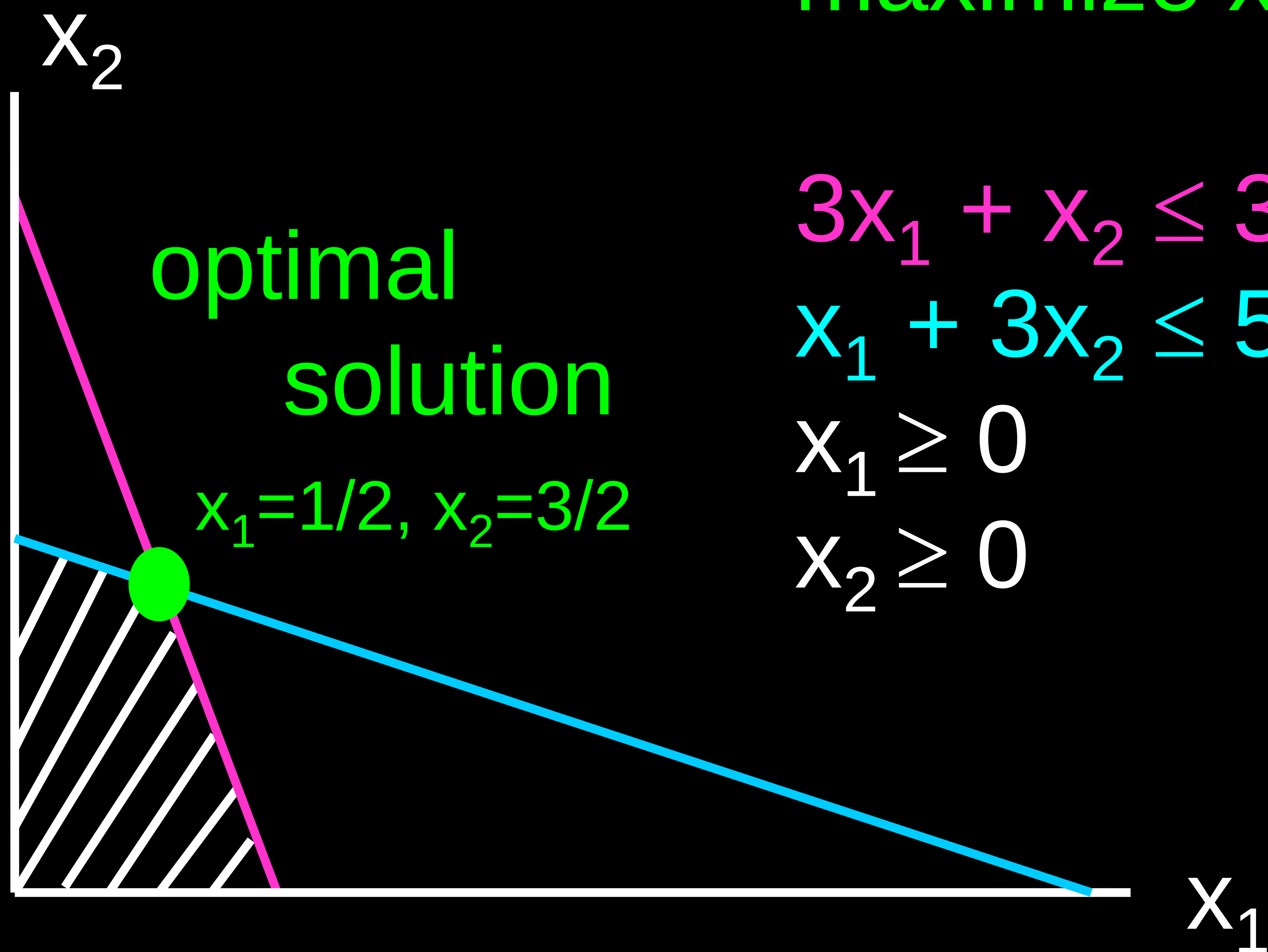
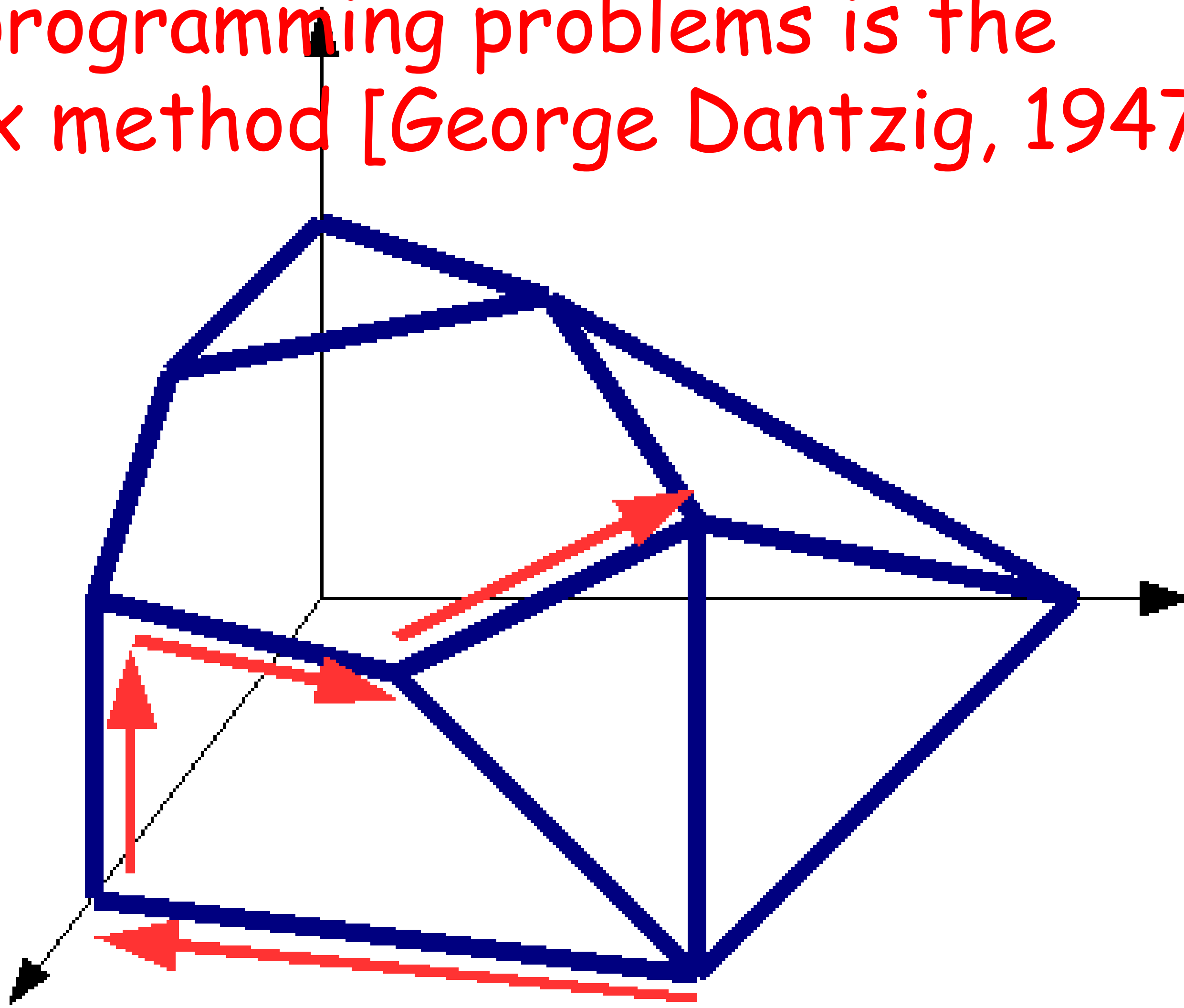


$$\text{maximize } x_1 + x_2$$

$$3x_1 + x_2 \leq 3$$
$$x_1 + 3x_2 \leq 5$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

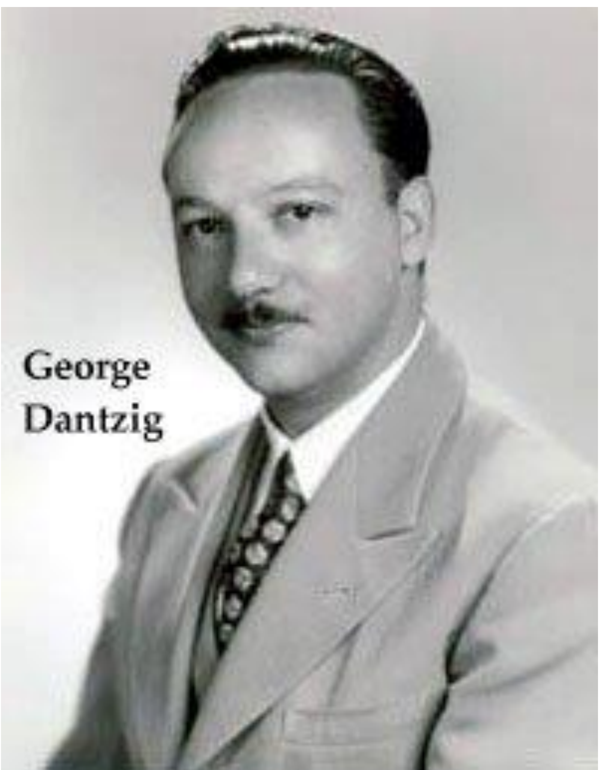


The algorithm we will study for solving linear programming problems is the Simplex method [George Dantzig, 1947].



http://wiki.hsc.com/wiki/Main/ConvexOptimization

In Dantzig's own words: During my first year at Berkeley I arrived late one day to one of Neyman's classes. On the blackboard were two problems which I assumed had been assigned for homework. I copied them down. A few days later I apologized to Neyman for taking so long to do the homework - the problems seemed to be a little harder to do than usual.



I asked him if he still wanted the work. He told me to throw it on his desk. I did so reluctantly because his desk was covered with such a heap of papers that I feared my homework would be lost there forever.

About six weeks later, one Sunday morning about eight o'clock, Anne and I were awakened by someone banging on our front door. It was Neyman. He rushed in with papers in hand, all excited: "I've just written an introduction to one of your papers. Read it so I can send it out right away for publication." For a minute I had no idea what he was talking about. To make a long story short, the problems on the blackboard which I had solved thinking they were homework were in fact two famous unsolved problems in statistics.

John von Neumann established the theory of duality also in 1947.

He made major contributions to a vast number of fields, including mathematics (set theory, functional analysis, ergodic theory, geometry, numerical analysis, and many other mathematical fields), physics (quantum mechanics, hydrodynamics, and fluid dynamics), economics (game theory), computer science (linear programming), and statistics. He is generally regarded as one of the greatest mathematicians in modern history.

http://en.wikipedia.org/wiki/John_von_Neumann

John von Neumann: Dec. 28, 1903 – Feb. 8, 1957.

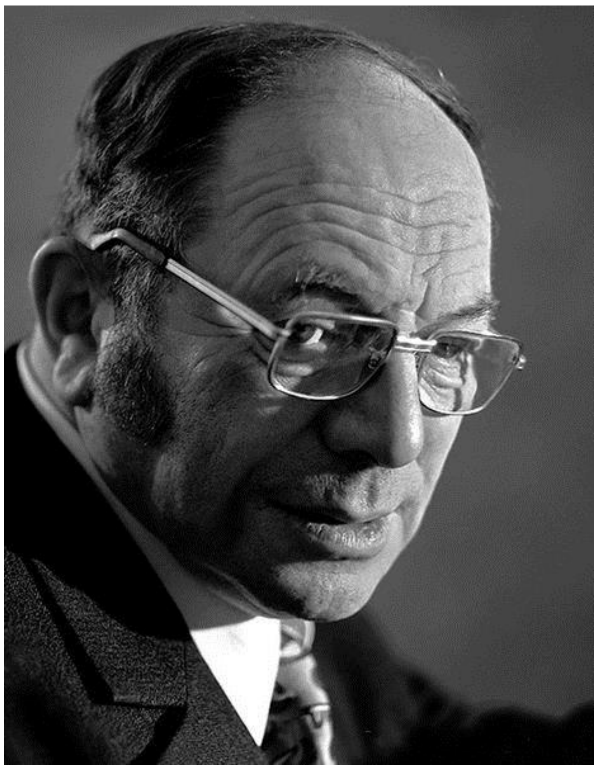
Van Neumann's contributions to computer science:

- mergeSort.
- established game theory as a mathematical discipline.
- contributions to mathematics of economics.
- introduced ideas leading to Karmarker's algorithm.
- developed a fast method for making pseudorandom numbers.
- first to describe a computer architecture where the data and the program are both stored in the computer's memory in the same address space.
- designed first template of a computer virus.

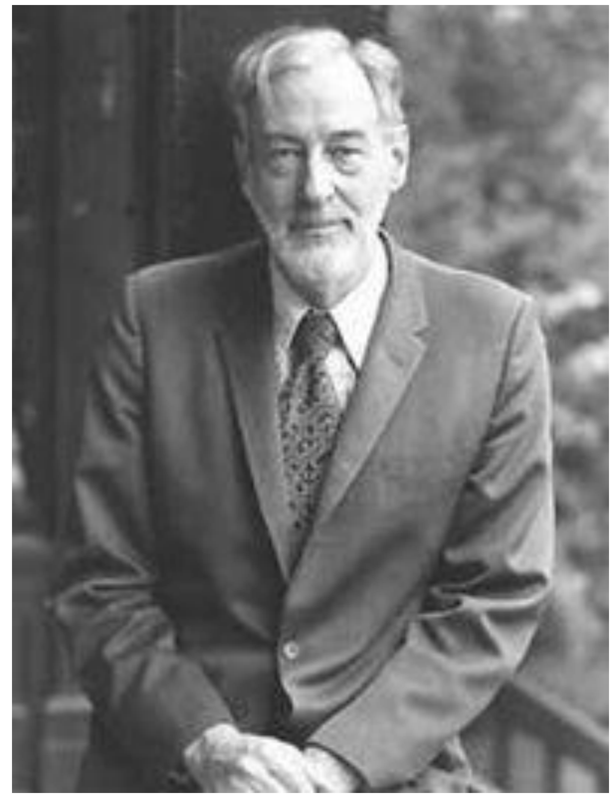
Other contributions:

- helped design nuclear bomb.
- member of committee responsible for choosing Hiroshima and Nagasaki as the first targets of the atomic bomb.
- oversaw computations related to the expected size of the bomb blasts, estimated death tolls, and the distance above the ground at which the bombs should be detonated for optimum shock wave propagation and thus maximum effect.

The Nobel prize in economics was awarded in 1975 to the mathematician Leonid Kantorovich (USSR) and the economist Tjalling Koopmans (USA) for their contributions to the theory of optimal allocation of resources.



Kantorovich: Jan. 19, 1912,- April 7, 1986.



Koopmans: Aug. 28, 1910- Feb. 26, 1985.

- Popularity sky rocketed with it was realized it could be used to solve problems in production management formerly tackled by hit-or-miss or intuitive approaches.
- Awareness grew of advantages of stating decision problems in well-defined, clear cut terms.

Leonid Khachiyan: May 3, 1952- April 29, 2005.

The Simplex method runs very fast in practice but has exponential worst case time. In 1979, Khachiyan presented the first polynomial time algorithm (the ellipsoid method) to solve linear programming problems (but it was not efficient in practice).

Leonid Khachiyan was a Soviet mathematician of Armenian descent who taught Computer Science at Rutgers University.

Karmarker in 1984 presented the first reasonably efficient algorithm for solving linear programs in polynomial time.

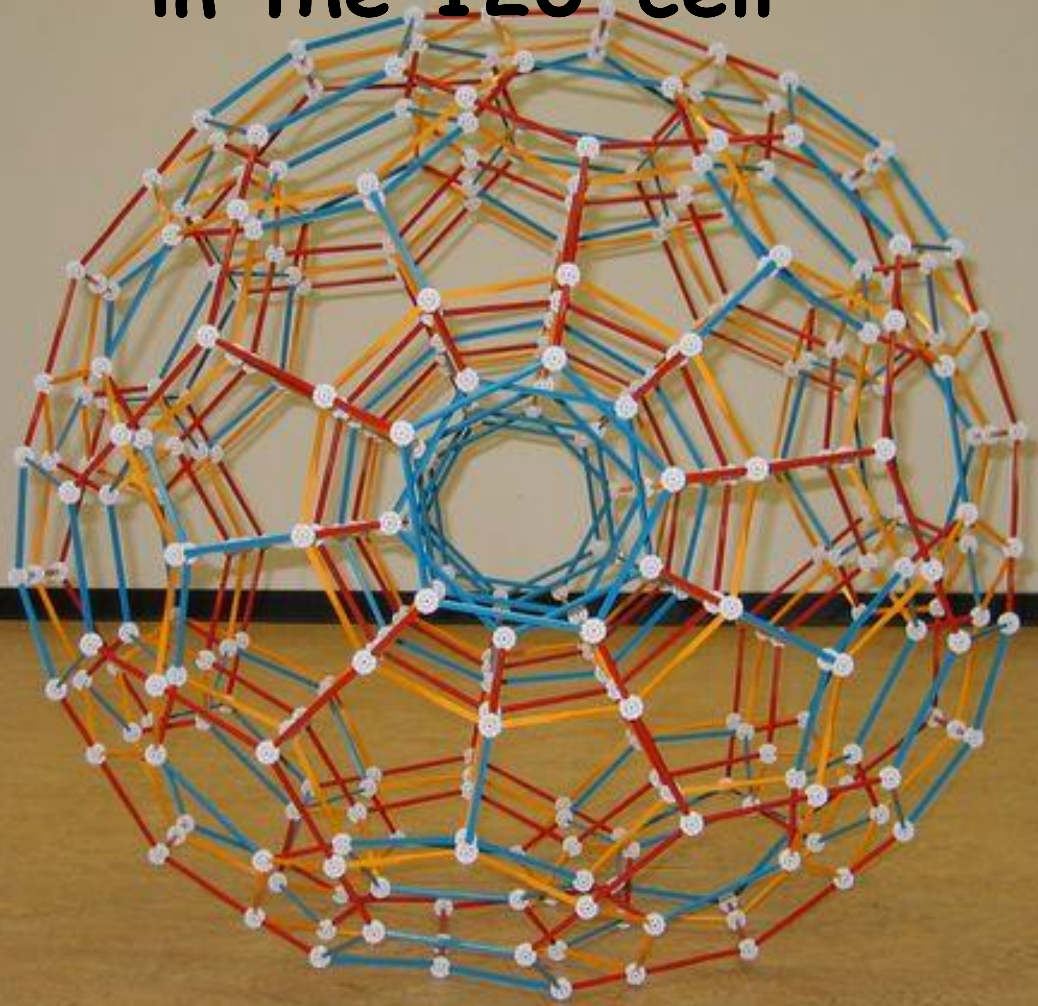
Our text: copyright 1983.

Narendra Karmarkar
1957-

At the time he discovered the algorithm, Narendra Karmarkar was employed by AT&T. AT&T applied for a patent on Karmarkar's algorithm. This left many mathematicians uneasy, such as Ronald Rivest (himself one of the holders of the patent on the RSA algorithm), who expressed the opinion that research proceeded on the basis that algorithms should be free. The patent was eventually granted but proved to be of limited commercial value.

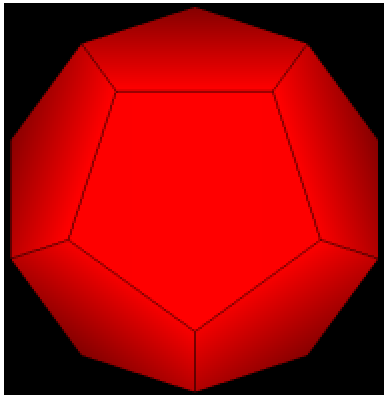
http://tetrahedral.blogspot.ca/2010/10/karmarkar-algorithm-algorithms-should.html

Finding a Maximum Independent Set in the 120-cell



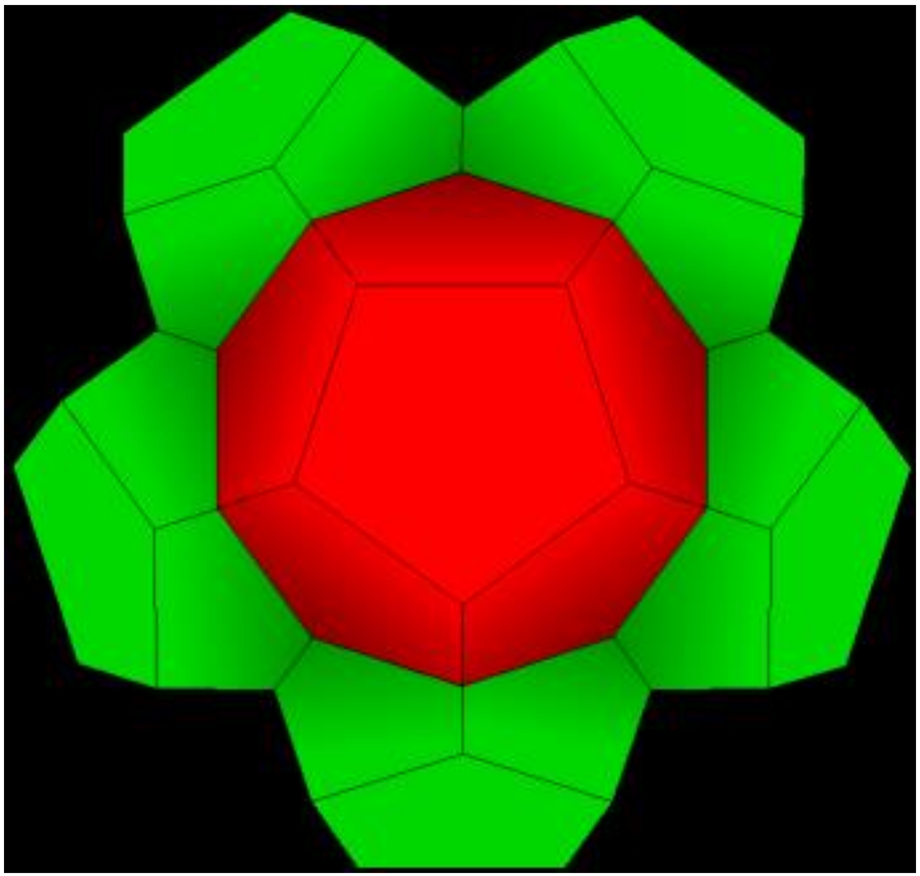
Sean Debroni, Erin Delisle, Michel Deza, Patrick Fowler, Wendy Myrvold, Amit Sethi, Benoit de La Vaissiere, Joe Whitney, Jenni Woodcock,

Start with a dodecahedron:

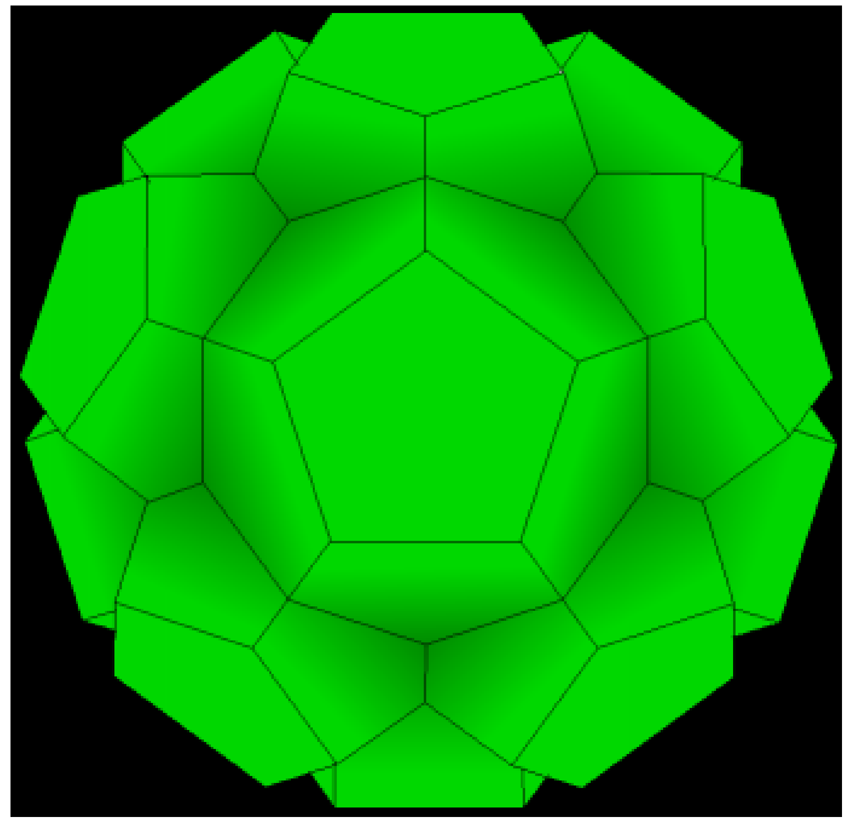


Pictures from: http://www.theory.org/geotopo/120-cell/

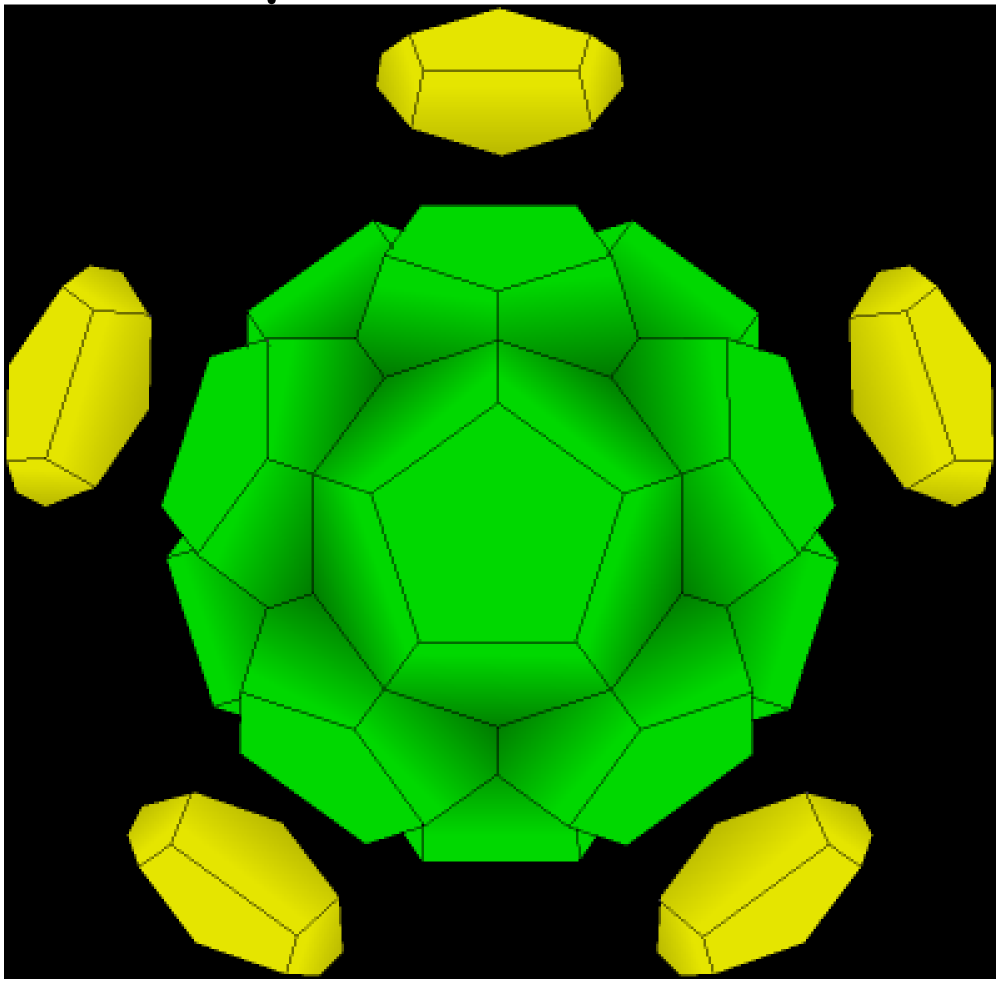
Glue 12 more on, one per face. After 6:



After 12: total 13



Add 20 more dodecahedra into the 20 dimples (total 33):



Keep going to get the 120-cell:

600 vertices,

4-regular,

girth 5,

720 5-cycles,

vertex transitive.

Notices of the AMS: Jan. 2001 Vol. 48 (1)

The Story of the 120-Cell

John Stillwell

One of the most beautiful objects in mathematics is the regular polytope in $\mathbb{R}^4$ whose boundary consists of 120 dodecahedral cells. This 120-cell is a rarity among rarities because it lives in three very special worlds. Its home is among the regular polytopes in $\mathbb{R}^4$, but it also lives in the remarkable sphere $\mathbb{S}^3$ and in the quaternions $\mathbb{H}$. And if this is not enough, the 120-cell encodes the symmetry of the icosahedron and the structure of the Poincaré homology sphere. All these facts have been known since the 1930s, but the story can be told more elegantly in contemporary language, and it can be *illustrated* better than ever before with the help of computer graphics. Moreover, the

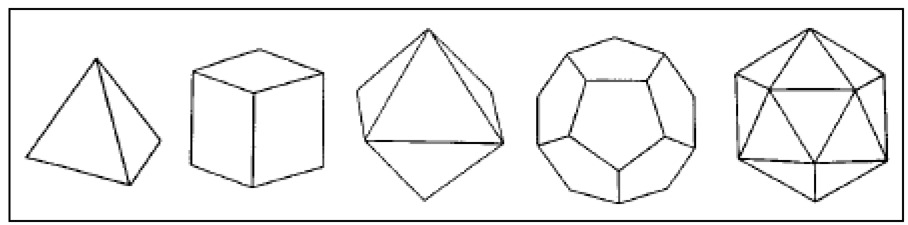


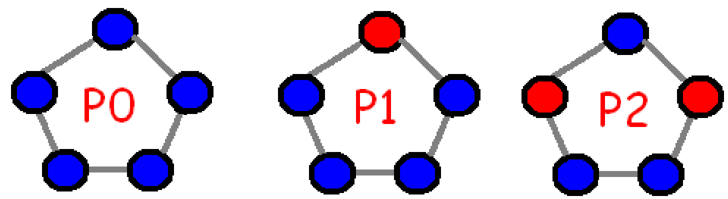
Figure 1. The five regular polyhedra.

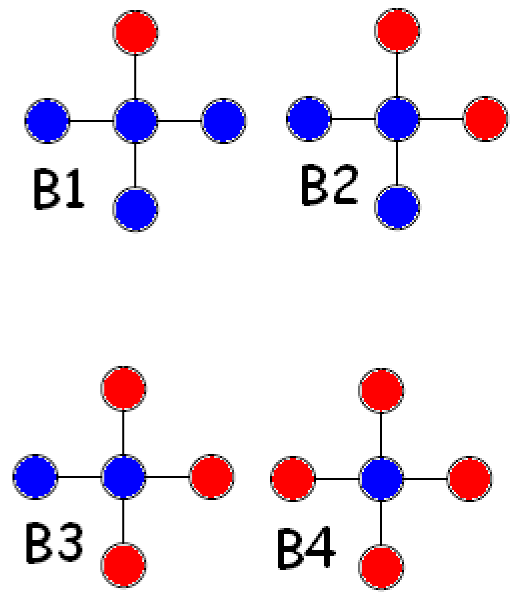
The Regular Polyhedra

The five regular polyhedra existed before human history (for example, in the form of crystals and viruses), and they certainly made an early appearance in the history of mathematics. They are the climax of Euclid's *Elements*.

LP UB of 221

P_0	P_1	P_2	B_1	B_2	B_3	B_4
25	64	631	0	253	126	0
26	62	632	0	254	124	1
27	60	633	0	255	122	2
28	58	634	0	256	120	3
26	62	632	1	251	127	0
27	60	633	1	252	125	1
28	58	634	1	253	123	2
29	56	635	1	254	121	3
27	60	633	2	249	128	0
28	58	634	2	250	126	1
29	56	635	2	251	124	2
28	58	634	3	247	129	0
29	56	635	3	248	127	1
30	54	636	3	249	125	2
29	56	635	4	245	130	0
30	54	636	4	246	128	1
30	54	636	5	243	131	0
31	52	637	5	244	129	1
31	52	637	6	241	132	0
32	50	638	7	239	133	0

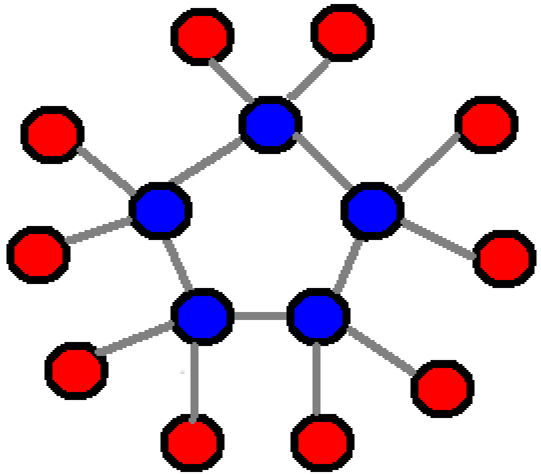


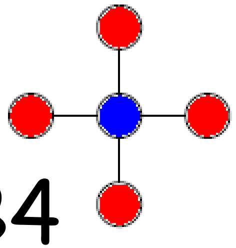


Finishing the problem:

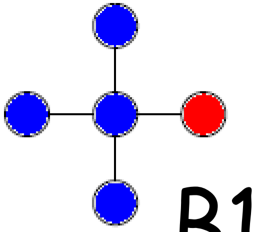
LP gives an upper bound of 221 on 120-cell or 110 on the antipodal collapse.

The resulting solutions indicate that if 221 is possible then there must be at least 25:





B4

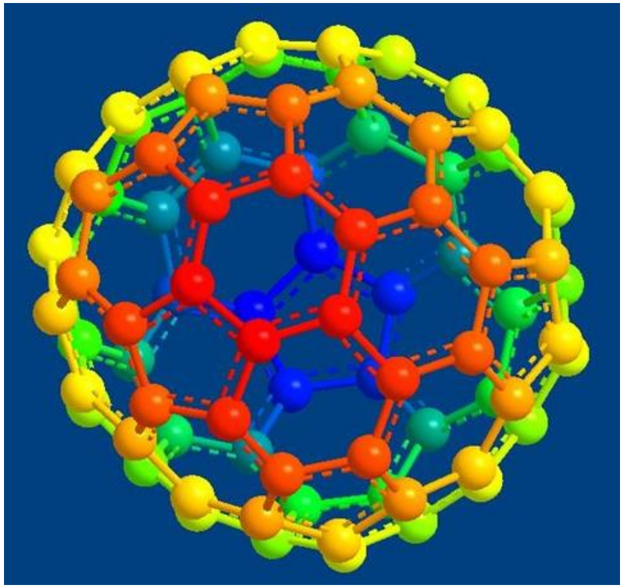


B1

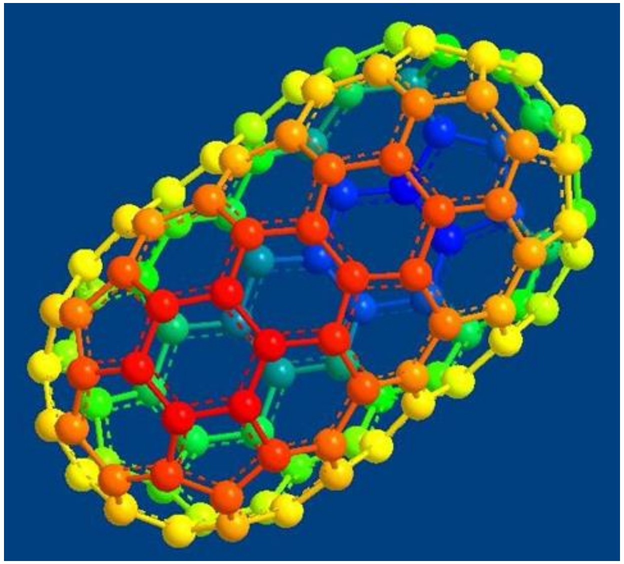
Also: B1 + 2 B4 ≤ 7

We planted 7 in all ways up to isomorphism then tried to extend to 25: not possible.

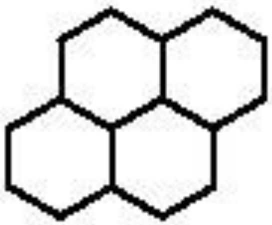
Fullerenes are all-carbon molecules that correspond to 3-regular planar graphs with all face sizes equal to 5 or 6.



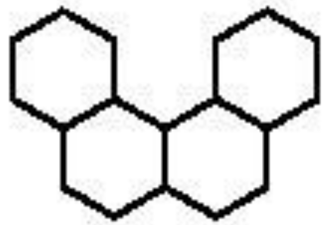
The Nobel Prize in Chemistry 1996 was awarded jointly to Robert F. Curl Jr., Sir Harold W. Kroto and Richard E. Smalley *"for their discovery of fullerenes"*.



Benzenoid:Having the six-membered ring structure or aromatic properties of benzene.

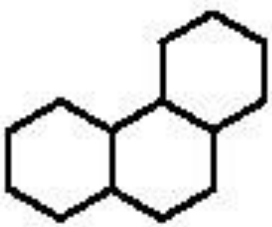


Pyrene

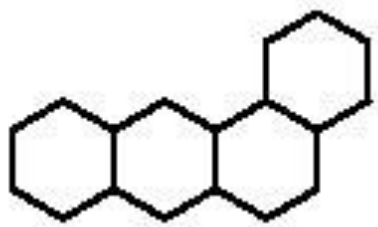


Benzo[c]phenanthrene

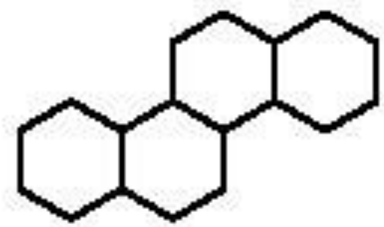
Triphenylene



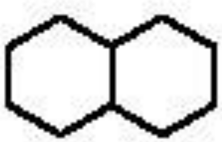
Phenanthrene



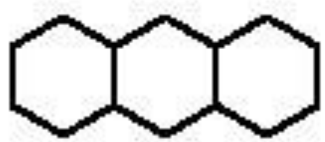
Benz[a]anthracene



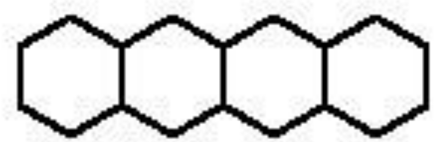
Chrysene



Naphthalene



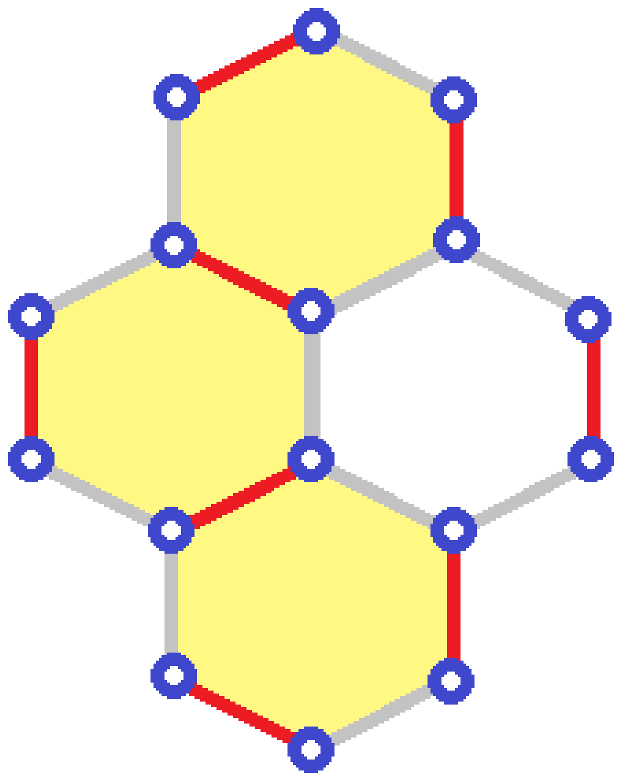
Anthracene

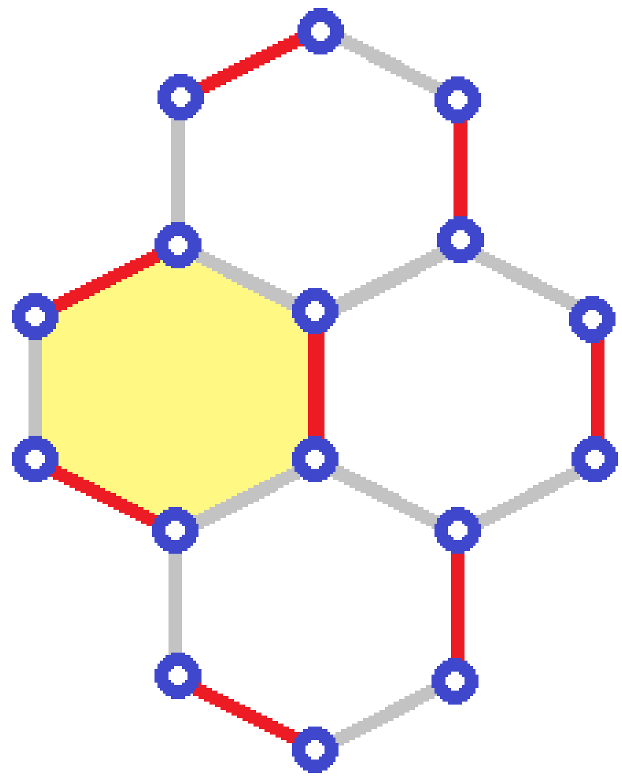


Tetracene

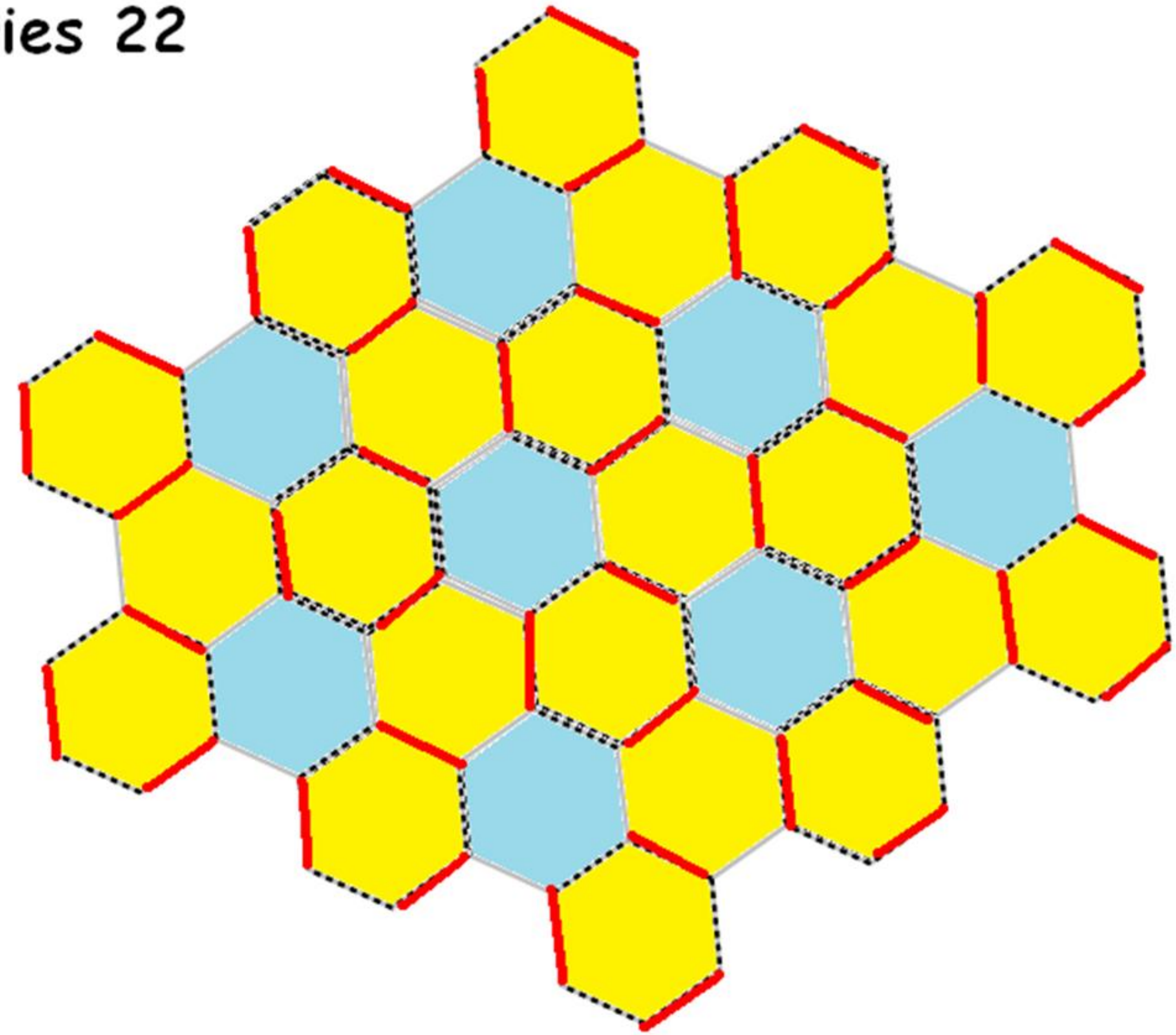
http://www.astrochem.org/sci/Cosmic_Complexity_PAHs.php

Matching: collection of disjoint edges.
Benzenoid hexagon: hexagon with 3 matching edges.
Fries number: maximum over all perfect matchings of the number of benzenoid hexagons.

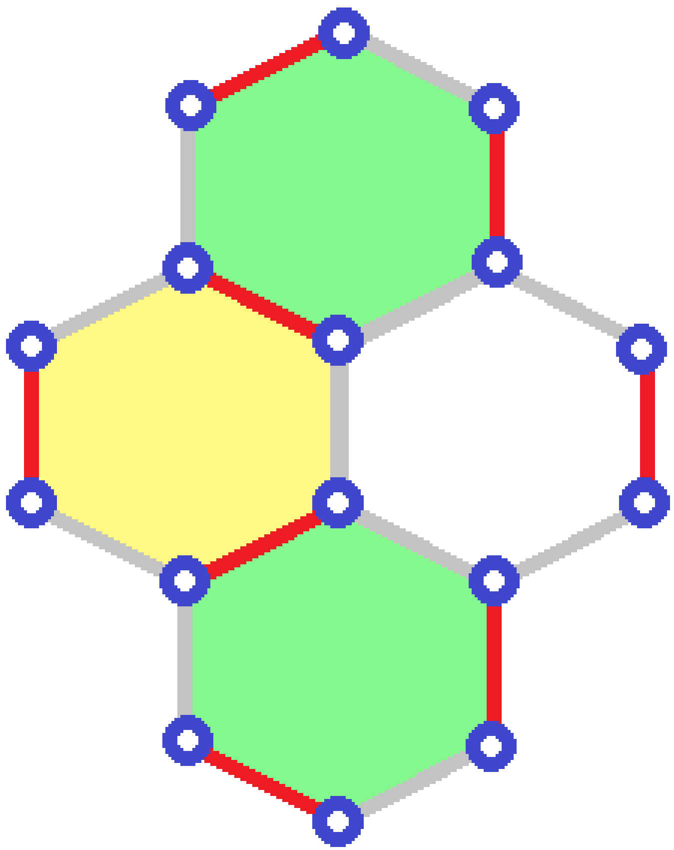


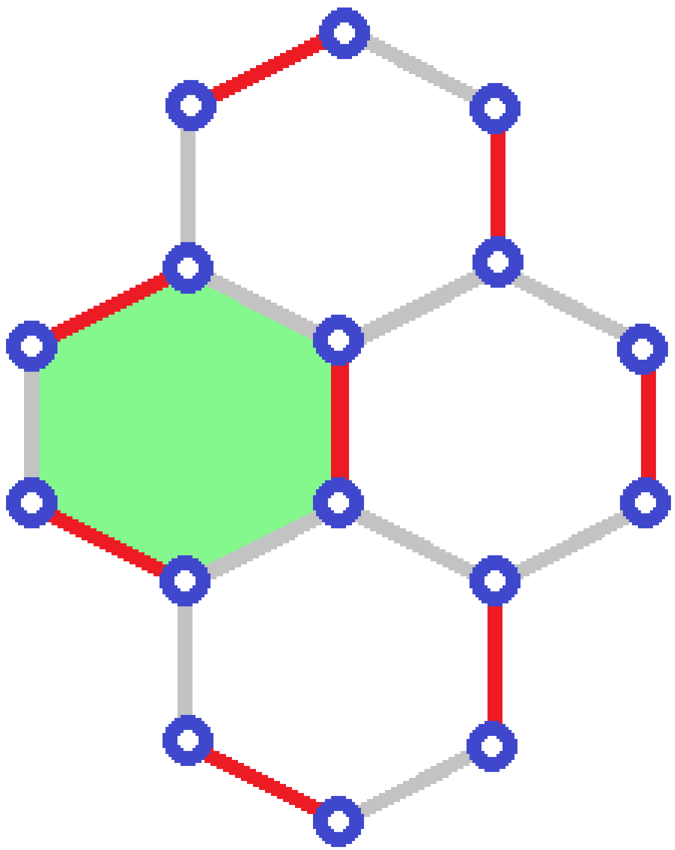


Fries 22

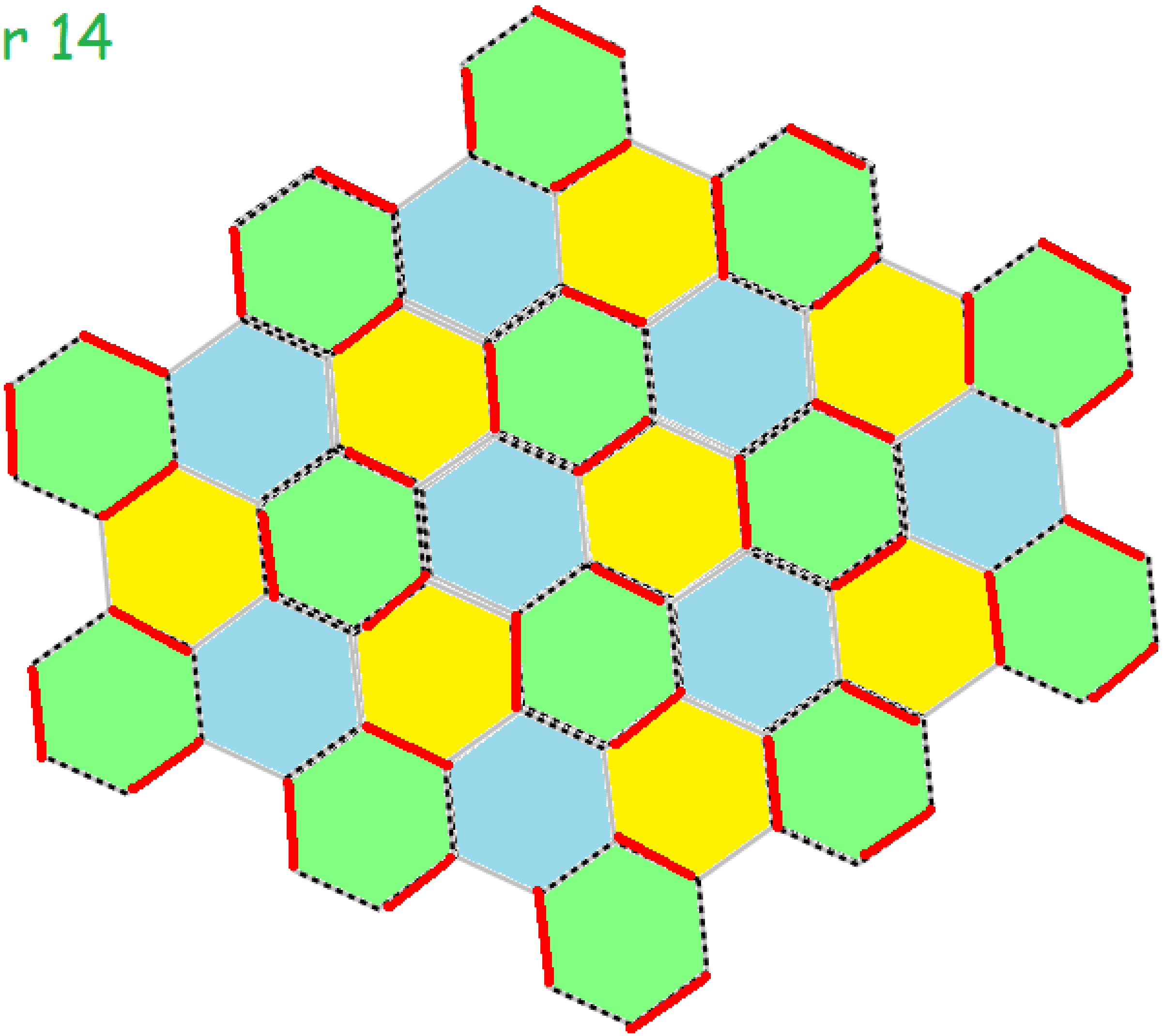


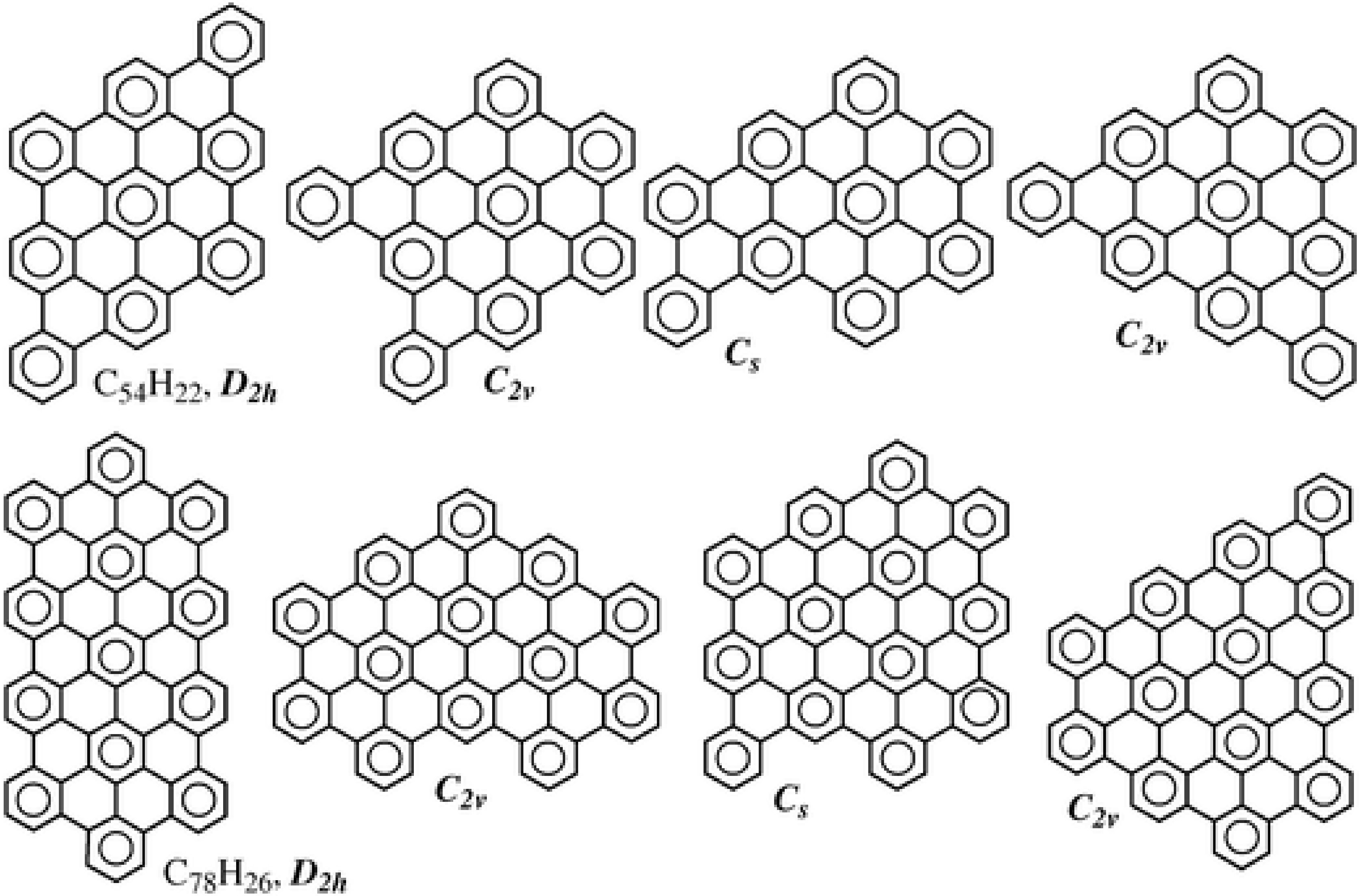
Clar number: maximum over all perfect matchings of the number of independent benzenoid hexagons.





Clar 14





http://www.springerimages.com/Images/RSS/1-10.1007_978-94-007-1733-6_8-24

It’s possible to find the Fries number and the Clar number using linear programming.

This is an example of a problem that is an integer programming problem where the integer solution magically appears when solving the linear programming problem.

http://www.javelin-tech.com/blog/2012/07/sketch-entities-splitting/magician-2/

The combinatorial curvature of a vertex v in a planar graph G is defined to be:

$$1 - \frac{degree(v)}{2} + \sum_{\substack{f:\, f \text{ is a face} \\ \text{containing } v}} 1/(face\ size\ of\ f)$$

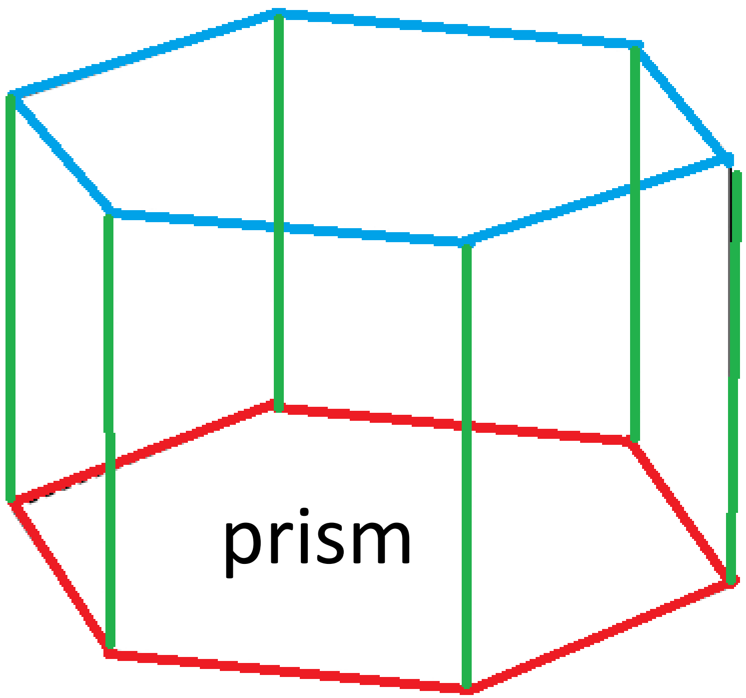
If v has degree 3 and face sizes are 5, 6, 6:

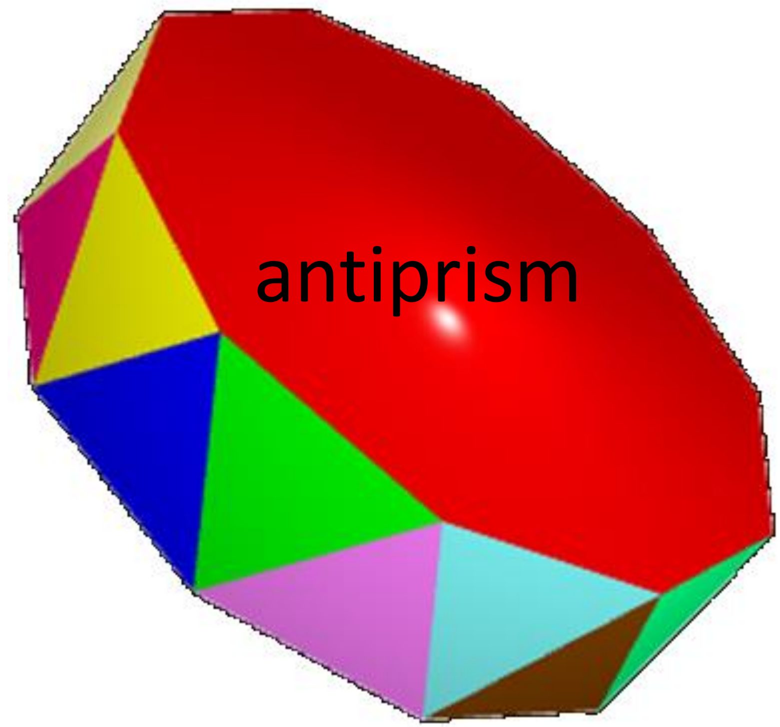
$$1 - \frac{3}{2} + \frac{1}{5} + \frac{1}{5} + \frac{1}{6} = \frac{30}{30} - \frac{45}{30} + \frac{6}{30} + \frac{6}{30} + \frac{5}{30} = \frac{6}{30}$$ positive

If v has degree 3 and face sizes are 6, 6, 7:

$$1 - \frac{3}{2} + \frac{1}{6} + \frac{1}{6} + \frac{1}{7} = \frac{42}{42} - \frac{63}{42} + \frac{7}{42} + \frac{7}{42} + \frac{6}{42} = -\frac{1}{42}$$ negative

Prisms and antiprisms maintain positive curvature and can have an unlimited number of vertices. Other positive curvature polyhedra have a bounded number of vertices.

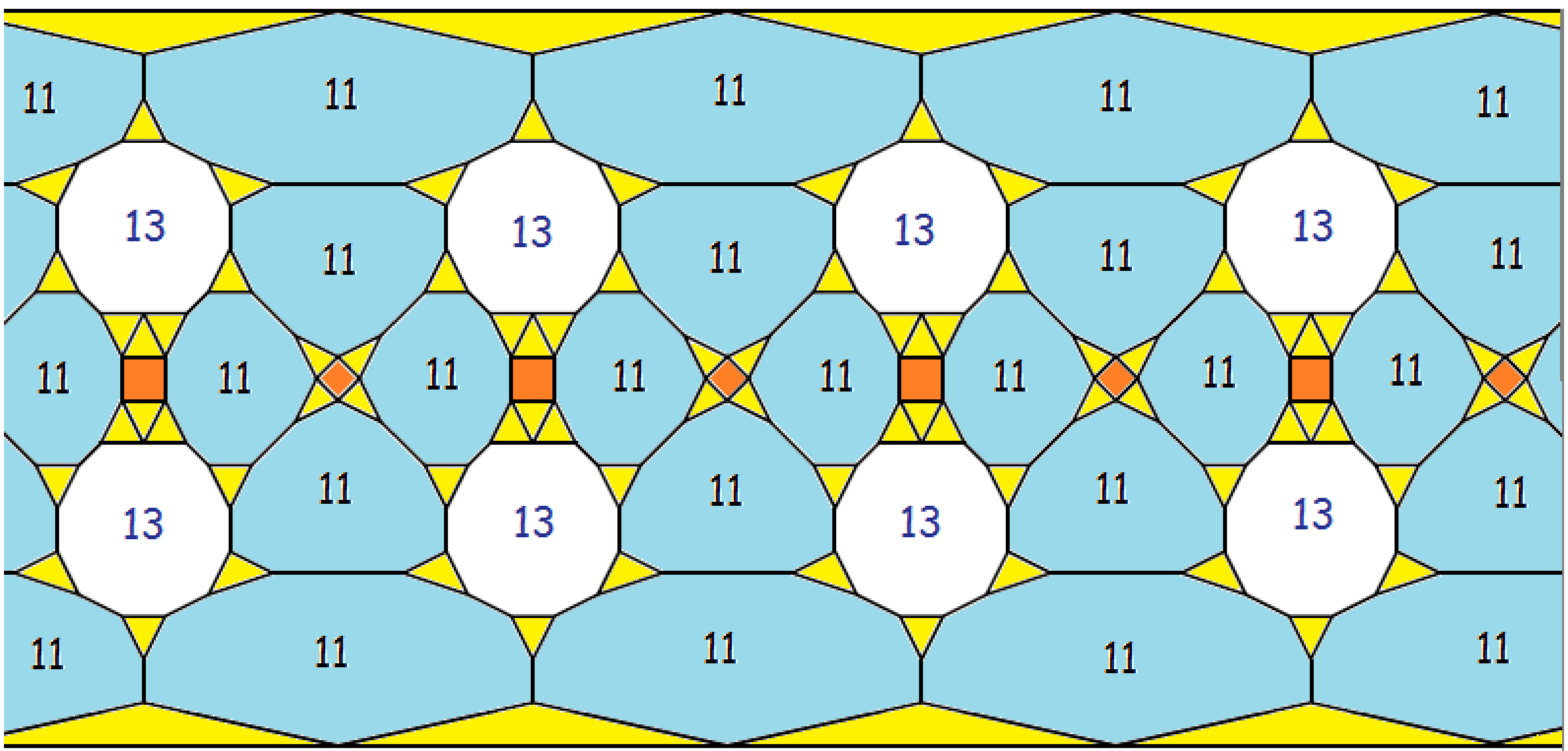




Prism:
Anti-prism: http://www.sacred-geometry.es/en/content/3d-antiprisms-and-deltohedrons

In Progress: bounding maximum size of polyhedra with positive curvature (ignoring prisms and antiprisms).

208 vertex graph found by Ruanui Nicholson and Jamie Sneddon:



Linear Programming can be used to create approximation algorithms for a wide variety of NP-hard problem including:

- travelling salesman problem,
- bin packing,
- vertex cover,
- network design problems,
- independent set,
- minimum arc feedback sets,
- integer multi-commodity flow,
- maximum satisfiability.