Formulate as an LP problem:

Toy company: produces planes and boats.
Selling price: Planes: $12, Boats $10.
Cost of raw materials: Planes $4, Boats $3.
A plane requires 3 hours to make and 1 hour to finish, while a boat requires 1 hour to make and 2 hours to finish.
The toy company knows it will not sell anymore than 30 planes per week.
The company cannot spend anymore than 120 hours per week making toys and 160 hours per week finishing toys.
The company wishes to maximize the profit it makes by choosing how many planes/boats to produce.

Announcements:

Now available from web page/connex:

Assignment #1: due Sept. 19 (or Sept. 23 with 10% late penalty)

Programming Project 1:due Sept. 26 (or Sept. 30 with a 10% late penalty).

NSERC scholarship applications

http://www.nserc.ca (look for student info)

Master's study: $17,500, due 4:30 pm Dec. 1.
Ph.D. Study: ($21,000-$25,000) due 4:30pm Oct. 1.

NSERC Grantcrafting Workshops:

Taught by Bradley Buckham.

Doctoral: Wed. Sept. 10, 1-3pm, Mac D110.

Master's:Tues. Oct. 7, 1-3pm, Mac D010.

Dr. Buckham can give you feedback on your grant applications.

Standard form (chapters 1-7):

Maximize $c^T x$

subject to

$Ax \leq b$

and $x \geq 0$

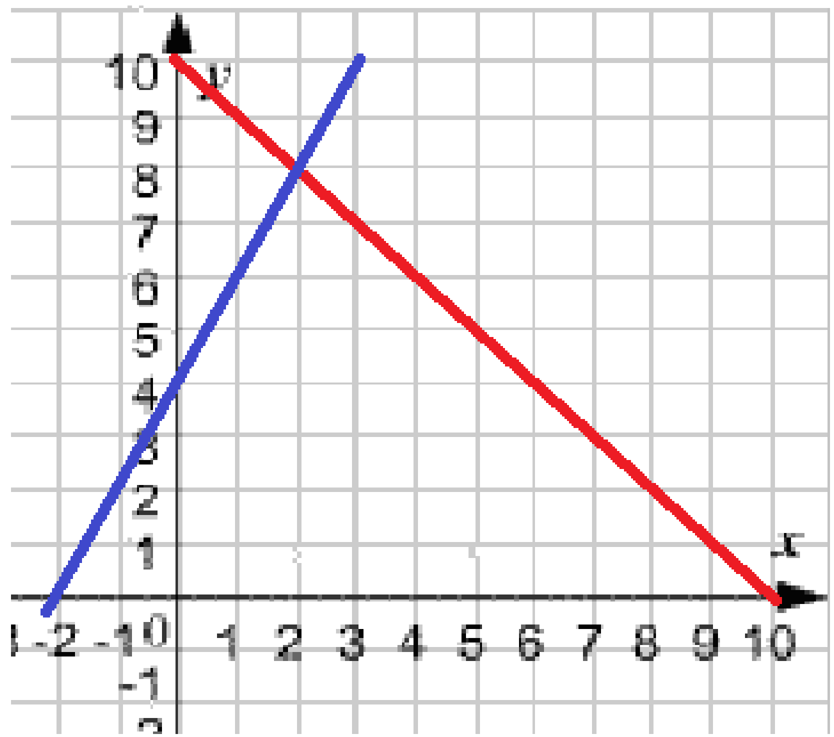
How to solve a sample LP problem:

Maximize $x_1 + 2\,x_2$

subject to

$$\begin{aligned} x_1 + \quad x_2 &\le 10 \\ -2\,x_1 + \quad x_2 &\le 4 \end{aligned}$$

$x_1, x_2 \ge 0$



1. Add slack variables to get equalities.
2. Initial dictionary has slacks on LHS of each equation.
3. Add objective function as last row of the dictionary.

Basic variables: those on the LHS.
Non-basic variables: the other variables.
Feasible solution: satisfies all the constraints.

Each dictionary corresponds to a
basic feasible solution:
Set non-basic variables to 0 and determine the values of the basic ones from the dictionary.

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While the z row (objective function) has a variable with a positive coefficient:

1. Choose a variable with a positive coefficient in the z row to enter the basis.

2. Examine each row of the dictionary to see which constraints are placed on the entering variable given that in a feasible solution, all variables have non-negative values.

3. Choose exiting variable to be the basic one for the equation giving the tightest constraint on the entering variable.

4. Rewrite the equation for the exiting variable so entering variable is on LHS (and the rest is on the RHS).

5. Substitute this new formula for the entering variable into the other equations.

This gives the next dictionary.

Transformation to get next dictionary: is call one pivot.

The Simplex method using LARGEST COEFFICIENT.

*************** Problem 1 *****

The initial dictionary:

X3 = 10.00- 1.00 X1 - 1.00 X2

X4 = 4.00+ 2.00 X1 - 1.00 X2

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z = -0.00+ 1.00 X1 + 2.00 X2

X2 enters. X4 leaves. z = -0.000000

After 1 pivot:

X3 = 6.00- 3.00 X1 + 1.00 X4

X2 = 4.00+ 2.00 X1 - 1.00 X4

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z = 8.00+ 5.00 X1 - 2.00 X4

X1 enters.   X3 leaves. z = 8.000000

After   2 pivots:

X1 = 2.00- 0.33 X3 + 0.33 X4

X2 = 8.00- 0.67 X3 - 0.33 X4

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z = 18.00- 1.67 X3 - 0.33 X4

The optimal solution: 18.000000

X1 =   2.0000   X2 =   8.0000

X3 =   0.0000   X4 =   0.0000

How long does this algorithm take in the worst case?

Can we choose an entering variable to make the algorithm terminate faster? How should we try to do that?

Can we prove that the solution is optimal at the end?

Can we prove that there always exists a basic feasible solution?

If there is more than one optimal solution, what can the solution space look like?

How can we analyze problems given that there could be small changes to the constraints (without starting from scratch)?

How can this be implemented in the computer?

How can numerical round off errors be mitigated?

How can we solve problems which are not in standard form?

What can we do if we do not have an initial feasible solution?