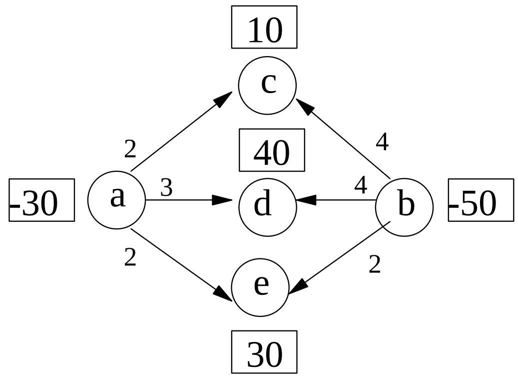
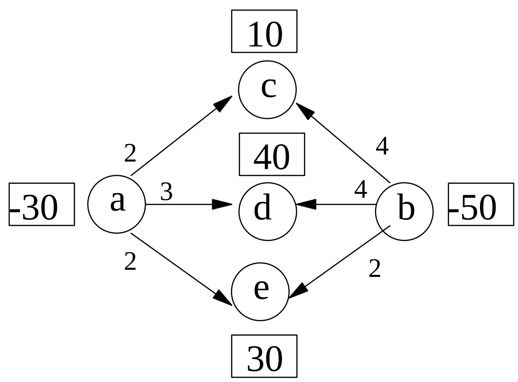
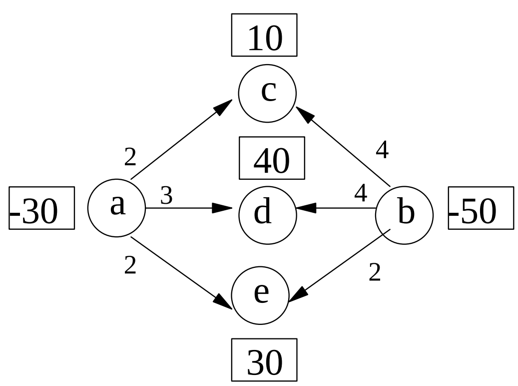
CSC 445/545 Test #2: Fri. Nov. 10, 2000

1. [25] Find an optimal solution to the transshipment problem where the network, the shipping costs and the demands are as shown below. You must show all your work for full marks. Start with arcs (a,e), (b,c) (b, d) and (b,e). **Compute fair prices starting at node a**.







2. Suppose Rita and Carl play a game where each hides either 5 cents or 10 cents. If they hide the same amount, Carl pays Rita the total amount hidden. Otherwise Rita pays Carl the total amount hidden.

(a) [5] What is the payoff matrix for Rita?

(b) [10] The problem I used as input to my program is:

Maximize $X3 - X4$

subject to

$$\begin{array}{rcrcrcrcr} -10\,X1 & + & 15\,X2 & + & X3 & - & X4 & \le & 0 \\ 15\,X1 & - & 20\,X2 & + & X3 & - & X4 & \le & 0 \\ X1 & + & X2 & & & & & \le & 1 \\ -X1 & - & X2 & & & & & \le & -1 \\ X1, & & X2, & & X3, & & X4 & \ge & 0 \end{array}$$

Explain why the solution to this problem gives Rita's optimal strategy.

(c) [5] The solution that I obtained for the LP problem from the previous page:

Maximize $X3 - X4$

subject to

$$\begin{array}{rcrcrcrcr} -10\,X1 & + & 15\,X2 & + & X3 & - & X4 & \le & 0 \\ 15\,X1 & - & 20\,X2 & + & X3 & - & X4 & \le & 0 \\ X1 & + & X2 & & & & & \le & 1 \\ -X1 & - & X2 & & & & & \le & -1 \\ X1, & & X2, & & X3, & & X4 & \ge & 0 \end{array}$$

is different from the solution in the back of the text. Check the 'prof' solution for correctness to determine who has the correct solution (the prof or the text). Explain what you are doing and show all your work.

Solution	Primal	Dual
Prof	(7/12, 5/12, 0, 5/12)	(7/12, 5/12, 0, 5/12)
Text	(2/3, 1/3, 0, 10/3)	(1/2, 1/2, 0, 5/2)

(d) [5] Based on your answer to (c), is the game described fair? Justify your answer.

3. [25] Suppose we are applying the revised Simplex method and have factorized the basis matrix B with basis header (x_5, x_4, x_7, x_8) as $B =$

$$\begin{vmatrix} 4 & 3 & -1 & 2 \\ 4 & 9 & 1 & 2 \\ 4 & 9 & 5 & 3 \\ 4 & 9 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} * \begin{vmatrix} 4 & 3 & -1 & 2 \\ 0 & 6 & 2 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 3 \end{vmatrix}$$

Suppose the first column of B is replaced by a column which corresponds to x_2 which equals $(3, 1, 0, 4)^T$.

Show the steps used to update the LU-factorization for this specific example using the approach that works in $O(n^2)$ time in general. At the end, indicate the new basis header, and L, U, and B as indicated by the header.

4. A garment factory makes dresses, shirts and jumpers. The requirements for each are:

Item	# Buttons	Yards of Fabric	Hours of Labour	Profit
Dress	5	4	3	20
Shirt	10	2	3	10
Jumper	2	3	2	15
Supply	100	100	70	

Use x_1 for the number of dresses, x_2 for the number of shirts, and x_3 for the number of jumpers.

(a) [5] Set up the linear programming problem which corresponds to maximizing profit subject to meeting the constraints on the supplies.

(b) [5] The final dictionary for this problem is:

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X4 =  10   + 3 X2 - 4 X5 + 7 X6
X1 =  10   - 5 X2 + 2 X5 - 3 X6
X3 =  20   + 6 X2 - 3 X5 + 4 X6
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z  =  500  + 0 X2 - 5 X5 + 0 X6
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Determine the solution to the dual from this last dictionary and interpret it in economic terms.

(c) [10] How does the optimal solution change given an extra t1 buttons, t2 yards of fabric and t3 hours of labour assuming t1, t2, and t3 are sufficiently small?

(d) [5] How much can each of t1, t2, and t3 increase (individually) before your answer to part (c) becomes invalid?