Name: ___________________________________

ID Number: ______________________________

CSC 445/545 Test #2

Thurs. Nov. 16, 2006

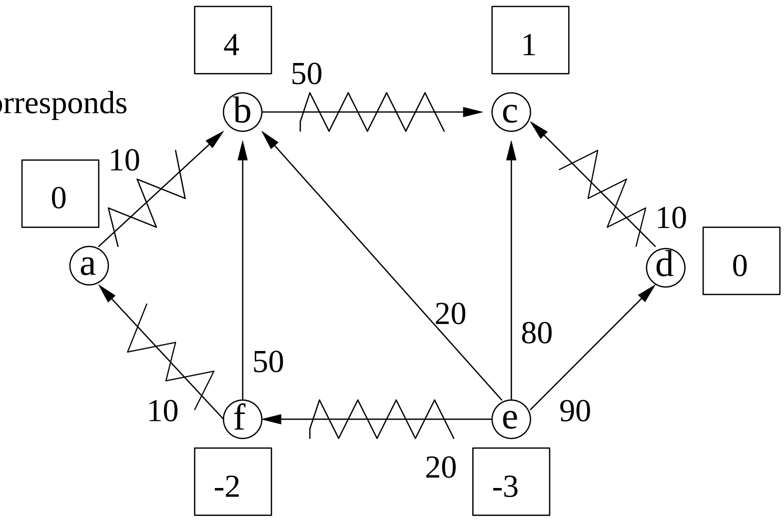
Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. You should have eight pages including this header page- the last page is blank.

1	20	
2	20	
3	20	
4	30	
5	10	
Total	100	

1. [A Transhipment problem]

(a) [4] Find the shipping schedule which corresponds to the indicated tree.

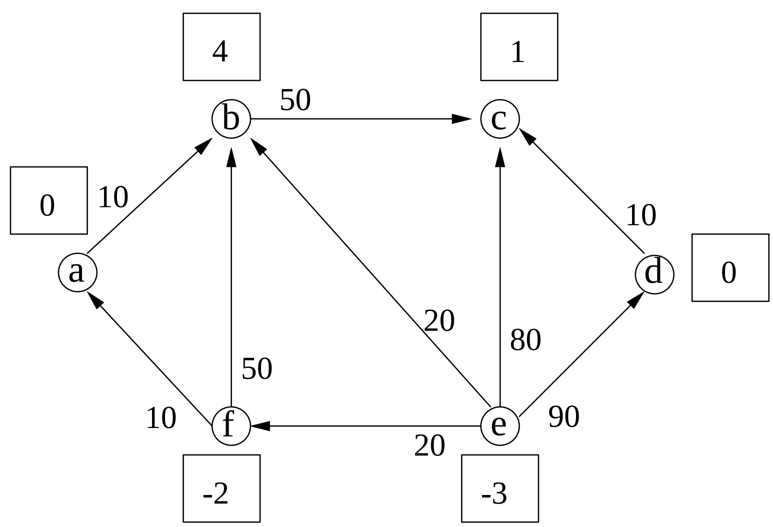


(b) [4] Compute the fair prices at each vertex. Start your computation with node *a*.

Node	a	b	c	d	e	f
Fair price	0					

(c) [4] Which arcs are eligible to enter?

(d) [4] Perform one pivot and show the resulting tree and schedule on this picture:



(e) [4] Is your schedule after the one pivot optimal or not? Justify your answer.

2. Consider this linear programming problem:

$$
\begin{array}{llllllll}
\text{Maximize} & 2\,x_1 & -\,1\,x_2 & +\,3\,x_3 & -\,1\,x_4 & & \\
\text{subject to} & 1\,x_1 & +\,1\,x_2 & +\,3\,x_3 & +\,1\,x_4 & \leq & 5 \\
 & 2\,x_1 & +\,0\,x_2 & +\,2\,x_3 & +\,0\,x_4 & \leq & 6 \\
 & -1\,x_1 & +\,1\,x_2 & +\,1\,x_3 & -\,1\,x_4 & \leq & 4
\end{array}
$$

$$x_1,\ x_2,\ x_3,\ x_4\ \geq 0$$

(a) [5] What is the dual of this problem?

(b) [15] Apply complementary slackness to determine if the solution (2, 0, 1, 0) is optimal or not. Show all your work explaining what you are doing at each step.

3. Consider the problem:

$$\begin{array}{lrcrcr} \text{Maximize} & x_1 & + & x_2 & & \\ \text{subject to} & 2x_1 & + & 2x_2 & \leq & 3 \\ & 2x_1 & - & 2x_2 & \leq & -3 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

(a) [5] Set up the initial (infeasible) dictionary for the 2-phase simplex method using x_0 to denote your aux. variable.

(b) [5] Do the first pivot to create the initial feasible dictionary for Phase 1.

(c) [10] The following dictionary corresponds to an optimal solution to the phase 1 problem (the aux. variable is X0):

```
X2 =  1.50+  0.00 X1 -  0.25 X3 +  0.25 X4
X0 =  0.00+  2.00 X1 +  0.50 X3 +  0.50 X4
----------------------------------------------
z  =  0.00-  2.00 X1 -  0.50 X3 -  0.50 X4
```

Create the initial feasible dictionary which should be used to commence Phase 2 (including the revised z row).

4. A hockey stick manufacturer is making hockey sticks according to the constraints in the following table:

Quantity	Item	Work	Metal	Wood	Net Profit
x_1	Wayne Gretzky Model	2	2	1	60
x_2	Todd Bertuzzi Model	4	1	0	50
x_3	Bobby Orr Model	0	1	2	40
	Amount available:	40	32	32	

For the optimal solution, all three types of sticks should be manufactured.

(a) [10] Use Jacobi's method to determine B^{-1}, (the inverse of the basis) for the optimal solution to this problem.

(b) [10] Determine the values for x_1, x_2 and x_3 at the optimal solution.

(c) [5] Suppose the factory has t_1 extra hours, t_2 extra units of metal, and t_3 extra units of wood where these are small enough so that the basis does not change. Give formulas for the revised values for x_1, x_2 and x_3.

(d) [5] What are the limits on t_1, t_2 and t_3 considered individually for this solution to be valid?

5. My graduate student is looking for the integer optimal solution to this problem P1:

$$\begin{array}{lrcrcr} \text{Maximize} & x_1 & + & 10x_2 & & \\ \text{subject to} & 14x_1 & + & 10x_2 & \le & 35 \\ & 0x_1 & + & 2x_2 & \le & 3 \\ & x_1, & & x_2 & \ge & 0 \end{array}$$

The student notes that $x_1 = 0$ and $x_2 = 1$ is an integral feasible solution which has $z = 10$ and wonders if there are any better integer optimal solutions. When the original problem P1 is solved the solution is: $z = 16.4$, $x_1 = 1.4$, and $x_2 = 1.5$.

(a) [4] Explain why it is sufficient for the student to find the integer optimal solution to each of P2 which is P1 plus the constraint $x_1 \ge 2$ and P3 which is P1 plus the constraint $x_1 \le 1$ and then take the best of these two solutions.

(b) [6] The student finds that the optimal solution to P2 is: $z = 9$, $x_1 = 2$, $x_2 = 0.7$, and that the optimal solution to P3 is: $z = 16$, $x_1 = 1$, $x_2 = 1.5$. Recall that we already know that $x_1 = 0$ and $x_2 = 1$ is a feasible integer solution which has $z = 10$. Which problems should the graduate student try to solve next using the separation technique? Draw the resulting computation tree and justify your answer.

Use this page if you need extra space. Clearly indicate the number of the question that you are answering.