Convert this problem into our standard form:

Minimize $3 x_1 + 4.5 x_2 - 6 x_3$

subject to

$1 x_1 - 2 x_2 - 3 x_3 \geq 6.2$

$4 x_1 + 5 x_2 - 6 x_3 \leq 5$

$7 x_1 + 8 x_2 - 9 x_3 = -3$

$x_2, x_3 \geq 0$

How would this problem be input to your program?

Announcements:

Assignment #1 has been posted: due Sept. 19 (or Sept. 23 with 10% late penalty)

Programming Project 1 has been posted: due Sept. 26 (or Sept. 30 with a 10% late penalty).

If you send me e-mail questions, use
CSC 445: meaningful subject or
CSC 545: meaningful subject

No office hour Tues. Sept. 16 at 1:30pm (dept. meeting).

Labor Shift Scheduling Problem

Unable to hire new police officers because of budget limitations, the Gotham City Police commissioner is trying to utilize the force better. The minimum requirements for police patrols for weekdays are noted below:

Time Period	Min
Midnight - 4:00 am	6
4:00 am - 8:00 am	4
8:00 am - noon	14
noon - 4:00 pm	8
4:00 pm - 8:00 pm	12
8:00 pm - Midnight	16

The police officers can start their shifts at the starting time of any of the above time periods. However they have to patrol for 8 hours. What is the minimum number of officers required to satisfy the requirements? Express this an LP problem.

June7/07 Raeside cartoon lampoons VPD, Times Colonist.

```
public class Repeat_Add

  public static void main(String [ ] args)
  {
    int i, next_print;

    float x, y;
    double error;

    x= 1.0f / 3;     // x= 1/3

    y=0;

   next_print=3;
```

```
      for (i=1; i <= 3000000; i++)
      {
            y= y+x;
            error= y- (double) i / 3 ;

            if (i == next_print)
            {
                  System.out.println(y + " should be " + (i/3)
                  + " Error =  "  +  error);
                  next_print= next_print * 10;
             }
       }
   }
}
```

Errors resulting from repeatedly adding 1/3 (edited to make this more readable).

```
1.0        should be 1       Error =  0.0
9.999999   should be 10      Error =  -9.5367431640625E-7
100.000175 should be 100     Error =  1.754760742187 5E-4
999.97644  should be 1000    Error =  -0.023559570312 5
9999.832   should be 10000   Error =  -0.16796875
100165.48  should be 100000  Error =  165.4765625
976144.56  should be 1000000 Error =  -23855.4375
```

The "error" would be zero if the mathematics was 100% precise. But it is not due to floating point errors. This is why we compare values to some small epsilon instead of zero to test if they are zero or not.

Another sample problem:

Maximize $5\ x_1 + 5\ x_2 + 3\ x_3$

subject to

$$
\begin{aligned}
x_1 + 3\ x_2 + x_3 &\le 3 \\
-x_1 + 3\ x_3 &\le 2 \\
2\ x_1 - x_2 + 2\ x_3 &\le 4 \\
2\ x_1 + 3\ x_2 - x_3 &\le 2
\end{aligned}
$$

$x_1, x_2, x_3 \ge 0$

How is this typed in as input?

Another sample problem: 3 4

Maximize $5 x_1 + 5 x_2 + 3 x_3$ 5 5 3

subject to

$x_1 + 3 x_2 + x_3$	$\leq$	3		3	1	3	1
$-x_1 + 3 x_3$	$\leq$	2		2	-1	0	3
$2 x_1 - x_2 + 2 x_3$	$\leq$	4		4	2	-1	2
$2 x_1 + 3 x_2 - x_3$	$\leq$	2		2	2	3	-1

$x_1, x_2, x_3 \geq 0$

```
The initial dictionary:


X4 =  3.00-  1.00 X1 -  3.00 X2 -  1.00 X3
X5 =  2.00+  1.00 X1 +  0.00 X2 -  3.00 X3
X6 =  4.00-  2.00 X1 +  1.00 X2 -  2.00 X3
X7 =  2.00-  2.00 X1 -  3.00 X2 +  1.00 X3
------------------------------------------
z  = -0.00+  5.00 X1 +  5.00 X2 +  3.00 X3
```

The program output will be in dictionary format.

How much does z increase if X1, X2, or X3 enters?

```
X1  enters.   X7  leaves.  z = -0.000000

After   1 pivot:

X4 =  2.00-  1.50 X2 -  1.50 X3 +  0.50 X7
X5 =  3.00-  1.50 X2 -  2.50 X3 -  0.50 X7
X6 =  2.00+  4.00 X2 -  3.00 X3 +  1.00 X7
X1 =  1.00-  1.50 X2 +  0.50 X3 -  0.50 X7
------------------------------------------
z  =  5.00-  2.50 X2 +  5.50 X3 -  2.50 X7
```

```
X3  enters.   X6  leaves.  z = 5.000000

After   2 pivots:
X4 =  1.00-  3.50 X2 +  0.50 X6 +  0.00 X7
X5 =  1.33-  4.83 X2 +  0.83 X6 -  1.33 X7
X3 =  0.67+  1.33 X2 -  0.33 X6 +  0.33 X7
X1 =  1.33-  0.83 X2 -  0.17 X6 -  0.33 X7
-----------------------------------------
z  =  8.67+  4.83 X2 -  1.83 X6 -  0.67 X7
```

```
X2  enters.   X5  leaves.  z = 8.666667

After   3 pivots:
X4 =  0.03+  0.72 X5 -  0.10 X6 +  0.97 X7
X2 =  0.28-  0.21 X5 +  0.17 X6 -  0.28 X7
X3 =  1.03-  0.28 X5 -  0.10 X6 -  0.03 X7
X1 =  1.10+  0.17 X5 -  0.31 X6 -  0.10 X7
-------------------------------------------
z  = 10.00-  1.00 X5 -  1.00 X6 -  2.00 X7
```

How could we argue that this must be the optimal solution?

LP problems can be:

1. infeasible,

2. unbounded, or

3. they have at least one basic (corresponding to a dictionary) optimal solution.