Name: ____________________________________

ID Number: ______________________________

CSC 445/545 Test #1

Thurs. Oct. 11, 2012

Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. You should have seven pages including this header page.

Question	Max	Marks
1	15	
2	25	
3	20	
4	20	
5	20	
Total	100	

1. Your first programming project only solves problems that are in our standard form:
Maximize $c^T x$ subject to $A x \le b$, $x \ge 0$.
Another student wants help in solving this problem:

$$\begin{array}{llrrrcl}
\text{Minimize} & 7 - & 2x_1 & -\ 3x_2 & +\ 4x_3 & & \\
\text{subject to} & & 5x_1 & -\ 6x_2 & +\ 7x_3 & \ge & 8 \\
 & & -0.9x_1 & -\ 0.8x_2 & +\ 0.7x_3 & = & 0.6
\end{array}$$

$$x_1,\ x_2 \ \ge\ 0$$

(a) [5] State an optimization problem in standard form that has the same solutions for x as the student's problem.

(b) [5] What would you type in as input to your computer program?

(c) [5] When your program finishes solving the problem you typed in, what do you need to do to get the final solution to the original problem?

2. Consider this problem:

$$
\begin{array}{lrcrcrcr}
\text{Maximize} & 0x_1 & + & 4x_2 & + & 8x_3 & & \\
\text{subject to} & -1x_1 & + & 0x_2 & - & 1x_3 & \leq & -3 \\
 & 3x_1 & + & 3x_2 & + & 4x_3 & \leq & 12 \\
 & -1x_1 & - & 2x_2 & - & 3x_3 & \leq & -6
\end{array}
$$

$$x_1,\ x_2,\ x_3 \geq 0$$

(a) [15] Set up the initial feasible dictionary for Phase 1 for this problem.

(b) [10] The final dictionary at the end of Phase 1 is:

$$
\begin{array}{rcrcrcrcrcr}
x_2 & = & 0.75 & - & 1.25x_4 & - & 0.5x_5 & - & 0.25x_6 & + & 2x_0 \\
x_1 & = & 2.25 & + & 0.25x_4 & - & 0.5x_5 & - & 0.75x_6 & + & 1x_0 \\
x_3 & = & 0.75 & + & 0.75x_4 & + & 0.5x_5 & + & 0.75x_6 & - & 2x_0 \\
\hline
z & = & 0 & + & 0x_4 & + & 0x_5 & + & 0x_6 & - & 1x_0
\end{array}
$$

What initial dictionary should be used to start Phase 2?

Recall that the objective function is:

Maximize $\quad 0x_1 \quad + \quad 4x_2 \quad + \quad 8x_3$

3. The problem a student is asked to solve by their COOP employer is:

(a) [6] What is the dual of this problem?

$$
\begin{array}{lrcrcl}
\text{Maximize} & -2x_1 & + & 4x_2 & & \\
\text{subject to} & & & x_2 & \leq & 1/2 \\
 & x_1 & - & x_2 & \leq & 0 \\
 & -5/3x_1 & + & x_2 & \leq & 0
\end{array}
$$

$$x_1,\ x_2 \geq 0$$

The student runs his/her computer program and the final dictionary is:

X1 = 0.3- 0.6 X3 + 0.6 X5
X4 = 0.2- 0.4 X3 - 0.6 X5
X2 = 0.5- 1.0 X3 + 0.0 X5
----
z = xxx- 2.8 X3 - 1.2 X5

The xxx represents a spot where the output is smudged with pizza stains and so it is unreadable.

The student guesses that the optimal solution to the primal problem is one of (1/3, 1/2) or (0.3, 1/2) since the lack of precision in the printed results makes it difficult to tell if the 0.3 really is 0.3 or if it is actually 1/3.

(b) [14] Apply duality theory to check the two solutions (1/3, 1/2) and (0.3, 1/2) to determine if they are correct solutions or not. Explain what you are doing at every step.

4. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

(a) [5] A linear programming problem can have an infinite number of optimal solutions.

True False

(b) [5] It is possible to find a primal linear programming problem that is unbounded such that its dual is also unbounded.

True False

(c) [5] No matter which pivoting rules are used, there are examples for with the Simplex method can get stuck in an infinite loop.

True False

(d) [5] For a feasible linear programming problem, it is not possible to have a final basic solution to the Phase 1 problem which has x_0 in the basis because having a feasible solution implies that x_0 is zero and hence it cannot be in the basis.

True False

5. For each part of this question, fill in the all the empty boxes with some constant terms and some coefficients for the variables in each dictionary so that the resulting dictionary has the requested properties.

(a) [6] The problem is clearly unbounded but a Simplex method using the maximum coefficient rule would still take at least one more pivot:

$x_1 =$		$+$		$x_2 +$		$x_4 +$		x_5
$x_3 =$		$+$		$x_2 +$		$x_4 +$		x_5
$x_6 =$		$+$		$x_2 +$		$x_4 +$		x_5
$z =$		$+$		$x_2 +$		$x_4 +$		x_5

(b) [8] The choice of the entering variable is:
x_2 and z increases by 1, if the minimum subscript rule is applied,
x_4 and z increases by 10, if the maximum coefficient rule is applied, and
x_5 and z increases by 100, if the maximum increase rule is applied.

$x_1 =$		$+$		$x_2 +$		$x_4 +$		x_5
$x_3 =$		$+$		$x_2 +$		$x_4 +$		x_5
$x_6 =$		$+$		$x_2 +$		$x_4 +$		x_5
$z =$		$+$		$x_2 +$		$x_4 +$		x_5

(c) [6] The next pivot must have x_4 entering and it is a degenerate pivot.

$x_1 =$		$+$		$x_2 +$		$x_4 +$		x_5
$x_3 =$		$+$		$x_2 +$		$x_4 +$		x_5
$x_6 =$		$+$		$x_2 +$		$x_4 +$		x_5
$z =$		$+$		$x_2 +$		$x_4 +$		x_5