Name: ________________________________

ID Number: ____________________________

CSC 445/545 Test #1

Thurs. Oct. 5, 2006

Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. You should have seven pages including this header page- the last page is blank.

1	20	
2	12	
3	23	
4	20	
5	25	
Total	100	

1. Consider the following problem:

$$\begin{array}{llllll}
\text{Maximize} & 1x_1 & -\ 5x_2 & +\ 2x_3 & & \\
\text{subject to} & 6x_1 & -\ x_2 & +\ x_3 & \le & 3 \\
 & 5x_1 & +\ x_2 & -\ x_3 & \le & 5 \\
 & 5x_1 & +\ x_2 & +\ 20x_3 & \le & 10
\end{array}$$

$x_1, x_2, x_3 \ \ge \ 0.$

(a) [8] Set up the initial dictionary for the Simplex method.

(b) [6] Fill in this chart to indicate where the first pivot would be using each of the pivoting strategies listed.

Pivoting Strategy	Entering Variable	Leaving Variable
Largest Coefficient		
Smallest Subscript		
Maximum Increase		

(c) [6] Justify your answers from (b). Include the computation of the increase to z for each possible entering variable.

2. The standard form for a LP problem is
Maximize $c^T x$ subject to $A x \leq b$, $x \geq 0$. How do you convert the following situations into this standard form?

1. [4] Minimize $c^T x$.

2. [4] Constraints of the form $f(x) \geq b$.

3. [4] Constraints $f(x) = b$.

3. The problem a student is asked to solve by their COOP employer is:

$$
\begin{array}{lrcrcl}
\text{Maximize} & -2x_1 & + & 4x_2 & & \\
\text{subject to} & & & & & \\
& & & x_2 & \leq & 1/2 \\
& x_1 & - & x_2 & \leq & 0 \\
& -5/3x_1 & + & x_2 & \leq & 0 \\
& x_1, & & x_2 & \geq & 0
\end{array}
$$

(a) [8] What is the dual of this problem?

[Problem 3, continued]

The student runs his/her computer program and the final dictionary is:

X1 = 0.3- 0.6 X3 + 0.6 X5
X4 = 0.2- 0.4 X3 - 0.6 X5
X2 = 0.5- 1.0 X3 + 0.0 X5
----
z = xxx- 2.8 X3 - 1.2 X5

where the xxx represents a spot where the output is smudged and so it is unreadable.

The student guesses that the optimal solution to the original problem is one of (1/3, 1/2) or (0.3, 1/2) since the lack of precision in the printed results makes it difficult to tell of the 0.3 really is 0.3 or if it is actually 1/3.

(b) [10] Go through the steps showing all your work of the process required to check if (0.3, 1/2) is an optimal solution.

(c) [5] What would you propose to give as a certificate to your supervisor in order that this person can verify that you actually have a correct solution to the problem?

4. One of the problems in my rough notes has a tableau representation (corresponding to the first dictionary) as
Maximize $c^T \ x$ where

$$c^T = [\ 5 \ \ 4 \ \ 3 \ \ 0 \ \ 0 \ \ 0\ ]$$

subject to $Ax = b$,

$$A = \begin{bmatrix} 2 & 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 2 & 0 & 1 & 0 \\ 3 & 4 & 2 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ 11 \\ 8 \end{bmatrix}$$

I wrote down in my notes that in the final iteration, the inverse of the basis is:

$$B^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 2 \end{bmatrix}$$

But then I lost the last page with the final dictionary.

(a) [10] Use B^{-1} to compute the updated A and b which (after several pivots) correspond to the optimal solution for this problem.

(b) [10] Use your answer from (a) plus a computation which determines the z row to find the final dictionary.

5. Circle true or false for each question and justify your answer. To get marks for a question, the justification must be correct.

(a) [5] The Simplex algorithm using the Maximum Coefficient rule with the Smallest Subscript rule used to break ties for entering and exiting variables results in a polynomial time algorithm for solving linear programming problems.

True **False**

(b) [5] If a problem does not have non-negativity constraints for its variables, it is not possible to solve the problem using the Simplex Method.

True **False**

(c) [5] Every linear programming problem has at least one feasible solution and such a solution can be found using Phase 1 of the Two-Phase Simplex method.

True **False**

(d) [5] Cycling means that the value of z increases and then it decreases and this pattern is repeated so that the program gets stuck in an infinite loop.

True **False**

(e) [5] There are examples of unbounded primal problems whose dual problems are also unbounded.

True **False**

Use this page if you need extra space. Clearly indicate the number of the question that you are answering.