UNIVERSITY OF VICTORIA

EXAMINATIONS- DECEMBER 1998

CSC 445/545

Instructor: Dr. W. Myrvold

Duration: 3 hours

TO BE ANSWERED ON THE PAPER.

Instructions:

Students **MUST** count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

This question paper has 9 pages plus this header page. The last page is blank in case you need more space.

Use only space provided on exam for answering questions. Closed book. No aids permitted.

IMPORTANT: Do any 6 of the 7 questions. Indicate here which question to omit because otherwise, I will grade questions 1-6:

Please omit Question # _______________.

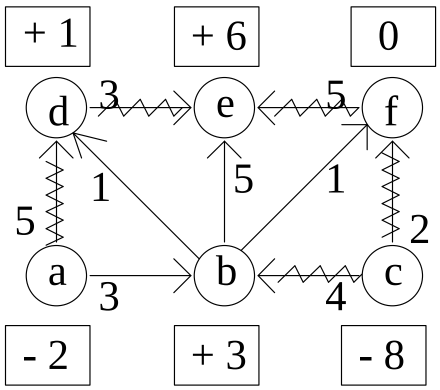
Name: _________________________

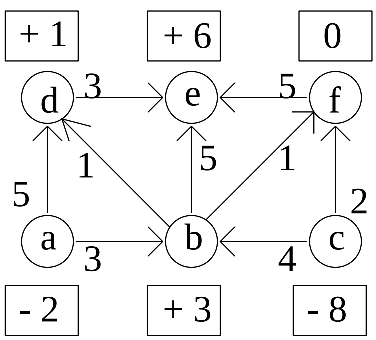
ID Number: _____________________

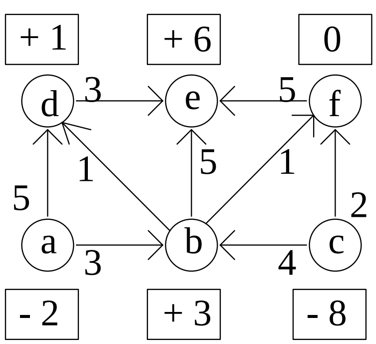
Question	**Value**	**Mark**
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	**120**	

1. [20] Solve the following transshipment problem starting with the given tree. Demands are given in the square boxes. The arcs are labeled with the shipping costs. Show all your work.

Compute prices starting at node *a*.







2. Consider the following LP problem:

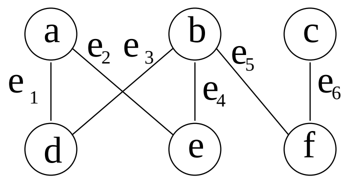
$$\begin{array}{lrrrcr}
\text{Maximize} & 5\,x_1 & +\,4\,x_2 & +\,3\,x_3 & & \\
\text{subject to} & 2\,x_1 & +\,3\,x_2 & +\,1\,x_3 & \le & 5 \\
 & 4\,x_1 & +\,1\,x_2 & +\,2\,x_3 & \le & 11 \\
 & 3\,x_1 & +\,4\,x_2 & +\,2\,x_3 & \le & 8
\end{array}$$

$x_1,\, x_2,\, x_3 \ \ge 0.$

(a) [10] Show how complementary slackness can be used to prove that (2.5, 0, 0) is not an optimal solution.

(b) [10] Show how complementary slackness can be used to prove that (2, 0, 1) is an optimal solution.

3.(a) [10] Set up the LP (do not solve) for finding a maximum matching in the following bipartite graph:



(b) [10] Indicate, using the above graph as an example, why it may not initially appear that the maximum matching problem in bipartite graphs can be solved using LP techniques.

4. Consider the following game. Rick (the row player) and Carla (the column player) each have two coins, a penny (1 cent) and a nickel (5 cents). Each player hides one of the two coins. Then both simultaneously guess what the other player has hidden. If exactly one player is correct, then they win the total amount hidden. Otherwise, it is a draw.

(a) [10] What is the payoff matrix from Carla's perspective? Explain thoroughly. Give the rows and columns appropriate labels.

(b) [10] Explain why for this particular problem, given that Rick has chosen a fixed strategy (p_1, p_2, p_3, p_4) where $p_1 + p_2 + p_3 + p_4 = 1$, that Carla always has a pure strategy that is optimal. Explain how to select this pure strategy.

5(a) [10] Formulate the following problem as an LP in standard form:

Maximize the minimum of $3x_1 + 2x_2$ and $x_1 + x_3$
subject to:

1. $|x_1 + x_3| \leq 0.5$ (the ||'s denote take the absolute value)
2. $x_1 + x_2 + x_3 = 1$
3. $-1 \leq x_1 \leq 0.75$
4. $x_2, \; x_3 \geq 0$

Explain what you are doing.

5.(b) Formulate the following as an LP problem:

[10] Consider the operation of a dealer of home heating oil. Suppose the dealer owns a storage tank with a capacity for 10,000 gallons of oil, that initially has 3000 gallons in it. Each month for the next three months, the dealer can sell up to 8000 gallons of oil per month, charging 92 cents per gallon in the first month, 95 cents per gallon in the second, and 97 cents per gallon in the third. Furthermore, the dealer can purchase up to 5000 gallons of oil each month either for distribution or for storage for later use. The cost to the dealer of this oil is 80 cents per gallon the first month, 82 cents per gallon during the second month and 85 cents per gallon during the third. How much oil should the dealer purchase, sell, and store each month to maximize profits? Assume that any oil left in the storage tank after the third month has a value of only 78 cents per gallon.

6. A furniture factory is manufacturing desks, chairs and bed frames according to the constraints in the following table:

Quantity	Item	Work	Metal	Wood	Net Profit
x_1	Desk	2	1	3	13
x_2	Chair	1	1	3	12
x_3	Bed frame	2	1	4	17
	Amount available:	225	117	420	

When I use my program to solve this problem, the last dictionary is:

X1 = 30.00+ 1.00 X2 - 2.00 X4 + 1.00 X6
X5 = 4.50- 0.50 X2 + 0.50 X4 + 0.00 X6
X3 = 82.50- 1.50 X2 + 1.50 X4 - 1.00 X6
----
z =1792.50- 0.50 X2 - 0.50 X4 - 4.00 X6

(a) [10] Determine the solution for the dual from this last dictionary and interpret it in economic terms.

(b) [10] Compute formulas for how the values of x_1 and x_3 change given that the amount of work, metal, and wood available increase by t_1, t_2, and t_3 respectively (assuming the changes are small). Your formulas should permit changes to all 3 resources at the same time.

7. [20] Suppose we are applying the revised Simplex method and have factorized the basis matrix B with basis header (x_5, x_6, x_7, x_8) as $B =$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 3 & 0 \\ 1 & 4 & 8 & 4 \\ 1 & 4 & 8 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} * \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 3 & 2 & -1 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

Suppose the first column of B is replaced by a column which corresponds to x_2 which equals $(2, 3, -1, 3)^T$.

Show the steps used to update the LU-factorization for this specific example using the approach that works in $O(n^2)$ time in general. At the end, indicate the new basis header, and L, U, and B as indicated by the header.

This page has been left blank in case you need more space. Clearly indicate the question you are answering.

The End.