Name: _____

ID Number: _____

CSC 445/545 Test #2

Monday Nov. 19, 2012

Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. You should have 8 pages including this header page.

Question	Topic	Max	Marks
1	Complementary Slackness	20	
2	Revised Simplex Method	20	
3	Integer Programming	20	
4	Curve Fitting	20	
	Total	80	

1. The problem a student is asked to solve by their COOP employer is:

(a) [6] What is the dual of this problem?

$$
\begin{array}{lrcrcl}
\text{Maximize} & -2x_1 & + & 4x_2 & & \\
\text{subject to} & & & x_2 & \leq & 1/2 \\
& x_1 & - & x_2 & \leq & 0 \\
& -5/3x_1 & + & x_2 & \leq & 0
\end{array}
$$

$$x_1, x_2 \geq 0$$

(b) [14] Apply complementary slackness to this problem to determine if (3/10, 1/2) is the correct solution. Explain what you are doing at every step.

2. [20] A student started solving this with the revised Simplex method:

Maximize $2\ x_1 + 4\ x_2 + 8\ x_3$

subject to

$0\ x_1 + 1\ x_2 + 2\ x_3 \leq\ 4$

$0\ x_1 + 1\ x_2 - 3\ x_3 \leq 12$

$1\ x_1 + 1\ x_2 + 1\ x_3 \leq 10$

$x_1,\ x_2,\ x_3 \geq 0$

and after several steps, had $z = 28$, $H_B^T = [2\ 5\ 1]$, $x_B^T = (4, 8, 6)$, and $H_N^T = [6\ 4\ 3]$. Compute the updated values for H_B^T, the current solution x_B and z after **ONE** more iteration. Show all your work.

The Revised Simplex method:

1. Solve $A_B^T\ y = c_B$ *for* y.

2. Compute $[c_N^T - y^T\ A_N]x_N$ *to get coefficients of non-basic variables.*

3. Solve for entering column d *in the current dictionary:* $d = A_B^{-1}a$ *where* a *is the entering column taken from the initial problem.*

3. The ultimate goal is to find an integer optimal solution to the problem which has this initial dictionary:

$$X4 = -5 - 2\,X1 + 2\,X2 - 3\,X3$$
$$X5 = -7 + 8\,X1 + 4\,X2 - 4\,X3$$
$$X6 = 8 - 2\,X1 + 0\,X2 - 1\,X3$$
$$X7 = 5 - 1\,X1 + 0\,X2 - 1\,X3$$
$$z = 0 + 4\,X1 - 1\,X2 - 1\,X3$$

You put this problem into the program you wrote and this was the final dictionary:

$$X2 = 6.5 + 1.0\,X3 + 0.5\,X4 - 0.5\,X6$$
$$X5 = 51.0 - 4.0\,X3 + 2.0\,X4 - 6.0\,X6$$
$$X1 = 4.0 - 0.5\,X3 + 0.0\,X4 - 0.5\,X6$$
$$X7 = 1.0 - 0.5\,X3 + 0.0\,X4 + 0.5\,X6$$
$$z = 9.5 - 4.0\,X3 - 0.5\,X4 - 1.5\,X6$$

IMPORTANT: In parts (a) and (b) below I am asking you just to tell me what to do next. I am not asking you to solve the problem.

(a) [6] What constraint(s) would you try adding next with your computer program if you were using the separation technique for integer programming?

(b) [14] Compute the Gomery cut for this equation:

$$X2 = 6.5 + 1.0\,X3 + 0.5\,X4 - 0.5\,X6$$

4. Consider the following three (x, y) data points: $(1, 1)$, $(4, 2)$, $(6, 5)$.

(a) [5] What problem would you solve (not in standard form) in order to find a linear approximation that minimizes the L_1-norm?

(b) [3] When you convert this problem to standard form, what will the objective function be?

(c) [5] When you convert this to standard form, which equations arise for the point which has $x = 6$ and $y = 5$?

[Question 4 continued]

Mary and Paul were asked by their boss to find a linear fit for some data points. They found both an L_1-fit and and L_∞-fit and plotted the results (see the next page). The blue points are the original 7 data points. The two fits are shown as red and green lines.

(d) [4] Your boss wants to know which line represents an L_∞-fit: the red one or the green one? Justify your answers based on how the L_1 and L_∞ fits are defined.

(e) [3] The red line currently looks like a much better fit for the data. What advice would you give your boss in order to make the approach indicated by the green line give a better approximation?

Replace this page with excel picture.

Use this page if you need more space. Please clearly indicate the question you are answering.