Manufacturing Microwaves

A company produces both small and large microwave ovens.

Machine time: 4 hours for small ovens and 3 hours for large ones. A total of 100 machine hours are available per day.

Assembly time: 2 hours for small ovens and 3 hours for large ones. There are 48 hours of assembly time available per day.

Market potential: 15 per day for small ovens and 20 per day for large ones.

The marketing department wants the company to produce at least 10 products per day, in any combination.

Each small oven contributes $2000 to profit and each large oven contributes $2500 to profit.

What is the LP problem?

How many small and large ovens should be produced per day to maximize profit?

x_1: Number of small microwaves to manufacture.

x_2: Number of large microwaves to manufacture.

Product	Machine Time	Assembly Time	Market Potential	Profit
Small	4	2	≤ 15	2000
Large	3	3	≤ 20	2500
Total	≤ 100	<= 48	≥ 10	

Maximize 2000 X1 + 2500 X2
subject to

$$
\begin{aligned}
4 X_1 + 3 X_2 &\le 100 \\
2 X_1 + 3 X_2 &\le 48 \\
X_1 &\le 15 \\
X_2 &\le 20
\end{aligned}
$$

And:

$$X_1 + X_2 \ge 10$$

Which in standard form is:

$$-X_1 - X_2 \le -10$$

$$X_1, X_2 \ge 0$$

Considering the units:

S= number of small ovens

L= number of large ovens

mach = number of hours of machine time

asmb= number of hours of assembly time

spot= pot for small ovens

lpot= pot for large ovens

mreq = minimum number of required ovens

Maximize

(profit/S)(S) + (profit/L)(L)

subject to

(mach/S)(S)+ (mach/L)(L) ≤ mach

(asmb/S)(S)+ (asmb/L)(L) ≤ asmb

(spot/S)(S)+ (spot/L)(L) ≤ spot

(lpot/S)(S)+ (lpot/L)(L) ≤ lpot

(mreq/S)(S)+ (mreq/L)(L) ≥ mreq

THE PRIMAL:What are dual units?

Maximize (profit/S)(S) + (profit/L) (L)

subject to

(mach/S)(S)+ (mach/L) (L) ≤ mach

(asmb/S)(S)+ (asmb/L) (L) ≤ asmb

(spot/S)(S)+ (spot/L) (L) ≤ spot

(lpot/S)(S)+ (lpot/L) (L) ≤ lpot

(mreq/S)(S)+ (mreq/L) (L) ≥ mreq

The dual problem:

Minimize

$$100\ Y_1 + 48\ Y_2 + 15\ Y_3 + 20\ Y_4 - 10\ Y_5$$

subject to

$$4\ Y_1 + 2\ Y_2 + Y_3 - Y_5 \geq 2000$$

$$3\ Y_1 + 3\ Y_2 + Y_4 - Y_5 \geq 2500$$

$$Y_1,\ Y_2,\ Y_3,\ Y_4,\ Y_5 \geq 0$$

The units for the dual:

Y_1: profit/mach

Y_2: profit/asmb

Y_3: profit/spot

Y_4: profit/lpot

Y_5: profit/(mreq decreasing)

Solving the primal problem:

The initial dictionary:

```
X3 = 100    - 4 X1    - 3 X2
X4 =  48    - 2 X1    - 3 X2
X5 =  15    - 1 X1    + 0 X2
X6 =  20    + 0 X1    - 1 X2
X7 = -10    + 1 X1    + 1 X2
----------------------------
z  =   0 +2000 X1 +2500 X2
```

Initial tableau:

```
   X1 X2 X3 X4 X5 X6 X7
[   4  3  1  0  0  0  0 ] [X1]   [100]
[   2  3  0  1  0  0  0 ] [X2]   [ 48]
[   1  0  0  0  1  0  0 ] [X3] =[ 15]
[   0  1  0  0  0  1  0 ] [X4]   [ 20]
[  -1 -1  0  0  0  0  1 ] [X5]   [-10]
                          [X6]
                          [X7]
```

The final dictionary:

```
X3 =      22 +  1    X4 +  2    X5
X2 =       6 -  0.33 X4 +  0.67 X5
X7 =      11 -  0.33 X4 -  0.33 X5
X6 =      14 +  0.33 X4 -  0.67 X5
X1 =      15 +  0    X4 -  1    X5
--------------------------------
z  =   45000 -833.33 X4 -333.33 X5
```

The optimal solution: 45000

The dual solution:

Y1= 0, Y2= 833.33, Y3= 333.33, Y4=0, Y5=0

The optimal strategy is to produce 15 small machines and 6 large machines.

The dual solution:

$Y_1 = 0, Y_2 = 833.33, Y_3 = 333.33, Y_4 = 0, Y_5 = 0$

Recall the units:

Y_2: profit/assembly

Y_3: profit/spot

From this, we can see that increasing the assembly hours by one unit increases profit by $833.33 and if there was potential to increase the number of small machines that could be sold by one unit, it would increase the profit by $333.33.

The corresponding tableau:

```
[0 0 1 -1.00 -2.00  0  0 ] [X1]   [ 22]
[0 1 0  0.33 -0.67  0  0 ] [X2]   [  6]
[0 0 0  0.33  0.33  0  1 ] [X3] = [ 11]
[0 0 0 -0.33  0.67  1  0 ] [X4]   [ 14]
[1 0 0  0.00  1.00  0  0 ] [X5]   [ 15]
                           [X6]
                           [X7]
```

How does the optimal strategy change:

t1 extra machine hours

t2 extra assembly hours

t3 more small machines can be sold

t4 more large machines can be sold

t5 decrease to min number of machines produced.

```
Initial tableau:
   X1 X2 X3 X4 X5 X6 X7
[  4  3  1  0  0  0  0 ] [X1]   [100]
[  2  3  0  1  0  0  0 ] [X2]   [ 48]
[  1  0  0  0  1  0  0 ] [X3] =[ 15]
[  0  1  0  0  0  1  0 ] [X4]   [ 20]
[ -1 -1  0  0  0  0  1 ] [X5]   [-10]
                         [X6]
                         [X7]
```

Final basis: X3, X2, X7, X6, X1

$$
\begin{array}{ccccccc} X1 & X2 & X3 & X4 & X5 & X6 & X7 \end{array}
$$

$$
\begin{bmatrix} 4 & 3 & 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \\ X6 \\ X7 \end{bmatrix}
=
\begin{bmatrix} 100 \\ 48 \\ 15 \\ 20 \\ -10 \end{bmatrix}
$$

The basis matrix B:

$$
\begin{array}{ccccc} X3 & X2 & X7 & X6 & X1 \end{array}
$$

$$
\begin{bmatrix} 1 & 3 & 0 & 0 & 4 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 \end{bmatrix}
\begin{bmatrix} X3 \\ X2 \\ X7 \\ X6 \\ X1 \end{bmatrix}
=
\begin{bmatrix} 100 \\ 48 \\ 15 \\ 20 \\ -10 \end{bmatrix}
$$

I computed the inverse of B using the following Matlab commands:

```
B= [1  3   0  0  4  ;
    0  3   0  0  2  ;
    0  0   0  0  1  ;
    0  1   0  1  0  ;
    0 -1   1  0 -1]
invB= inv(B)
```

The inverse is:

1	-1.00	-2.00	0	0
0	0.33	-0.67	0	0
0	0.33	0.33	0	1
0	-0.33	0.67	1	0
0	0.00	1.00	0	0

The inverse is:

1	-1.00	-2.00	0	0
0	0.33	-0.67	0	0
0	0.33	0.33	0	1
0	-0.33	0.67	1	0
0	0.00	1.00	0	0

The final dictionary:

X3	=	22	+	1	X4	+	2	X5
X2	=	6	-	0.33	X4	+	0.67	X5
X7	=	11	-	0.33	X4	-	0.33	X5
X6	=	14	+	0.33	X4	-	0.67	X5
X1	=	15	+	0	X4	-	1	X5

The inverse is hiding in here.

If the update to b is
[t1 t2 t3 t4 t5]^T and the ti's are not too big, the change to the optimal solution is:

```
[ 1   -1.00  -2.00  0    0 ]  [t1 ]
[ 0    0.33  -0.67  0    0 ]  [t2 ]
[ 0    0.33   0.33  0    1 ]  [t3 ]
[ 0   -0.33   0.67  1    0 ]  [t4 ]
[ 0    0.00   1.00  0    0 ]  [t5 ]
```

So the new solution would be:

```
X3= 22 +t1 - 1.00 t2 -2.00 t3
X2=  6     + 0.33 t2 -0.67 t3
X7= 20     + 0.33 t2 +0.33 t3 +   + t5
X6= 14     - 0.33 t2 +0.67 t3 + t4
X1= 15     + 0.00 t2 +1.00 t3
```

In terms of the original variables:

X1= 15 + 0.00 t2 + 1.00 t3

X2= 6 + 0.33 t2 - 0.67 t3

1. Increase assembly hours by 3(t2=3):

X1= 15, X2= 7,

profit increases by 833.33 * 3 = 2500

2. Increase number of small units that can be sold by 3 (t3=3):

X1= 18, X2= 4,

profit increases by 333.33 * 3 = 1000

3. Increase both by 3 (t2=3, and t3=3):

X1= 18, X2= 5,

profit increases by 833.33*3 + 333.33*3 = 3500

If there is too much of an increase to t2 (assembly hours), there is no longer any benefit:

There are two equations that impose a feasibility constraint on t2:

(1) $X3 = 22 + 1\ t1 - 1.00\ t2 - 2.00\ t3 + 0\ t4 + 0\ t5$

(2) $X6 = 14 + 0\ t1 - 0.33\ t2 + 0.67\ t3 + 1\ t4 + 0\ t5$

(1) X3= 22 +1 t1 -1.00 t2 -2.00 t3
(2) X6= 14 +0 t1 -0.33 t2 +0.67 t3+ t4
The constraints are:
(1) t2 <= 22 (tightest)
(2) t2 <= 42

Consider (1):
X3= 22 + 1 t1 - 1.00 t2 - 2.00 t3
If t2 > 22, then constraint 1 is violated:

(at most 100 machine hours)
$4 X1 + 3 X2 \leq 100$

X1= 15, X2 > 6 + 22/3

If there is too much of an increase to t3 (potential for small machines), there is no longer any benefit:

There are two equations that impose a feasibility constraint on t3:

(1)X2= 6 + 0 t1 + 0.33 t2 - 0.67 t3 + 0 t4 + 0 t5

(2) X3= 22 + 1 t1 - 1.00 t2 - 2.00 t3 + 0 t4 + 0

(1)X2= 6 + 0 t1 + 0.33 t2 - 0.67 t3
(2)X3= 22 + 1 t1 - 1.00 t2 - 2.00 t3

The constraints are:

(1) t3 <= 9 (tightest)
(2) t3 <= 11

If t3 > 9, then the value of X2 would go negative.

This means that the basis is no longer valid: X2 would exit the basis, and no large machines would be made.