CSC 445/545 Test #1: Fall 2005

1. I ran the Simplex method on two sample problems using my program and the final dictionaries are as given below. What (if anything) can you say about the duals of these two problems?

(a) [10 marks]

X3	=	0.83	-	0.50 X2	-	0.21 X4	+	0.04 X6
X5	=	0.67	+	0.00 X2	+	0.33 X4	+	0.33 X6
X1	=	3.83	-	0.50 X2	+	0.04 X4	-	0.21 X6
z	=	25.00	-	8.00 X2	-	1.25 X4	-	0.75 X6

(b) [10 marks]

X3	=	16.00	+	3.00 X2	-	2.00 X4	+	1.00 X6
X5	=	88.00	+	14.00 X2	-	10.00 X4	+	5.00 X6
X1	=	26.00	+	3.00 X2	-	3.00 X4	+	1.00 X6
z	=	274.00	+	45.00 X2	-	33.00 X4	+	14.00 X6

2. [15 marks] Set up the initial feasible dictionary for Phase 1 for the following problem:

$$\text{Maximize} \quad 5x_1 - 3x_2 + 2x_3$$

subject to

$$\begin{array}{rcrcrcr} 0x_1 & + & 0x_2 & - & 1x_3 & \leq & -4 \\ -1x_1 & + & 5x_2 & - & 1x_3 & \leq & -7 \\ 1x_1 & + & 1x_2 & + & 1x_3 & \leq & 8 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array}$$

(b) [10 marks] The final dictionary at the end of Phase 1 is:

$$\begin{array}{rcrcrcrcrcr} x_1 & = & 3 & + & 5x_2 & - & 1x_4 & + & 1x_5 & + & 0x_0 \\ x_3 & = & 4 & + & 0x_2 & + & 1x_4 & + & 0x_5 & - & 1x_0 \\ x_6 & = & 1 & - & 6x_2 & + & 0x_4 & - & 1x_5 & + & 2x_0 \\ \hline z & = & 0 & + & 0x_2 & + & 0x_4 & + & 0x_5 & - & 1x_0 \end{array}$$

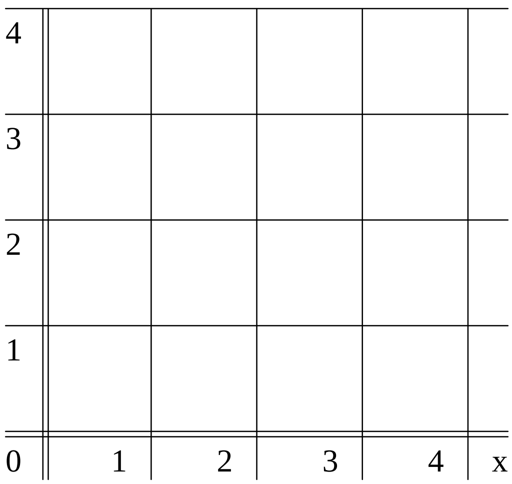
Set up the initial dictionary to start Phase 2.

3. Consider the following set of constraints:

$$\begin{array}{rcrcr} x & + & y & \leq & 4 \\ -x & + & 2y & \leq & 2 \end{array}$$

$$
\begin{array}{lcl}
x & \leq & 3 \\
x, \quad y & \geq & 0
\end{array}
$$

(a) [10 marks] Show these constraints and then shade in the area which represents the feasible region for these constraints.



(b) [5 marks] What are the corners of the feasible region?

(c) [5 marks] Given the constraints from part (a), could a solution of (2.5, 1.5) be a final solution represented by a dictionary from the Simplex method at terminination for some objective function? If you answer yes, give an example of such an objective function. If no, justify your answer.

(d) [5 marks] Given the constraints from part (a), could a solution of (2, 0) be an optimal solution for some objective function? If you answer yes, give an example of such an objective function. If no, justify your answer.

4. Consider the linear programming problem:

$$
\begin{array}{lccccccccc}
\text{Maximize} & 1x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & & \\
\text{subject to} & & & & & & & & & \\
 & 1x_1 & + & 1x_2 & + & 1x_3 & + & 1x_4 & \leq & 10 \\
 & 1x_1 & + & 0x_2 & + & 1x_3 & + & 0x_4 & \leq & 1 \\
 & 2x_1 & + & 0x_2 & + & 3x_3 & + & 1x_4 & \leq & 8 \\
 & 1x_1 & + & 3x_2 & + & 1x_3 & + & 0x_4 & \leq & 7 \\
 & x_1, & & x_2, & & x_3, & & x_4 & \geq & 0
\end{array}
$$

(a) [10 marks] What is the dual?

(b) [20 marks] Apply complementary slackness to determine if (0, 2, 0, 8) is an optimal solution to the problem from part (a). Show all your work.