CSC 445/545 Test #1

Fri. Oct. 6, 2000

1(a). [15] Set up the initial feasible dictionary for Phase 1 for the following problem:

Maximize $3x_1 - 2x_2 + 1x_3$

subject to

$$
\begin{array}{rcrcrcr}
1x_1 & + & 3x_2 & - & 2x_3 & \leq & -3 \\
3x_1 & - & 1x_2 & - & 1x_3 & \leq & -7 \\
-1x_1 & + & 3x_2 & + & 0x_3 & \leq & -5 \\
x_1, & & x_2, & & x_3 & \geq & 0
\end{array}
$$

(b) [10] The final dictionary at the end of Phase 1 is:

$$
\begin{array}{rcrcrcrcrcr}
x_4 & = & 36 & + & 10x_2 & + & 2x_5 & + & 5x_6 & - & 6x_0 \\
x_1 & = & 5 & + & 3x_2 & + & 0x_5 & + & 1x_6 & - & 1x_0 \\
x_3 & = & 22 & + & 8x_2 & + & 1x_5 & + & 3x_6 & - & 4x_0 \\
\hline
z & = & 0 & + & 0x_2 & + & 0x_5 & + & 0x_6 & - & 1x_0
\end{array}
$$

Set up the initial dictionary to start Phase 2.

2(a). [15] I ran the Simplex method on a problem. The program generated two pages of output. Unfortunately, I lost the second page. The last dictionary on the first page is given below. Continue applying the Simplex method until termination.

$$
\begin{array}{rcrcrlcrlcrl}
x_2 & = & 1 & + & 1 & x_1 & - & 1 & x_4 & + & 2 & x_5 \\
x_3 & = & 2 & - & 1 & x_1 & + & 0 & x_4 & - & 1 & x_5 \\
x_6 & = & 2 & - & 3 & x_1 & + & 2 & x_4 & - & 4 & x_5 \\
\hline
z & = & 10 & + & 1 & x_1 & - & 2 & x_4 & + & 0 & x_5
\end{array}
$$

(b) [10] What (if anything) can you say about the optimal solution to the dual problem given what you have done for part (a)?

3.(a) [25] Apply complementary slackness to determine if (5, 0, 4, 0) is an optimal solution to:

$$\text{Maximize} \quad 4x_1 + 5x_2 + 1x_3 + 1x_4$$

subject to

$$\begin{array}{rcrcrcrcl}
1x_1 & - & 1x_2 & - & 1x_3 & + & 3x_4 & \le & 1 \\
5x_1 & + & 3x_2 & + & 1x_3 & + & 4x_4 & \le & 55 \\
-1x_1 & + & 3x_2 & + & 2x_3 & - & 5x_4 & \le & 3 \\
x_1, & & x_2, & & x_3, & & x_4 & \ge & 0
\end{array}$$

4. Circle true or false for each question and justify your answer. To get marks for a question, the justification must be correct.

(a) [5] The Simplex Method with an appropriate pivoting rule leads to a fast algorithm that runs in polynomial time for all inputs.

True **False**

(b) [5] If a problem does not have non-negativity constraints for its variables, it is not possible to solve the problem using the Simplex Method.

True **False**

(c) [5] Two dictionaries for a problem which have the same basis variables can correspond to two different basic feasible solutions.

True **False**

(d) [5] All linear programming problems have an optimal solution, and furthermore, there must be an optimal solution that is a basic feasible solution.

True **False**

(e) [5] If the primal problem is unbounded, then the dual problem must be infeasible.

True **False**