Using the smallest subscript rules (Bland's rule) to avoid cycling, where would you pivot in this dictionary? That is, which variable enters and which one exits?

```
X1 =   0   - 1   X6 +   2 X2 - 1 X4 + 4 X5
X3 =   0   - 1   X6 +   2 X2 - 1 X4 - 0 X5
X7 =   3   + 1   X6 +   0 X2 + 0 X4 - 9 X5
X9 =   0   + 1   X6 +   0 X2 + 0 X4 - 2 X5
X8 =   0   + 0   X6 -   1 X2 + 1 X4 - 3 X5
-------------------------------------------
z =   0    + 10 X6 -   1 X2 + 0 X4 + 1 X5
```

Recall: As long as $b_i \geq 0$ for $i = 1, 2, ..., m$,

setting $x_i = 0$ for $i = 1, 2, ... n$ (original variables)

and assigning the slack variables according to the dictionary:

$x_{n+i} = b_i$ for $i = 1, 2, ... m$

gives an initial feasible solution.

What can we do if one or more b_i's are < 0?

Maximize $x_1 + x_2$ subject to:

$$-x_1 - x_2 \le -3$$
$$3x_1 - 4x_2 \le 2$$
$$x_1, \quad x_2 \ge 0$$

Initial dictionary:

```
X3 = - 3.0 + 1.0 X1 +  1.0 X2
X4 =   2.0 - 3.0 X1 +  4.0 X2
------------------------------
z =    0.0 + 1.0 X1 +  1.0 X2
```

Problem: $X_3 = -3$ violates non-negativity constraints.

Maximize $-x_0$ subject to:

$$-x_1 - x_2 - x_0 \le -3$$

$$3x_1 - 4x_2 - x_0 \le 2$$

$$x_1, x_2, x_0 \ge 0$$

It's easier for the program to use X5 instead of X0:

```
X3= -3.0 +1.0 X1 + 1.0 X2 + 1.0 X5
X4=  2.0 -3.0 X1 + 4.0 X2 + 1.0 X5
-----------------------------------
z =  0.0 +0.0 X1 + 0.0 X2 - 1.0 X5
```

The problem is still not feasible. What can we do to fix it?

```
X3= -3.0 +1.0 X1 + 1.0 X2 + 1.0 X5
X4=  2.0 -3.0 X1 + 4.0 X2 + 1.0 X5
----------------------------------
z =  0.0 +0.0 X1 + 0.0 X2 - 1.0 X5
```

Auxillary problem:

Minimize x_0 (Maximize $-x_0$)

subject to:

$$- x_1 - x_2 - x_0 \leq -3$$
$$3 x_1 - 4 x_2 - x_0 \leq 2$$

$$x_1, x_2, x_0 \geq 0$$

At the optimal solution:

What does it mean if $x_0=0$?

What does it mean if $x_0 > 0$?

Choose the variable to exit to be the one with the equation with the smallest b_i: the new variable x_0 (X5) enters:

```
X3 = -3.00+ 1.00 X1 + 1.00 X2 + 1.00 X5
X4 =  2.00- 3.00 X1 + 4.00 X2 + 1.00 X5
---------------------------------------
z =    0.00+ 0.00 X1 + 0.00 X2 - 1.00 X5
```

```
X5 enters. X3 leaves. z = -0.000000
```

```
X5 = 3.00- 1.00 X1 - 1.00 X2 + 1.00 X3
X4 = 5.00- 4.00 X1 + 3.00 X2 + 1.00 X3
--------------------------------------
z = -3.00+ 1.00 X1 + 1.00 X2 - 1.00 X3
```

Now proceed as usual for the phase 1 problem:

```
X1 enters. X4 leaves. z = -3.000000

After 1 pivot:
X5 = 1.75- 1.75 X2 + 0.75 X3 + 0.25 X4
X1 = 1.25+ 0.75 X2 + 0.25 X3 - 0.25 X4
--------------------------------------
z = -1.75+ 1.75 X2 - 0.75 X3 - 0.25 X4

X2 enters. X5 leaves. z = -1.750000
```

```
X2 enters. X5 leaves. z = -1.750000

After 2 pivots:
X2 = 1.00+ 0.43 X3 + 0.14 X4 - 0.57 X5
X1 = 2.00+ 0.57 X3 - 0.14 X4 - 0.43 X5
--------------------------------------
z = -0.00+ 0.00 X3 + 0.00 X4 - 1.00 X5

The optimal solution: -0.000000
X1 =  2.0000 X2 =  1.0000 X3 =  0.0000
X4 =  0.0000 X5 =  0.0000
```

The original problem:

Maximize $x_1 + x_2$ subject to:

$- x_1 - x_2 \leq -3$

$3 x_1 - 4 x_2 \leq 2 \quad x_1, x_2 \geq 0$

The auxillary problem:

Maximize $-x_0$ subject to:

$- x_1 - x_2 - x_0 \leq -3$

$3 x_1 - 4 x_2 - x_0 \leq 2$

$x_1, x_2, x_0 \geq 0$

Why is this solution feasible for the original problem?

$x_1=2,\ x_2=1,\ x_3=0,\ x_4=0,\ x_0=0$

The last dictionary of Phase 1:

```
X2 = 1.00+ 0.43 X3 + 0.14 X4 - 0.57 X5
X1 = 2.00+ 0.57 X3 - 0.14 X4 - 0.43 X5
--------------------------------------
z = -0.00+ 0.00 X3 + 0.00 X4 - 1.00 X5
```

Our original objective function:
Maximize x1 + x2

How do we add this to the dictionary?

z= x1 + x2

and from the last phase 1 dictionary:

X1 = 2.00+ 0.57 X3 - 0.14 X4 - 0.43 X5

X2 = 1.00+ 0.43 X3 + 0.14 X4 - 0.57 X5

So for the orginal problem:

X1 = 2.00+ 0.57 X3 - 0.14 X4

X2 = 1.00+ 0.43 X3 + 0.14 X4

Therefore:

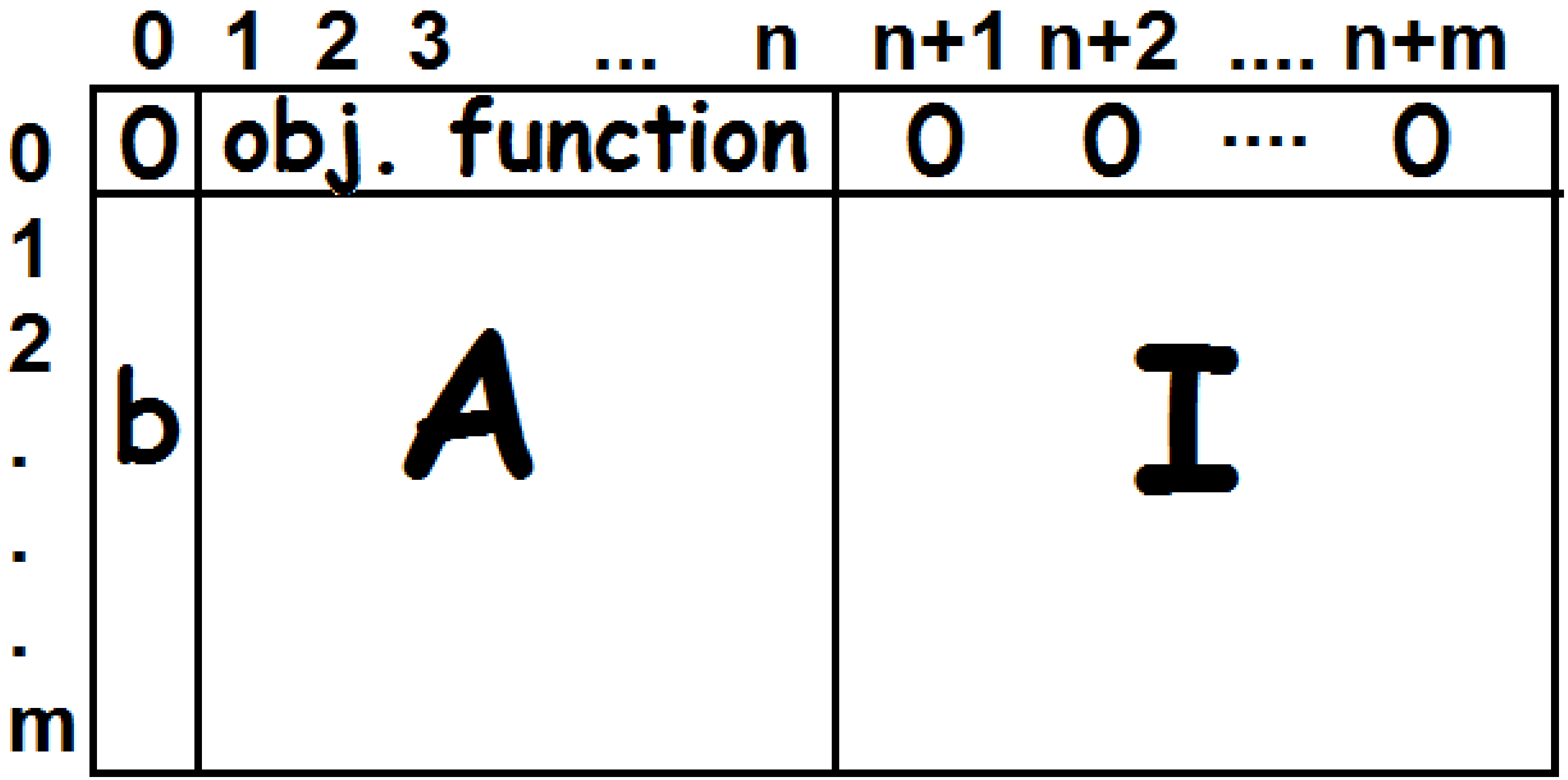
z = 3.00+ 1.00 X3 + 0.00 X4

Change the z row from the last dictionary of Phase 1 and get rid of the variable x_0 (X5):

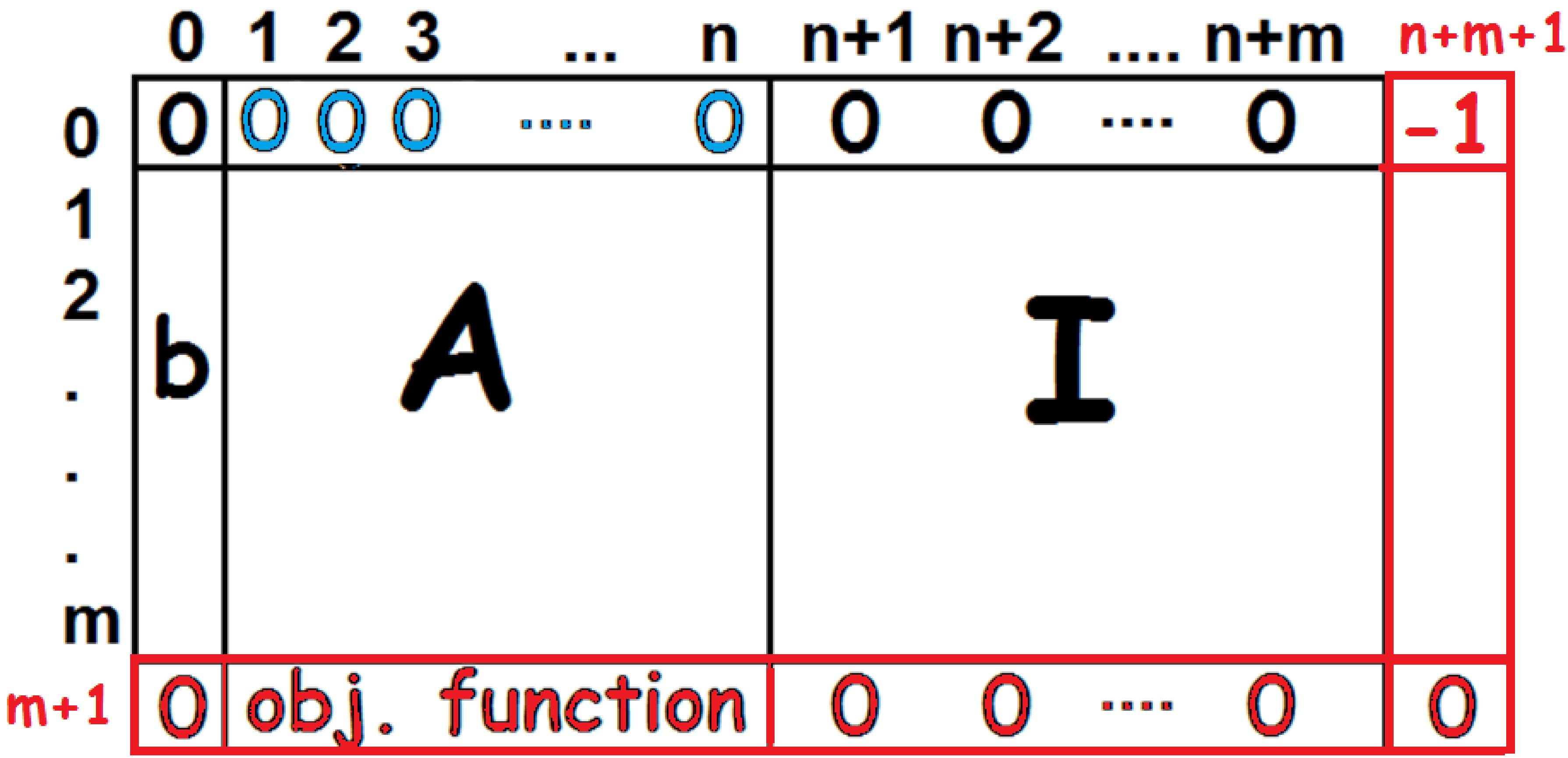
```
X2 = 1.00+ 0.43 X3 + 0.14 X4
X1 = 2.00+ 0.57 X3 - 0.14 X4
-----------------------------
z  = 3.00+ 1.00 X3 + 0.00 X4
```

This is a basic feasible solution that we can use to start the Simplex method for the original problem (Phase 2).

An approach you could take in your program. If you used:



For phase 1, use this:



If your pivots include row m+1 then your objective function will already be ready to use when you finish phase 1.

```
X2 = 1.00 + 0.43 X3 + 0.14 X4
X1 = 2.00 + 0.57 X3 - 0.14 X4
-----------------------------
z  = 3.00 + 1.00 X3 + 0.00 X4
```

Conclusion: original problem is unbounded.

Maximize $x_1 + x_2$ subject to:

$$
\begin{aligned}
- x_1 - x_2 &\le -3 \\
3 x_1 - 4 x_2 &\le 2 \\
x_1, \quad x_2 &\ge 0
\end{aligned}
$$

x_3 is the slack on the first equation. We can make x_3 as big as we want because there is no bound on how big we can make x_2.

Another example:

Maximize $x_1 + x_2$ subject to:

$$
\begin{aligned}
2\,x_1 - x_2 &\le -5 \\
0\,x_1 + 1\,x_2 &\le 3 \\
x_1,\quad x_2 &\ge 0
\end{aligned}
$$

Set up and solve the Phase 1 problem.

Minimize x_0 (Maximize $-x_0$)

$$2\,x_1 - x_2 - x_0 \leq -5$$
$$0\,x_1 + 1\,x_2 - x_0 \leq 3$$

$$x_0, x_1, x_2 \geq 0$$

The initial dictionary:

$$x_3 = -5 - 2\,x_1 + x_2 + x_0$$
$$x_4 = 3 + 0\,x_1 - 1\,x_2 + x_0$$

$$z = 0 - x_0$$

The initial dictionary:

$x_3 = -5 - 2\,x_1 + x_2 + x_0$

$x_4 = 3 + 0\,x_1 - 1\,x_2 + x_0$

$z = 0 - x_0$

Pivot to make it feasible:

X0 enters. X3 leaves. z = 0.0

X0 = 5 + 2 X1 - 1 X2 + 1 X3

X4 = 8 + 2 X1 - 2 X2 + 1 X3

----

z = -5 - 2 X1 + 1 X2 - 1 X3

X2 enters. X4 leaves. z = -5.00
After   1 pivot:

```
X0 = 1.0+ 1.0 X1 + 0.5 X3 + 0.5 X4
X2 = 4.0+ 1.0 X1 + 0.5 X3 - 0.5 X4
----------------------------------
z = -1.0- 1.0 X1 - 0.5 X3 - 0.5 X4
```

The optimal solution: -1.000000

What can we conclude from this?

The original problem:

Maximize $x_1 + x_2$ subject to:

$2x_1 - x_2 \leq -5$

$0x_1 + 1x_2 \leq 3$

$x_1, x_2 \geq 0$

The auxillary problem:

Minimize x_0

$2x_1 - x_2 - x_0 \leq -5$

$0x_1 + 1x_2 - x_0 \leq 3$

$x_0, x_1, x_2 \geq 0$

The solution:

$x_0 = 1$, $x_1 = 0$, $x_2 = 4$.