CSC 445/545: Midterm Exam
Fall 1998

1. Consider the following problem:

$$\begin{array}{lrrrrr} \text{Maximize} & 3x_1 & + \ 4x_2 & + \ 5x_3 & & \\ \text{subject to} & 2x_1 & - \ x_2 & + \ x_3 & \leq & 3 \\ & 5x_1 & + \ x_2 & - \ 0.1\,x_3 & \leq & 5 \\ & 10x_1 & + \ x_2 & + \ 20x_3 & \leq & 10 \end{array}$$

$x_1, x_2, x_3 \geq 0.$

(a) [5] Set up the intial dictionary for the Simplex method.

(b) [5] Fill in this chart to indicate where the first pivot would be using each of the pivoting strategies listed.

Pivoting Strategy	Entering Variable	Leaving Variable
Maximum Increase		
Largest Coefficient		
Smallest Subscript		

(c) [10] Justify your answers from (b). Include the computation of the increase to z for each possible entering variable.

2.(a) [10] Solve the following linear programming problem with the Simplex method. Use the **Largest Coefficient Rule** to decide where to pivot.

$$\begin{array}{lrrrr} \text{Maximize} & 3x_1 & + \ 2x_2 & & \\ \text{subject to} & x_1 & - \ 0.5\,x_2 & \leq & 2 \\ & & x_2 & \leq & 4 \end{array}$$

$x_1, x_2 \geq 0.$

(b) [5] Give the dual of the problem from part (a):

(c) [5] Obtain the optimal solution to the dual from the final dictionary for the primal problem that you computed for part (a). Explain what you are doing.

3. Consider this LP problem:

$$\begin{array}{lrrrrr} \text{Maximize} & 7x_1 & + \ 3x_2 & + \ 4x_3 & & \\ \text{subject to} & x_1 & & + \ x_3 & \leq & 2 \\ & x_1 & + \ x_2 & & \leq & 4 \end{array}$$

$x_1, x_2, x_3 \geq 0.$

(a) [10] Apply complementary slackness to determine if (2, 2, 0) is an optimal solution to this problem. Explain what you are doing.

(a) [10] Apply complementary slackness to determine if (0, 3) is an optimal solution to the dual of this problem. Explain what you are doing.

4. [10] The standard form for a LP problem is

$$\text{Maximize} \quad c^T x \qquad \text{subject to } A\,x \le b, \;\; x \ge 0.$$

How do you convert the following situations into this standard form?
Assume $f(x)$ is a linear function.

1. Minimize $c^T x$.
2. Constraints of the form $f(x) \ge b$.
3. Constraints of the form $f(x) = b$.
4. Maximize $c^T x + c_0$ for constant c_0.
5. A variable x_i with no constraint that $x_i \ge 0$.

5. [15] Create the intial feasible dictionary used to start phase 1 of the 2-phase Simplex method for the following problem. Show your work.

$$\begin{array}{lrrrl}
\text{Maximize} & -4x_1 & -\ 5x_2 & -\ 7x_3 & \\
\text{subject to} & -x_1 & -\ 2x_2 & -\ 2x_3 & \le -3 \\
 & -2x_1 & -\ 3x_2 & -\ 3x_3 & \le -4 \\
 & -x_1 & & -\ x_3 & \le -2
\end{array}$$

$x_1,\ x_2,\ x_3 \ \ge\ 0.$

6. [15] Show that the set of optimum solutions to a linear program forms a *convex set*. This means that if x_1 and x_2 are both optimum solutions, then so is $x_3 = \lambda x_1 + (1-\lambda)x_2$, where $0 \le \lambda \le 1$.