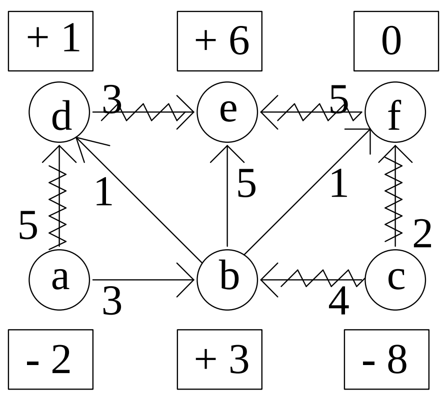
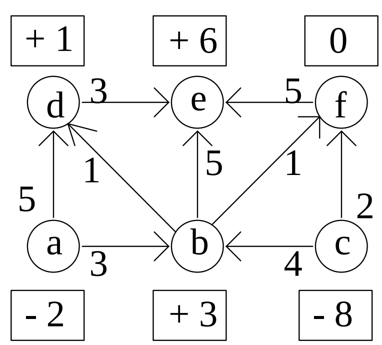
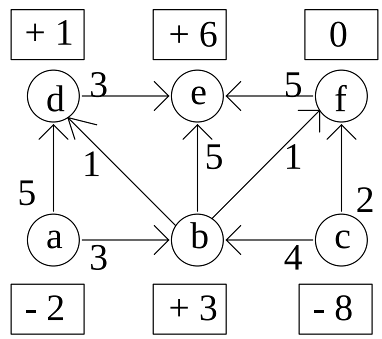
CSC 445/545 Test #2: Thurs. Nov. 17, 2005

1. [20] Solve the following transhipment problem starting with the given tree. Demands are given in the square boxes. The arcs are labeled with the shipping costs. Show all your work.
Compute prices starting at node *a*.







2. Suppose we are applying the revised Simplex method and have factorized the basis matrix B with basis header (x_1, x_6, x_7) as $B = L\,U$ as follows:

$$\begin{vmatrix} 1 & -3 & 2 \\ 1 & 3 & 4 \\ 1 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} * \begin{vmatrix} 1 & -3 & 2 \\ 0 & 6 & 2 \\ 0 & 0 & 3 \end{vmatrix}$$

Suppose the first column of B is replaced by a column which corresponds to x_2 which equals $a = (2, 3, -1)^T$.

(a) [10] First solve $L\ d = a$ for d.

(b) [10] Show the steps used to update the LU-factorization for this specific example using the approach that works in $O(n^2)$ time in general. At the end, indicate the new basis header, and L, U, and B as indicated by the header.

3. A hockey stick manufacturer is making hockey sticks according to the constraints in the following table:

Quantity	Item	Work	Metal	Wood	Net Profit
x_1	Wayne Gretzky Model	2	2	1	60
x_2	Todd Bertuzzi Model	4	1	0	50
x_3	Bobby Orr Model	0	1	2	40
	Amount available:	40	32	32	

For the optimal solution, all three types of sticks should be manufactured.

(a) [10] Use Jacobi's method to determine B^{-1}, (the inverse of the basis) for the optimal solution to this problem.

(b) [10] Determine the values for x_1, x_2 and x_3 at the optimal solution.

(c) [5] Suppose the factory has t_1 extra hours, t_2 extra units of metal, and t_3 extra units of wood where these are small enough so that the basis does not change. Give formulas for the revised values for for x_1, x_2 and x_3.

(d) [5] What are the limits on t_1, t_2 and t_3 considered individually for this solution to be valid?

4. The ultimate goal is to find an integer optimal solution to:
Maximize $x_1 + x_2$
subject to
$2x_1 + 4x_2 \leq 15$
$-1x_1 + 3x_2 \leq 7$
$x_1, x_2 \geq 0.$

You put this problem into the program you wrote and the final dictionary is:
X1 = 7.50- 2.00 X2 - 0.50 X3
X4 = 14.50- 5.00 X2 - 0.50 X3
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z = 7.50- 1.00 X2 - 0.50 X3
IMPORTANT: In parts (a) and (b) below I am asking you just to tell me what to do next. I am not asking you to solve the problem.

(a) [10] What problem(s) would you try next with your computer program if you were using the separation technique for linear programming?

(b) [10] Compute the Gomery cut for this equation:
X1 = 7.50- 2.00 X2 - 0.50 X3
and tell me what problem you would try next with your program if applying the Gomery cut method.