CSC 445/545: Final exam- Mon. Dec. 7, 1992

The exam is 3 hours long. You should have 6 questions on 8 pages plus 2 blank pages at the end. Closed book. No calculators or other aids permitted.

Question	1	2	3	4	5	6
Marks	20	20	15	25	10	15

1.(a) [10] Convert the following LP to standard form:

Min $-3x_1 + 2x_2 + x_3$

subject to

$$
\begin{array}{rcrcrcl}
x_1 & + & x_2 & & & \leq & 3 \\
 & & 2x_2 & + & 4x_3 & \geq & 5 \\
x_1 & + & x_2 & + & x_3 & = & 2 \\
 & & 0 & \leq & x_1 & \leq & 4 \\
 & & & & x_2 & \geq & 0
\end{array}
$$

(b). [10] Find necessary and sufficient conditions to make the following LP

(i) unbounded,

(ii) infeasible,

(iii) have an optimal solution.

$$\begin{aligned} \text{Max} \quad & x_1 + x_2 \\ \text{subject to} \quad & \\ & s\,x_1 + t\,x_2 \leq 1 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

2.(a) [10] Draw the feasible region for the following LP and use your picture to find the optimum value.

$$\text{Max} \quad 4x_1 + 5x_2$$

subject to

$$\begin{aligned} 2x_1 + x_2 &\leq 9 \\ x_1 &\leq 4 \\ x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(b) [5] How many iterations does the Simplex method take on the above LP for each of the following pivot rules:

	Pivot Rule	Number of Pivots
(i)	Largest Increase in Objective Function	
(ii)	Largest Coefficient	
(iii)	Smallest Subscript	

(c) [5] Which of the above three rules is best for general problems? Give a short discussion summarizing what you know.

3. Let E be an eta-matrix with eta column in column i equal to $c = (c_1, c_2, \cdots, c_n)^T$.

(a) [5] What is the inverse of E?

(b) [10] Explain how an eta-factorization of B_k^{-1} can be used to solve the systems $y\, B_k = c_{B_k}$ and $B_k\, d = a$ from the revised Simplex method. Assume $B_k = E_1\, E_2 \cdots E_k$ for some eta matrices $E_1, \cdots, E_k$.

4. A company makes two types of fertilizer called Hi-phosphate and Lo-phosphate. Three basic raw materials are used in manufacturing these fertilizers as given in the following table:

	Tons of material required to make 1 ton		max amount of raw materials
Raw material	Hi-Phosphate	Lo-Phosphate	available per month (tons)
1	2	1	1500
2	1	1	1200
3	1	0	500
Net profit (per ton)	$15	$10	

Let x_1 denote the amount of Hi-phosphate fertilzer produced and x_2 the amount of Lo-phosphate fertilizer.

(a) [10] The company wants to maximize profit. State the primal and the dual problems.

(b) [5] Suppose the last dictionary for the Simplex method is:

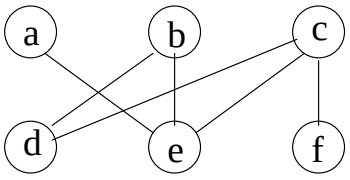
X2 = 900.00+ 1.00 X3 - 2.00 X4
X5 = 200.00+ 1.00 X3 - 1.00 X4
X1 = 300.00- 1.00 X3 + 1.00 X4
---
z =13500.00- 5.00 X3 - 5.00 X4

Read the dual variable values from this dictionary.

(c) [5] Give an economic interpretation for the values of the dual variables obtained for part (b).

(d) [5] Solve for the dual variables using complementary slackness. Show all your work.

5.(a) [5] Set up the LP (do not solve) for finding a maximum matching in the following bipartite graph:



(b) [5] Suppose somebody claimed that $\{ e_3, e_6 \}$ is a maximum weight matching, where $e_3 = (b, e)$, and $e_6 = (c, f)$. Give an argument in terms of vertex covers that demonstrates that this claim is incorrect.

6. [15] Suppose iron ore is produced at two mines- mine A and mine B, and this ore is in demand by three car manufacturing plants, Plant 1, Plant 2, and Plant 3. The supplies, demands, and shipping costs are summarized in the following table.

	Shipping Costs			
	Plant 1	Plant 2	Plant 3	Amt. of ore available (tons)
Mine A	9	16	28	103
Mine B	14	29	19	197
Amt. of ore required	71	133	96	

The current shipping schedule is:

	Plant 1	Plant 2	Plant 3
Mine A	7	0	96
Mine B	64	133	0

You have been hired by the company to find an optimal shipping schedule. Solve this as a transshipment problem. Show all your work. At each step, clearly indicate the computation of the fair prices and the updated schedule on two separate figures.

Clearly indicate the question you are answering.

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