A company cannot spend anymore than 120 hours per week making toys and 160 hours per week finishing toys.

	Sell	Materials	Make (hours)	Finish (hours)	Max sold
Plane (X1)	$12	$4	3	1	30
Boat (X2)	$10	$3	1	2	--

Maximize 8 X1 + 7 X2
subject to
$3\ X1 + 1\ X2 \leq 120$
$1\ X1 + 2\ X2 \leq 160$
$1\ X1 + 0\ X2 \leq 30$
$X1,\ X2 \geq 0$

Solve for y using complementary slackness then give an economic interpretation of the dual variables.

Solution: (16, 72)

After 3 pivots:

$$X2 = 72 + 0.20\ X3 - 0.60\ X4$$
$$X5 = 14 + 0.40\ X3 - 0.20\ X4$$
$$X1 = 16 - 0.40\ X3 + 0.20\ X4$$

$$z = 632 - 1.80\ X3 - 2.60\ X4$$

One more making hour: $1.80.
One more finishing hour: $2.60
It would not help to be able to sell more planes.

What are B and B^{-1}?

Maximize 8 X1 + 7 X2

subject to

3 X1 + 1 X2 ≤ 120

1 X1 + 2 X2 ≤ 160

1 X1 + 0 X2 ≤ 30

X1, X2 ≥ 0

Use last dictionary to get B^{-1}:

X2= 72 + 0.2 X3 − 0.6 X4

X5= 14 + 0.4 X3 − 0.2 X4

X1= 16 − 0.4 X3 + 0.2 X4

$$B = \begin{array}{c} \begin{matrix} X2 & X5 & X1 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$B^{-1} = \begin{array}{c} \begin{matrix} X3 & X4 & X5 \end{matrix} \\ \begin{bmatrix} -0.2 & 0.6 & 0 \\ -0.4 & 0.2 & 1 \\ 0.4 & -0.2 & 0 \end{bmatrix} \end{array}$$

```
Maximize 8 X1 + 7 X2
subject to
3 X1 + 1 X2 ≤  120 + t1  // making
1 X1 + 2 X2 ≤  160 + t2  // finishing
1 X1 + 0 X2 ≤   30 + t3  // max planes
X1, X2 ≥ 0
```

$$\begin{bmatrix} X2 \\ X5 \\ X1 \end{bmatrix} = B^{-1} b = \begin{bmatrix} -0.2 & 0.6 & 0 \\ -0.4 & 0.2 & 1 \\ 0.4 & -0.2 & 0 \end{bmatrix} \begin{bmatrix} 120 + t1 \\ 160 + t2 \\ 30 + t3 \end{bmatrix}$$

```
X2= 72 - 0.2 t1 + 0.6 t2 // boats
X5= 14 - 0.4 t1 + 0.2 t2 + t3
X1= 16 + 0.4 t1 - 0.2 t2 //planes
```

Announcements:

CSC 545 only:

Survey Paper- due on Fri. Oct. 17 at 11:55pm.

Accepted until Tues. Oct. 21 at 11:55pm with a 10% late penalty.

Make sure you use 12 point font, at least 1.5 spacing and your .pdf file is uploaded.

Both classes:

Programming project 2:

Due at 11:55pm on Fri. Oct. 24.

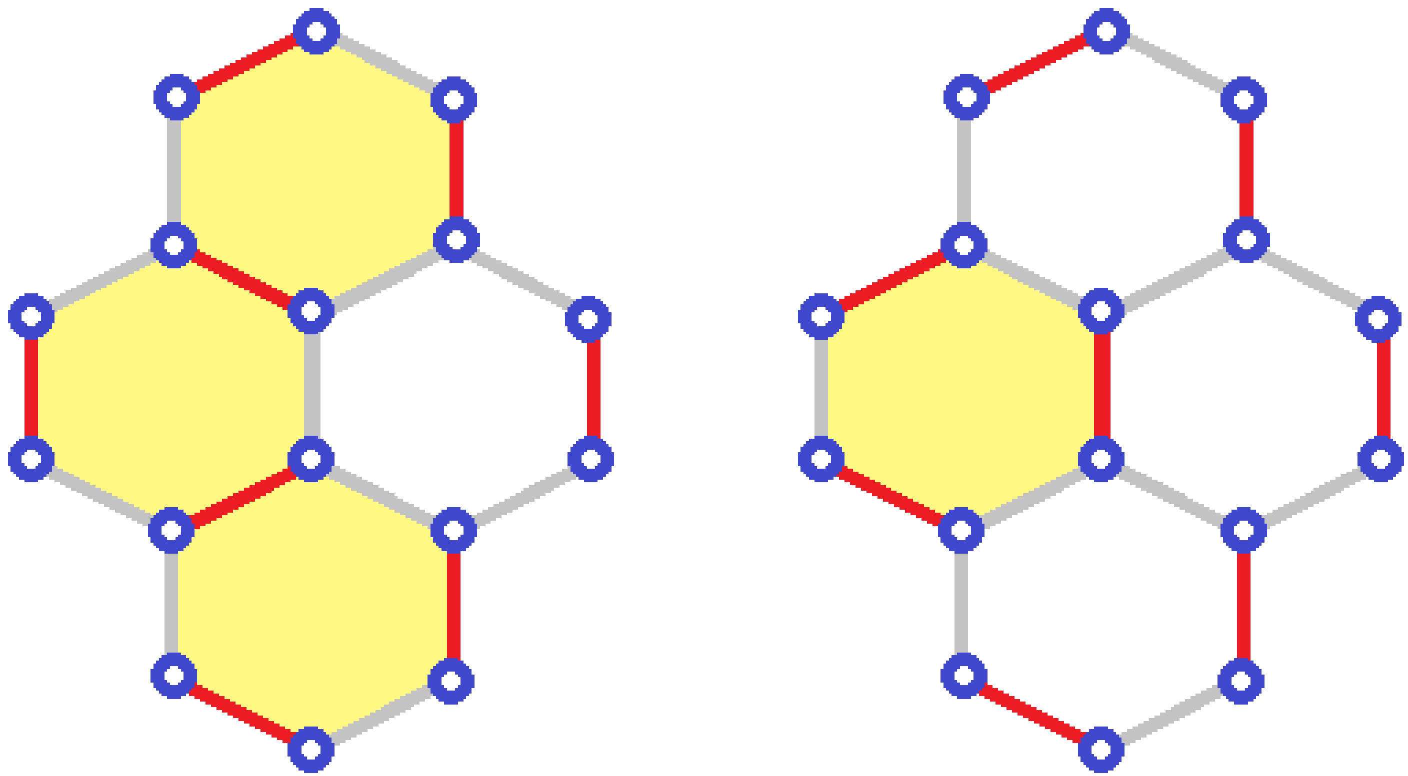
Accepted with 10% penalty until Tues. Oct. 28 at 11:55pm.

Two applications of linear programming to chemistry:

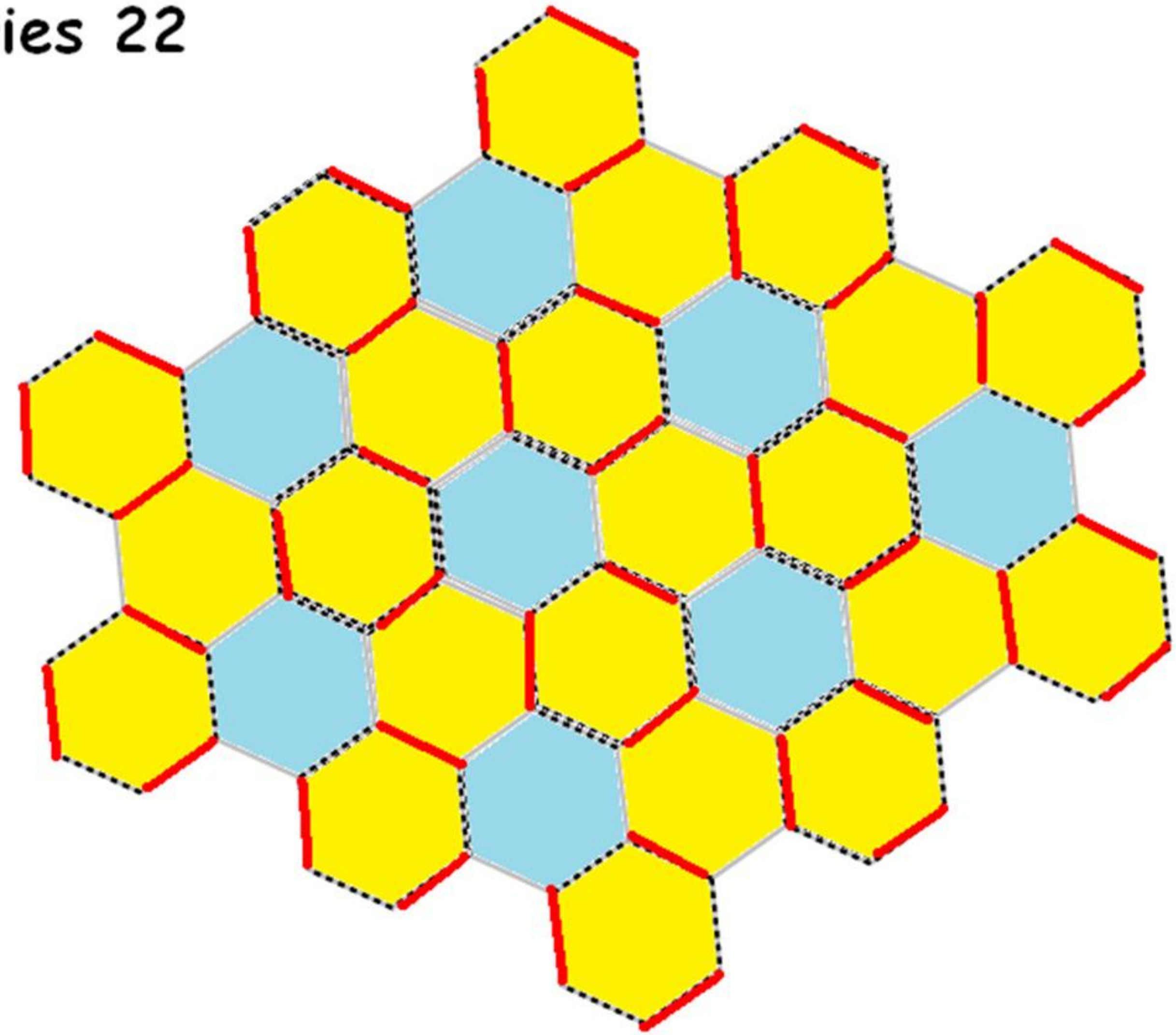
Finding the Clar number and the Fries number of a benzenoid in polynomial time using a LP.

The desired solutions are integer but it has been proven that the basic feasible solutions are all integral so integer programming tactics are not required.

Matching: collection of disjoint edges.
Benzenoid hexagon: hexagon with 3 matching edges.
Fries number: maximum over all perfect matchings of the number of benzenoid hexagons.



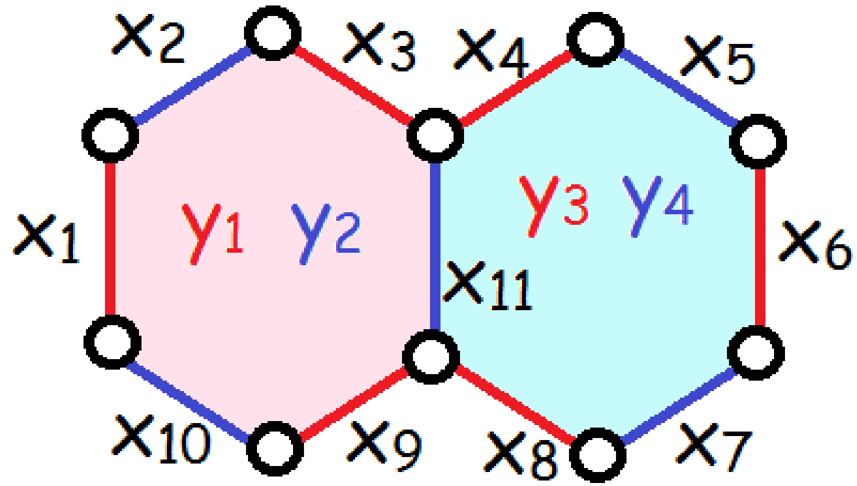
Fries 22



Proof that the solutions to the LP are integral:

```
@ARTICLE{LP_Fries,
author = {Hernan G. Abeledo and Gary W. Atkinson},
title = {Polyhedral Combinatorics of Benzenoid Problems},
journal = {Lect. Notes Comput. Sci},
year = {1998},
volume = {1412},
pages = {202--212}
}
```

One variables x_e for each edge e.
Two variables per hexagon.
All variables are constrained to be between 0 and 1.



y_1= 1 pink hex. is benzenoid because of red edges.
y_2= 1 pink hex. is benzenoid because of blue edges.
y_3= 1 aqua hex. is benzenoid because of red edges.
y_4= 1 aqua hex. is benzenoid because of blue edges.

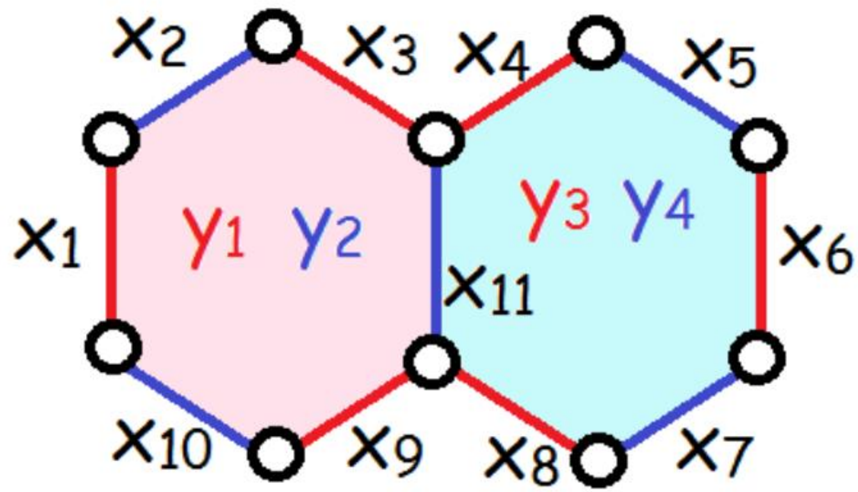
Maximize $y_1 + y_2 + y_3 + y_4$

Maximize $y_1 + y_2 + y_3 + y_4$

To get a perfect matching:

For each vertex, the number of edges incident sums to 1:

$x_1 + x_2 = 1$
$x_2 + x_3 = 1$
$x_3 + x_4 + x_{11} = 1$
…



To ensure benzenoid hexagons:

Red edges of pink:

$x_1 - y_1 \geq 0$
$x_3 - y_1 \geq 0$
$x_9 - y_1 \geq 0$

Blue edges of pink:

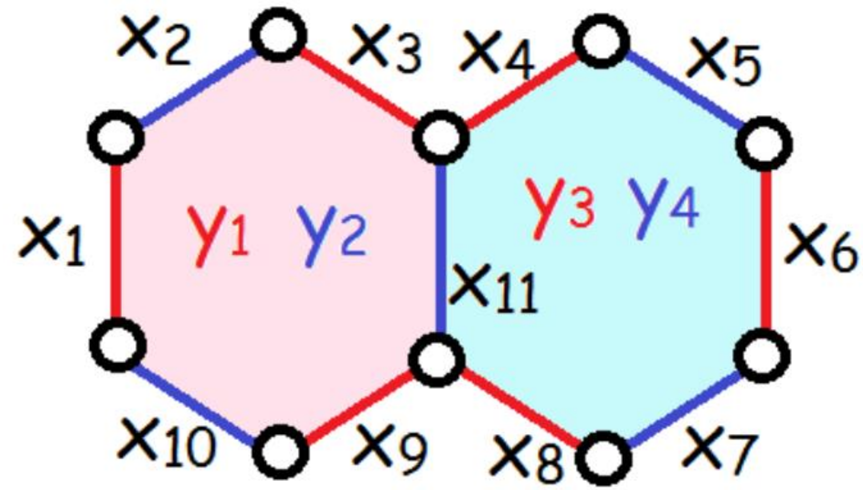
$x_2 - y_2 \geq 0$
$x_{10} - y_2 \geq 0$
$x_{11} - y_2 \geq 0$

Red edges of aqua:

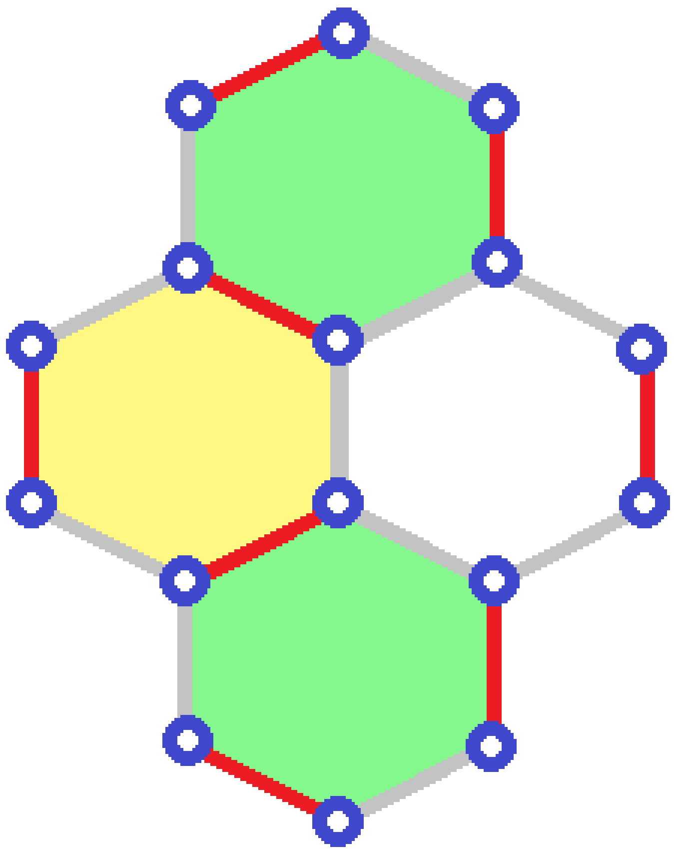
$x_4 - y_3 \geq 0$
$x_6 - y_3 \geq 0$
$x_8 - y_3 \geq 0$

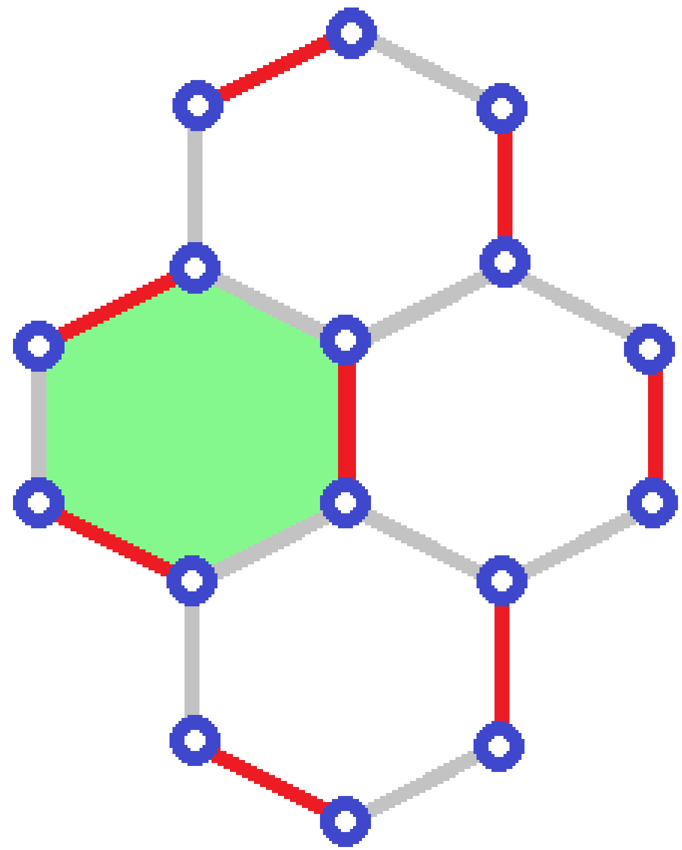
Blue edges of aqua:

$x_5 - y_4 \geq 0$
$x_7 - y_4 \geq 0$
$x_{11} - y_4 \geq 0$

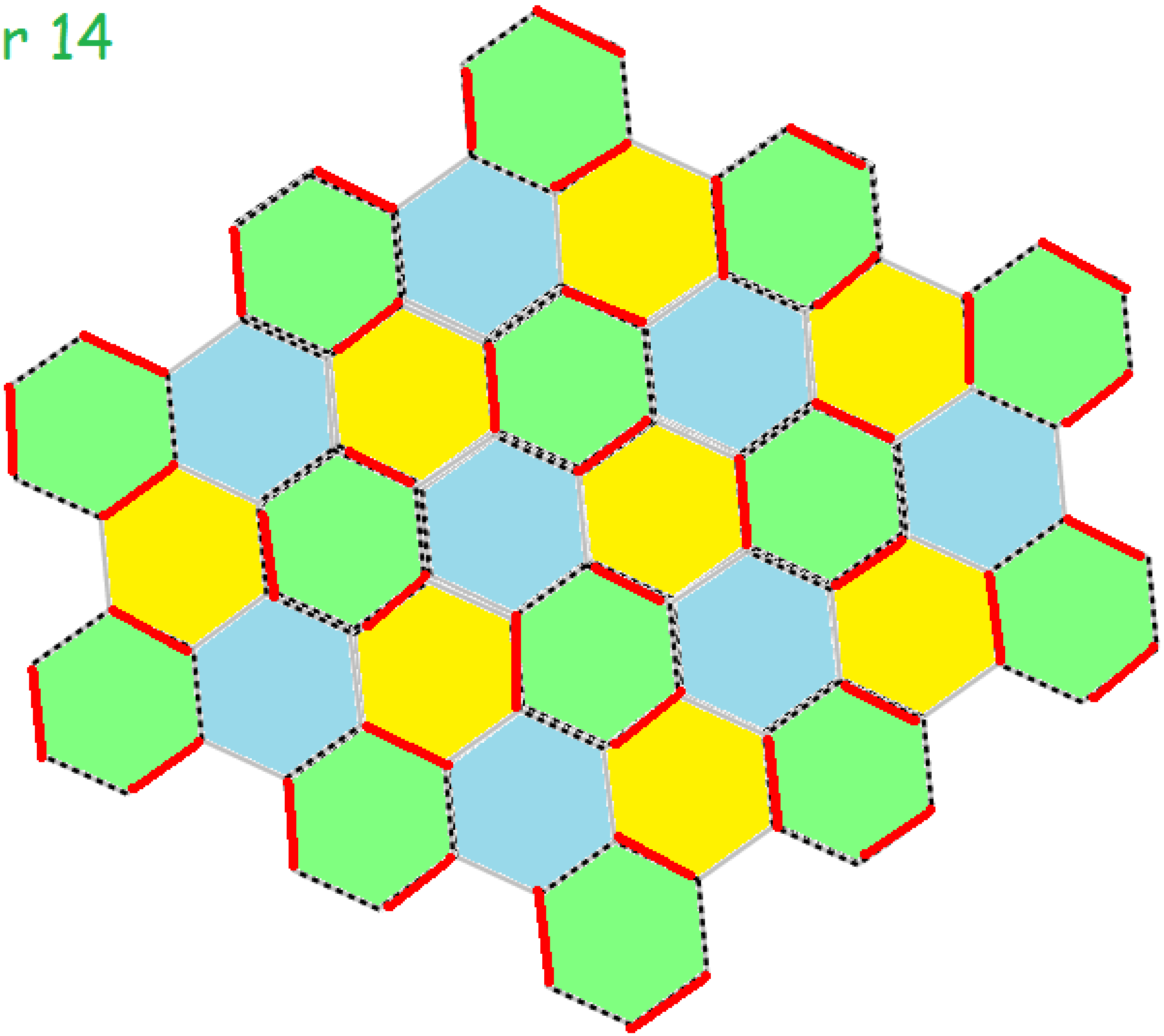


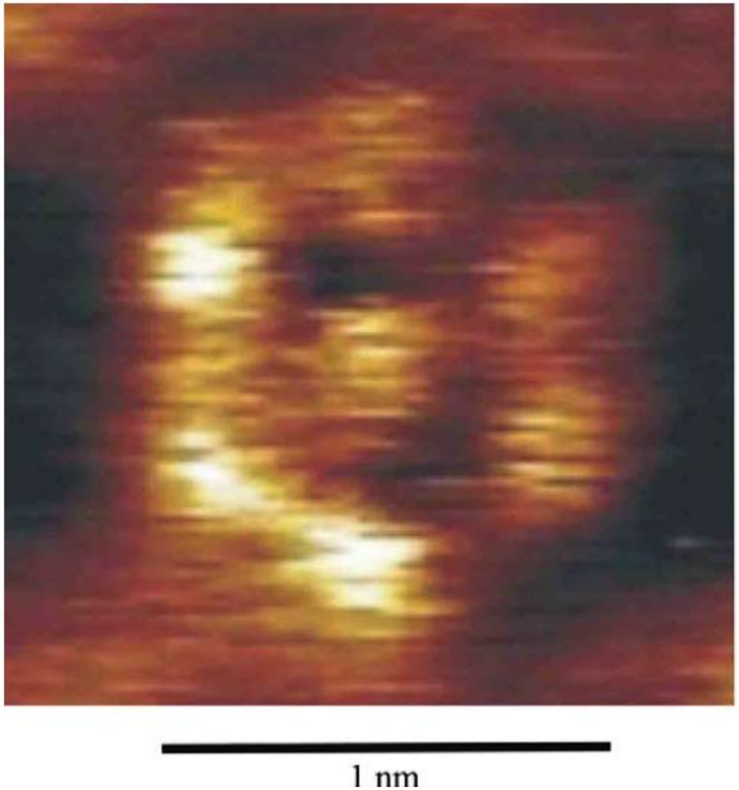
Clar number: maximum over all perfect matchings of the number of independent benzenoid hexagons.

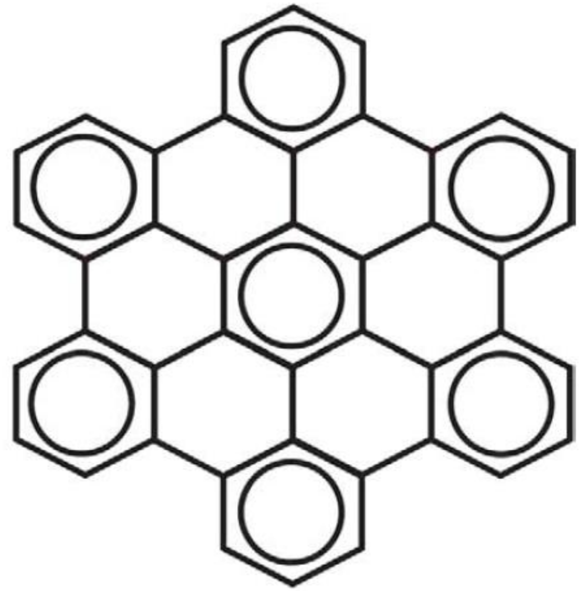




Clar 14



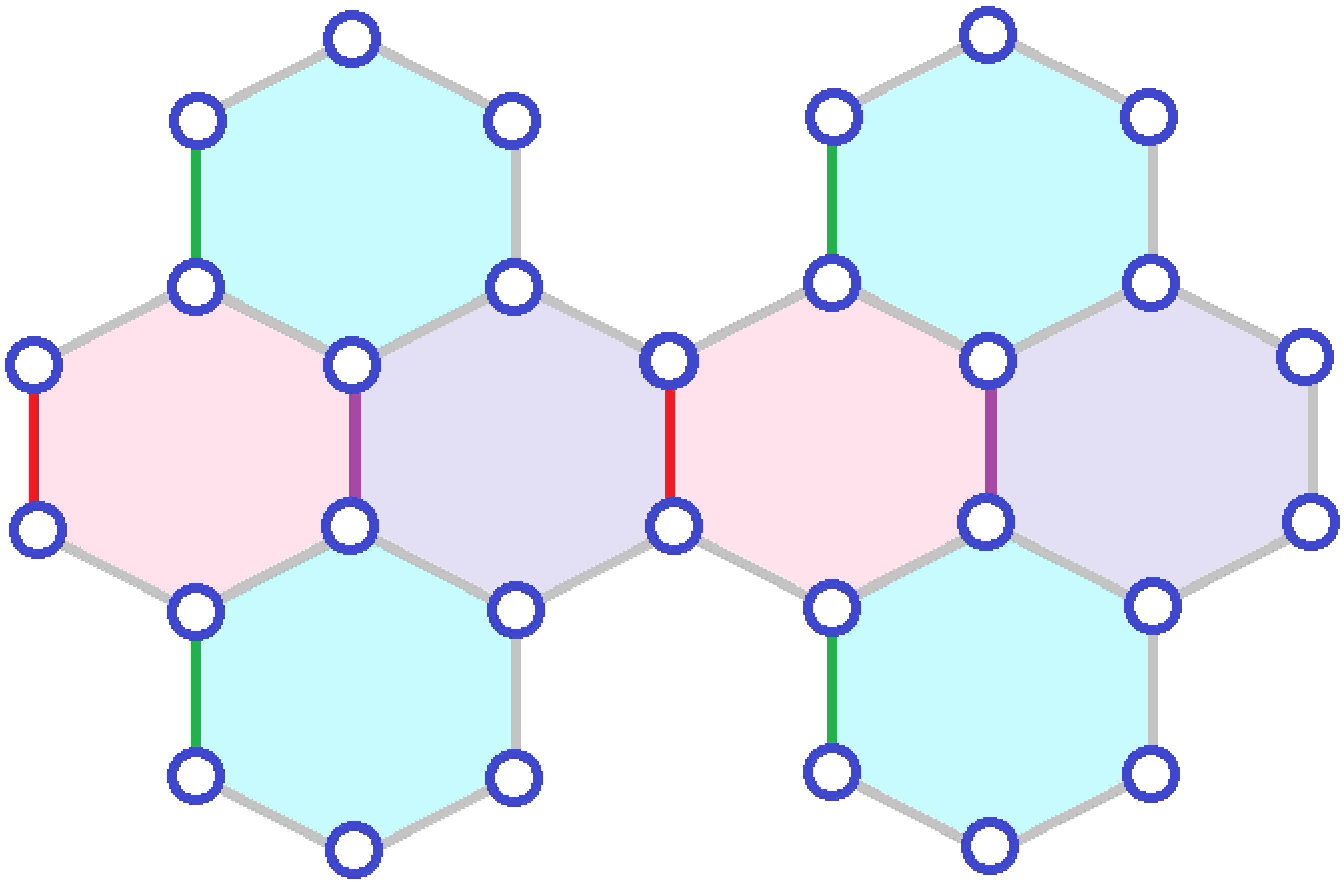




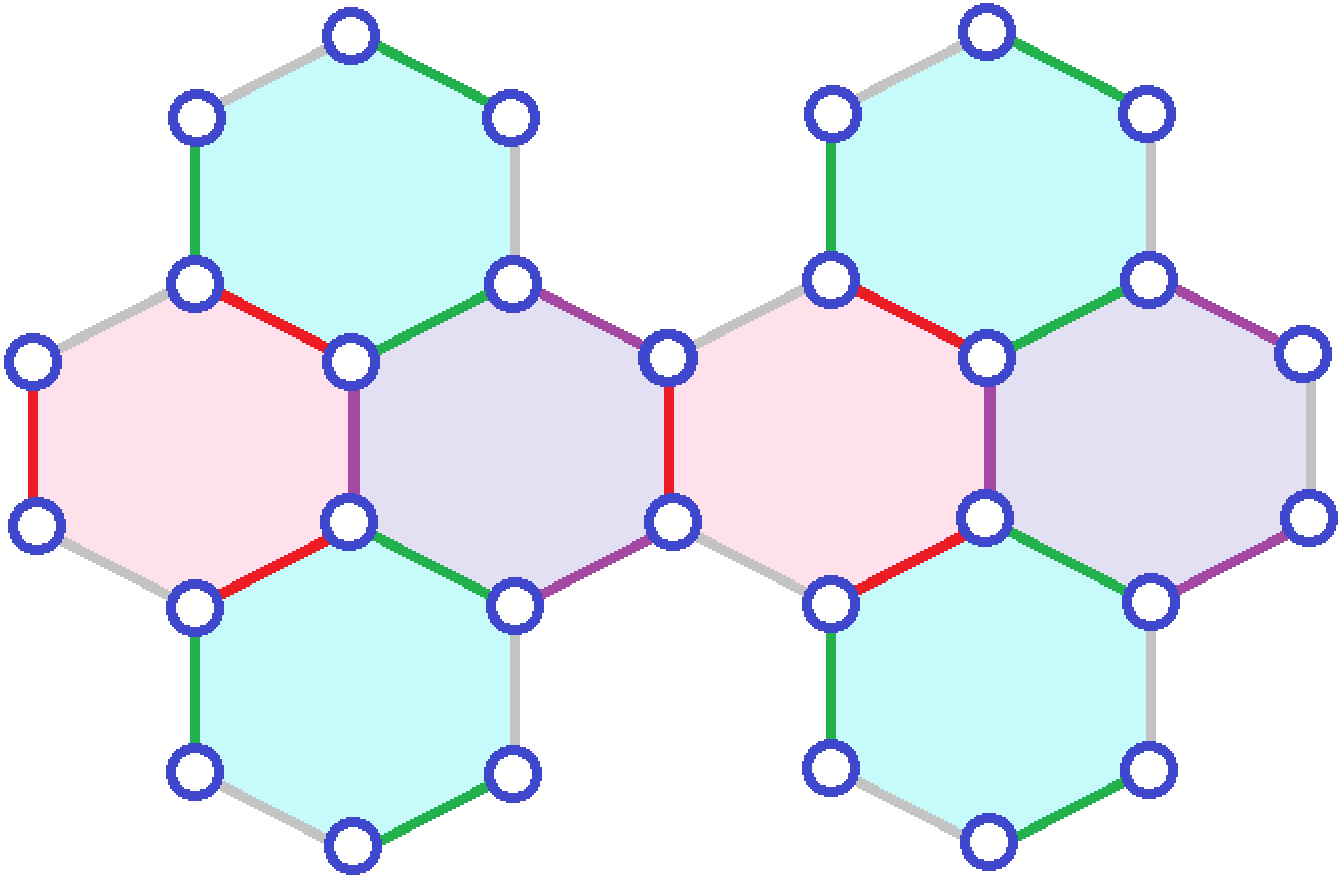
Scanning Tunnelling Microscopy
image of hexabenzocoronene

from
I Gutman, Ž Tomović, K Müllen, J P Rabe,
Chemical Physics Letters 397 (2004) 412–416

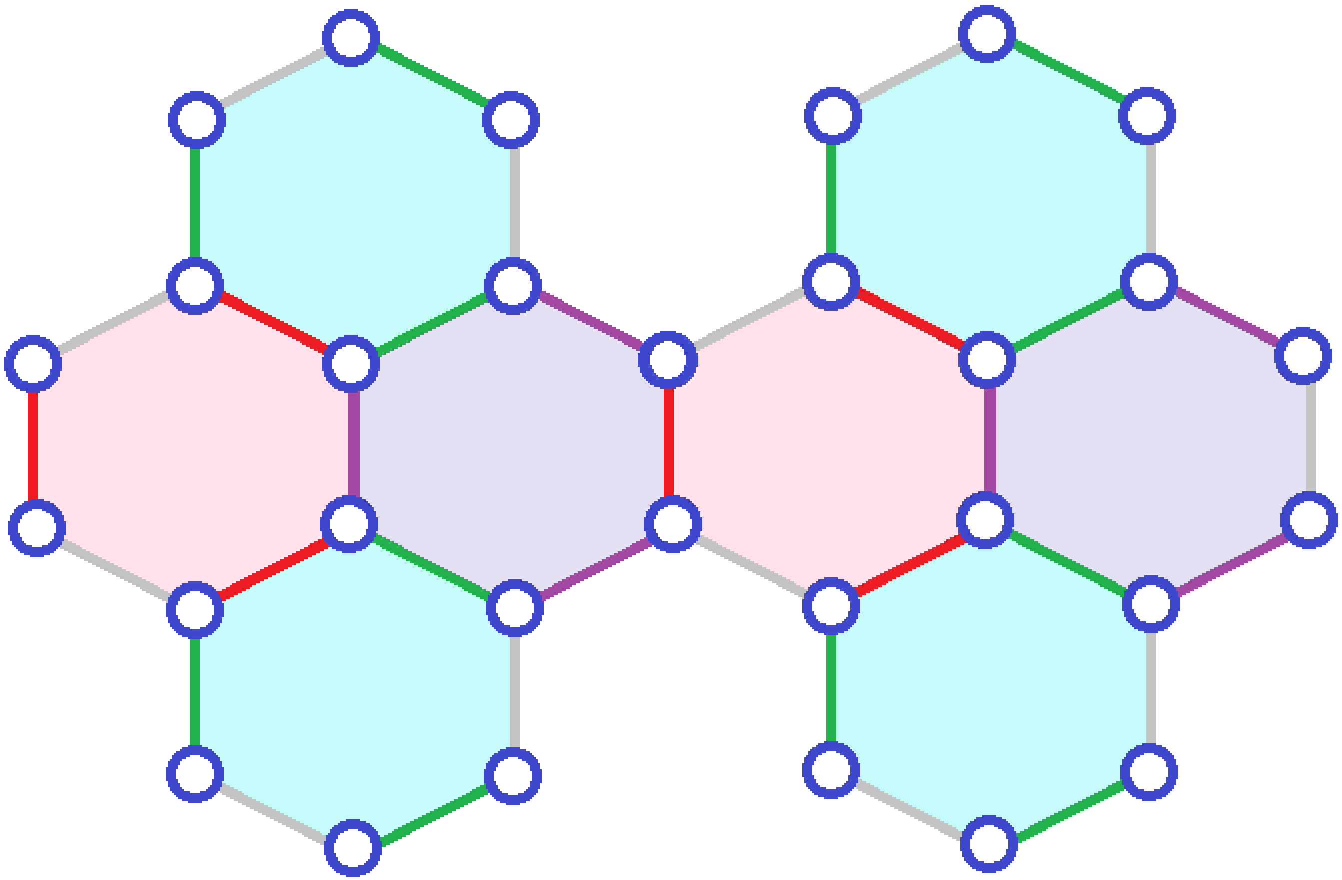
LP for the Clar number: The LH edge of each hexagon is its canonical edge.



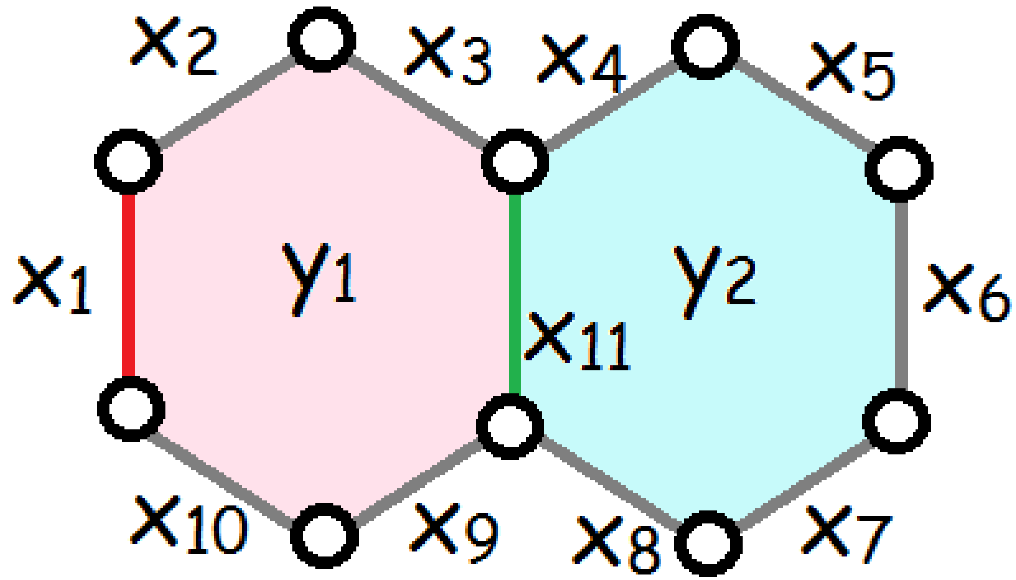
The corresponding matching is the canonical matching for the hexagon.



If a hexagon is Clar, assume it realizes its canonical matching.



Two variables x_e, z_e for each edge e. One variable y_h for each hexagon h. All variables are constrained to be between 0 and 1.



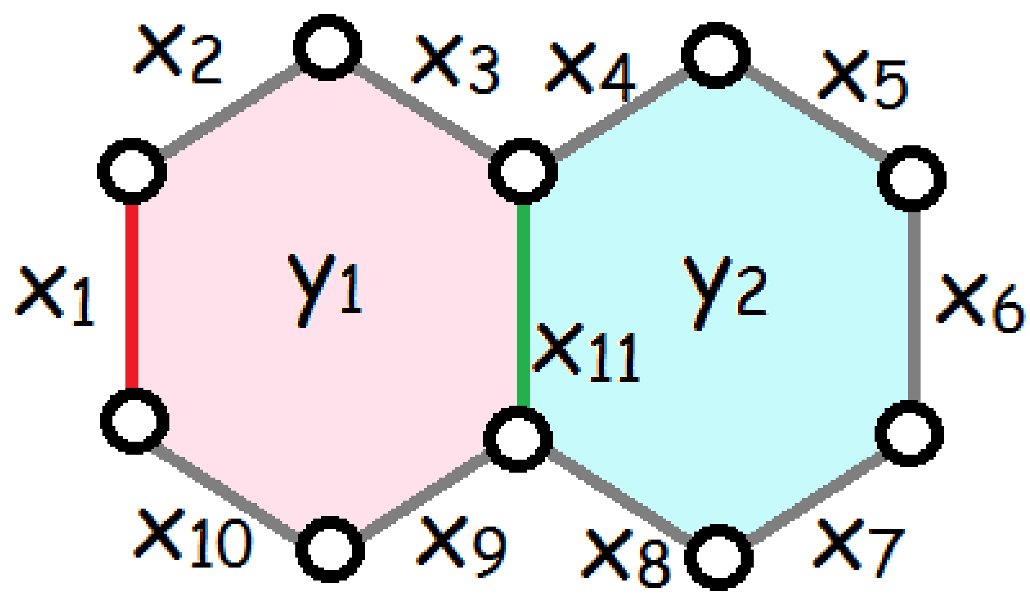
z_e= 1 if e is a perfect matching edge.
x_e = 1 if e is a matching edge not in a benzenoid hexagon.
y_h = 1 if h is an independent benzenoid hexagon and 0 otherwise.

Maximize $y_1 + y_2$

To get a perfect matching:

For each vertex, the number of edges incident sums to 1:

$z_1 + z_2 = 1$
$z_2 + z_3 = 1$
$z_3 + z_4 + z_{11} = 1$
...



To get the independent benzenoid hexagons:

For the pink hexagon:

$x_1 + y_1 - z_1 = 0$

$x_3 + y_1 - z_3 = 0$

$x_9 + y_1 - z_9 = 0$

If $y_1 = 1$ then

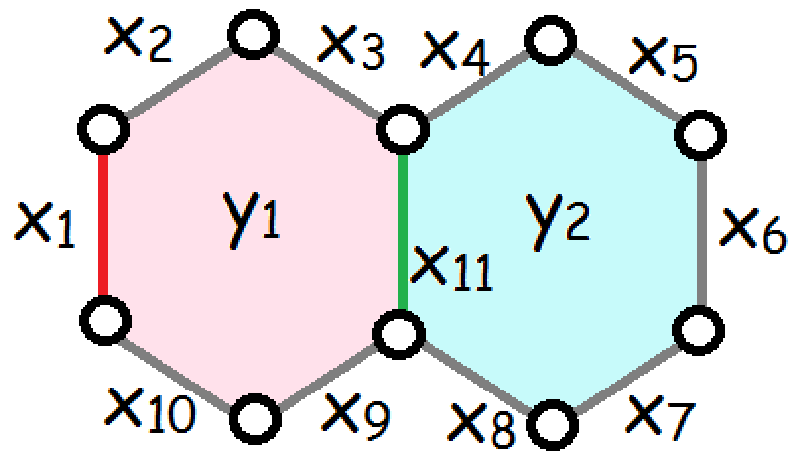
$z_1 = z_3 = z_9 = 1 \Longrightarrow$

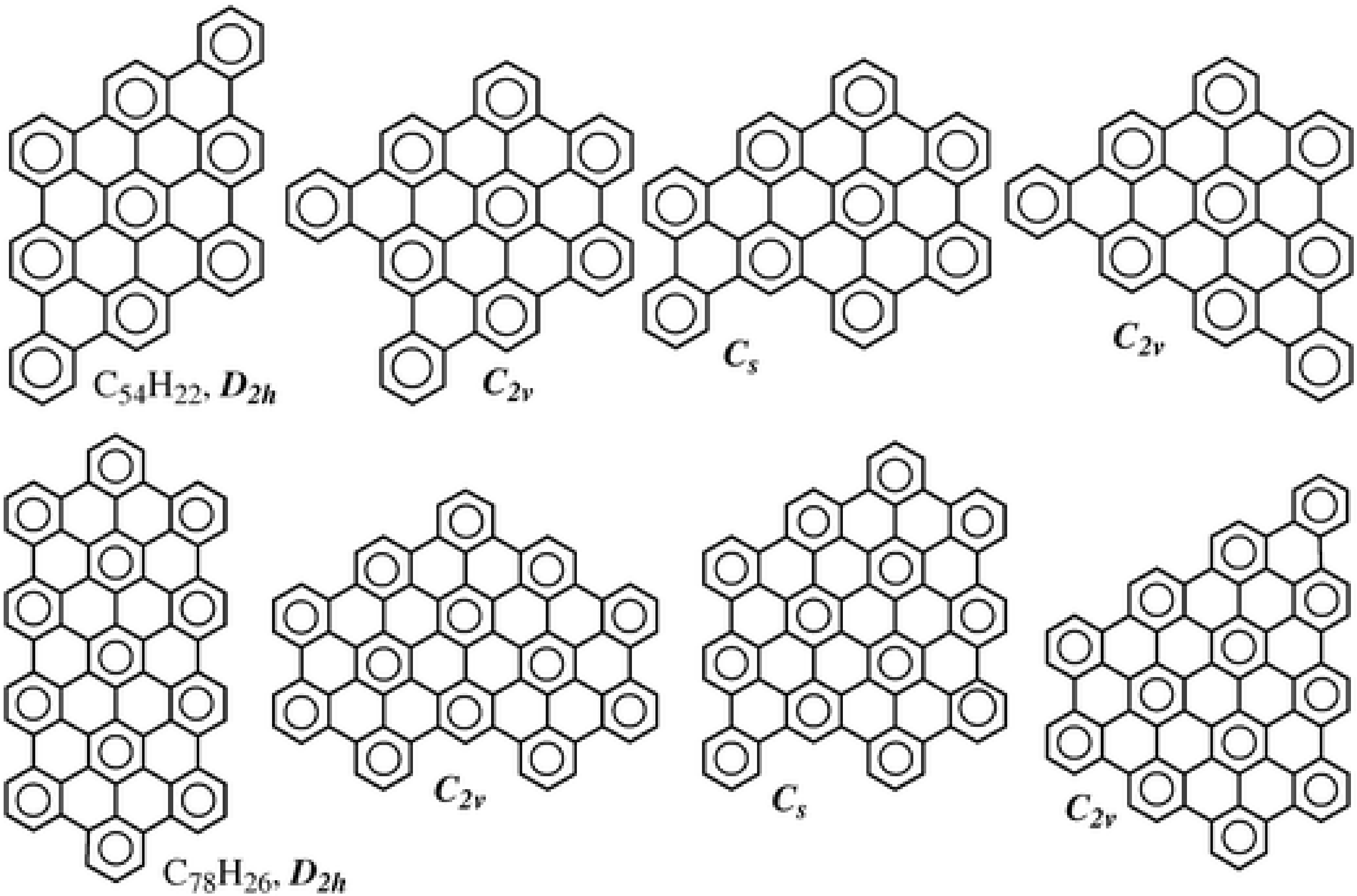
$z_2 = z_{11} = z_{10} = 0$

If $y_1 = 1$, $y_2 = 0$

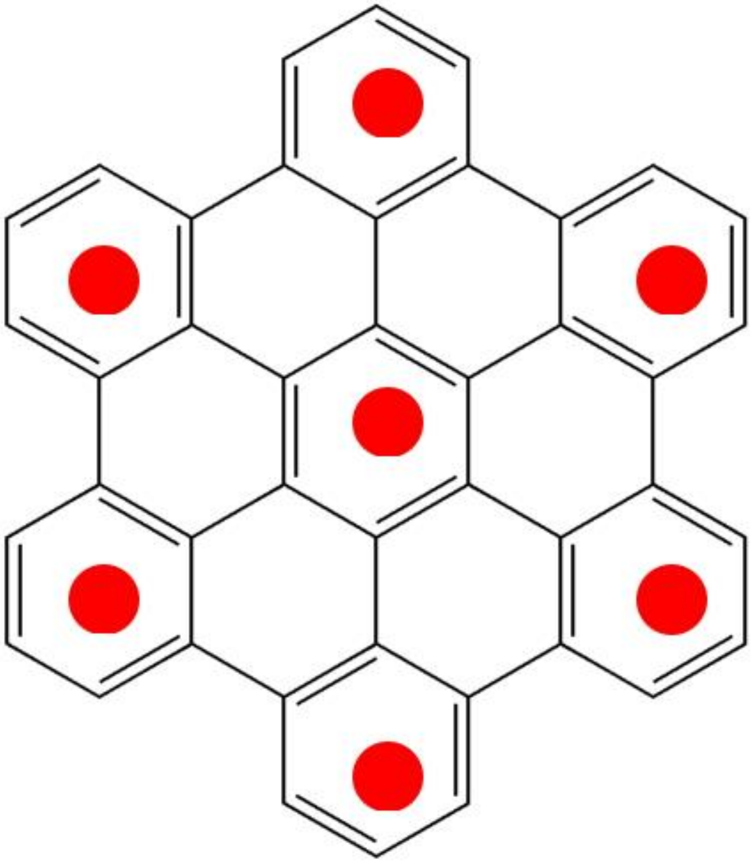
since

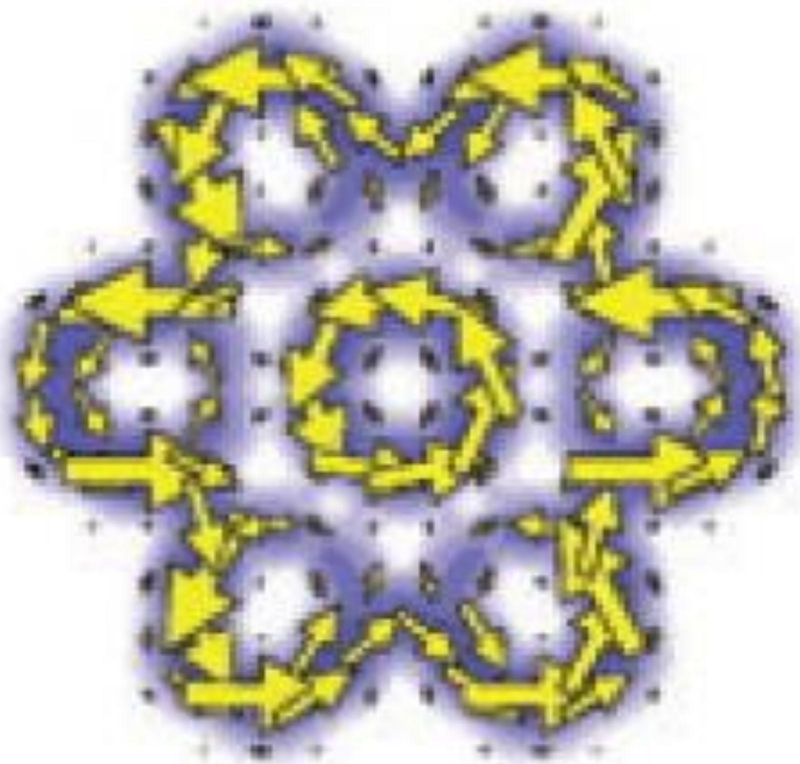
$x_{11} + y_2 - z_{11} = 0$





http://www.springerimages.com/Images/RSS/1-10.1007_978-94-007-1733-6_8-24





It's possible to find the Fries number and the Clar number using linear programming.

This is an example of a problem that is an integer programming problem where the integer solution magically appears when solving the linear programming problem.

http://www.javelin-tech.com/blog/2012/07/sketch-entities-splitting/magician-2/