1. Set up the phase 1 dictionary for this problem and make the first pivot:

Maximize $X_1 + X_2 + 2X_3 + X_4$

subject to

$$
\begin{aligned}
-X_1 \quad\quad\quad + X_3 + 2X_4 &\le -3 \\
-X_1 + X_2 \quad\quad\quad\quad\quad\quad &\le -7 \\
-X_1 + 2X_2 \quad\quad + X_4 &\le -5
\end{aligned}
$$

$X_1, X_2, X_3, X_4 \ge 0$

The programming project is due Friday (or Tuesday for a 10% penalty). Don't forget to upload your programs to connex.

Make sure your input/output are standard input and standard output (not hard coded or typed in file names.

Available from course web page (and coming to connex soon):

Assignment #2 is now available: Due Fri. Oct. 3 at the beginning of class.

Grad project description: Topic selection Wed., Survey Paper- due on Fri. Oct. 17

Grad Project Topics

On Wed. at the beginning of class, each grad student will draw a number that determines the order in which the students can select one of these topics. Each topic can be chosen by at most 3 students. The optimization techniques:

1. Ant Colony algorithms
2. Genetic algorithms
3. Hill Climbing (Random restart)/Iterated Greedy Algorithm
4. Neural Networks
5. Simulated annealing
6. Tabu Search
7. Approximation Algorithms (if > 18 students)

The equations:

$$-X_1 + X_3 + 2X_4 - x_0 \le -3$$
$$-X_1 + X_2 - x_0 \le -7$$
$$-X_1 + 2X_2 + X_4 - x_0 \le -5$$

Phase 1: Before pivoting to make feasible.

X5 = -3 + X1 - X3- 2 X4 + X0
X6 = -7 + X1 - X2 + X0
X7 = -5 + X1 -2 X2 - X4 + X0
----
z = 0 - X0

Taking the first pivot:

X0 enters. X6 leaves. z = -0.000000

The initial dictionary:

```
X5 = 4+ 0 X1 + 1 X2 - 1 X3 - 2 X4 + 1 X6
X0 = 7- 1 X1 + 1 X2 + 0 X3 + 0 X4 + 1 X6
X7 = 2+ 0 X1 - 1 X2 + 0 X3 - 1 X4 + 1 X6
----------------------------------------
z = -7+ 1 X1 - 1 X2 + 0 X3 + 0 X4 - 1 X6
```

The optimal solution to the phase 1 problem is:

```
After  1 pivot:
X5= 4 +1 X2 -1 X3 -2 X4 +1 X6 + 0 X0
X1= 7 +1 X2 +0 X3 +0 X4 +1 X6 - 1 X0
X7= 2 -1 X2 +0 X3 -1 X4 +1 X6 + 0 X0
-------------------------------------
z = 0 +0 X2 +0 X3 +0 X4 +0 X6 - 1 X0
```

2. Set up the initial dictionary for the Phase 2 problem. Recall the objective function is:

Maximize $X_1 + X_2 + 2\ X_3 + X_4$

Answer:

```
X5 = 4 +1 X2 -1 X3 -2 X4 +1 X6
X1 = 7 +1 X2 +0 X3 +0 X4 +1 X6
X7 = 2 -1 X2 +0 X3 -1 X4 +1 X6
------------------------------
z  = 7 +2 X2 +2 X3 +1 X4 +1 X6
```

The final dictionary is:

```
X5 =  6 -1 X3 - 3 X4 + 2 X6 - 1 X7
X1 =  9 +0 X3 - 1 X4 + 2 X6 - 1 X7
X2 =  2 +0 X3 - 1 X4 + 1 X6 - 1 X7
-----------------------------------
z =  11 +2 X3 - 1 X4 + 3 X6 - 2 X7
```

3. What can you conclude about this problem?

A problem that could cause grief for a computer implementation (small example found by Fay Ye):

Maximize $X_1 + X_2$

subject to

$1\ X_1 + 1\ X_2 \leq \ 1.5$

$1\ X_1 - 1\ X_2 \leq \ -1.5$

$X_1,\ X_2 \geq 0$

Phase 1 Problem:

Maximize $-x_0$

subject to

$1\ x_1 + 1\ x_2 \ - x_0 \leq 1.5$

$1\ x_1 - 1\ x_2 \ - x_0 \leq -1.5$

$x_0,\ x_1,\ x_2 \geq 0$

Initial dictionary:

$x_3 = 1.5 - 1\ x_1 - 1\ x_2 + x_0$

$x_4 = -1.5 - 1\ x_1 + 1\ x_2 + x_0$

$z = 0.0 - x_0$

Initial dictionary:

$x_3 = \ \ 1.5 - 1\ x_1 - 1\ x_2 + x_0$

$x_4 = -1.5 - 1\ x_1 + 1\ x_2 + x_0$ (*)

\-----------------------------

$z = \ \ 0.0 \qquad\qquad - x_0$

Pivot to make feasible:

X5 enters. X4 leaves. z = 0.0

X3 = 3.0+ 0.0 X1 - 2.0 X2 + 1.0 X4

X0 = 1.5+ 1.0 X1 - 1.0 X2 + 1.0 X4

\-------------------------------------

z = -1.5- 1.0 X1 + 1.0 X2 - 1.0 X4

```
X3 = 3.0+ 0.0 X1 - 2.0 X2 + 1.0 X4
X0 = 1.5+ 1.0 X1 - 1.0 X2 + 1.0 X4
-------------------------------------
z = -1.5- 1.0 X1 + 1.0 X2 - 1.0 X4
```

X2 enters. X3 leaves. z = -1.5
After 1 pivot:

```
X2 = 1.5 + 0.0 X1 - 0.5 X3 + 0.5 X4
X0 = 0.0 + 1.0 X1 + 0.5 X3 + 0.5 X4
------------------------------------
z =  0.0 - 1.0 X1 - 0.5 X3 - 0.5 X4
```

The optimal solution: 0
X1= 0, X2= 1.5, X3= 0, X4= 0

Problem: X0 is still in the basis!

```
X2 = 1.5 + 0.0 X1 - 0.5 X3 + 0.5 X4
X0 = 0.0 + 1.0 X1 + 0.5 X3 + 0.5 X4
-----------------------------------
z =  0.0 - 1.0 X1 - 0.5 X3 - 0.5 X4
```

What happens if you leave z there?

Use any variable with non-zero coefficient in the X0 row to pivot it out.

Problem: X0 is still in the basis!

```
X2 = 1.5 + 0.0 X1 - 0.5 X3 + 0.5 X4
X0 = 0.0 + 1.0 X1 + 0.5 X3 + 0.5 X4
------------------------------------
z =  0.0 - 1.0 X1 - 0.5 X3 - 0.5 X4
```

Use any variable with non-zero coefficient in the X0 row to pivot it out. X1 enters. X5 leaves.

```
X2 =  1.5 - 0.5 X3 + 0.5 X4 + 0.0 X0
X1 = -0.0 - 0.5 X3 - 0.5 X4 + 1.0 X0
---------------------------------------
z =   0.0 + 0.0 X3 + 0.0 X4 - 1.0 X0
```

X2 = 1.5 - 0.5 X3 + 0.5 X4 + 0.0 X5
X1 = 0.0 - 0.5 X3 - 0.5 X4 + 1.0 X5
----
z = 0.0 + 0.0 X3 + 0.0 X4 - 1.0 X5

Recall: $z = x_1 + x_2$

The new objective function row is:
X1 + X2 =
1.5 - 0.5 X3 + 0.5 X4 + 0.0 X5
0.0 - 0.5 X3 - 0.5 X4 + 1.0 X5
==============================
1.5 - 1.0 X3 + 0 X4

Finishing the problem (Phase 2):

The initial dictionary:

```
X2 = 1.5 - 0.5 X3 + 0.5 X4
X1 = 0.0 - 0.5 X3 - 0.5 X4
------------------------------
z =  1.5 - 1.0 X3 + 0.0 X4
```

The optimal solution: 1.5

X1= 0.0, X2= 1.5, X3=0.0, X4=0.0