Symmetric Monotone Venn Diagrams with Seven Curves

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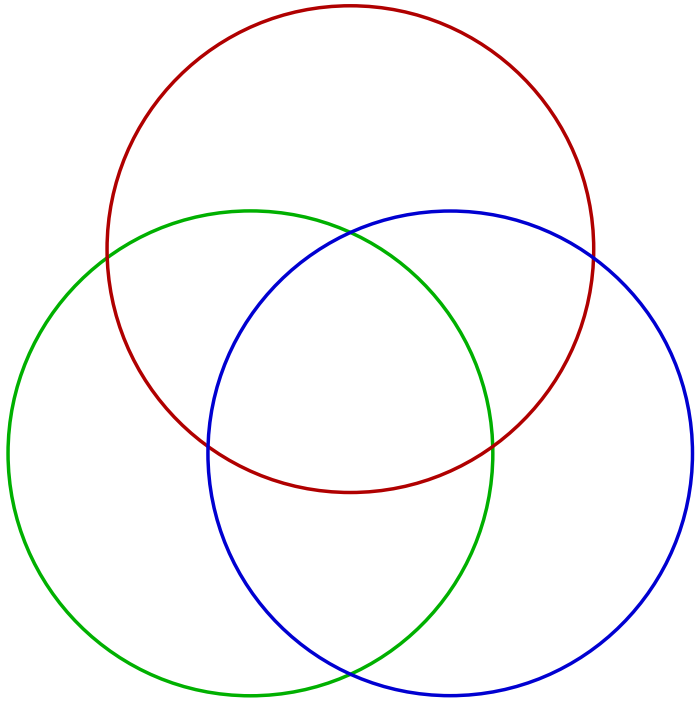
University of Victoria, BC, Canada

Fifth International Conference on Fun with Algorithms

Definitions

- A collection of n simple closed curves $C_1, C_2, \cdots, C_n$
- Let X_i denotes interior or exterior of curve C_i



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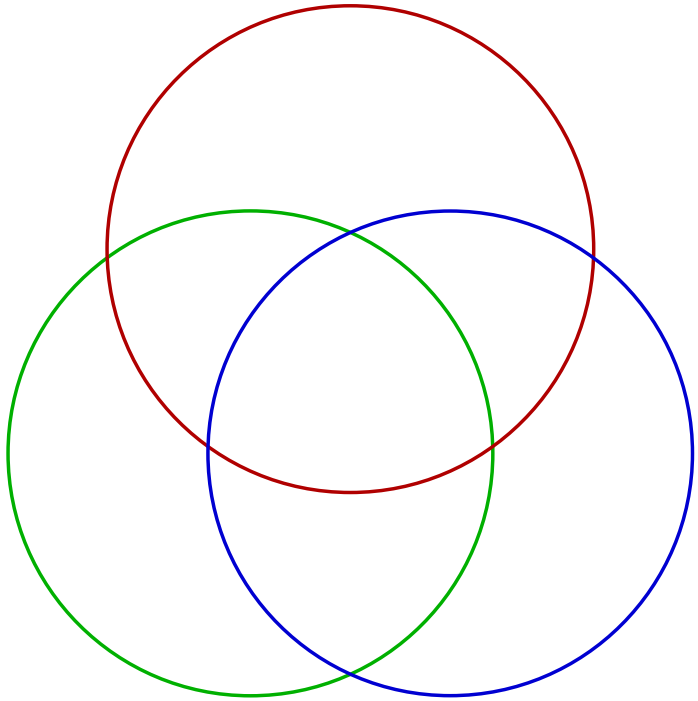

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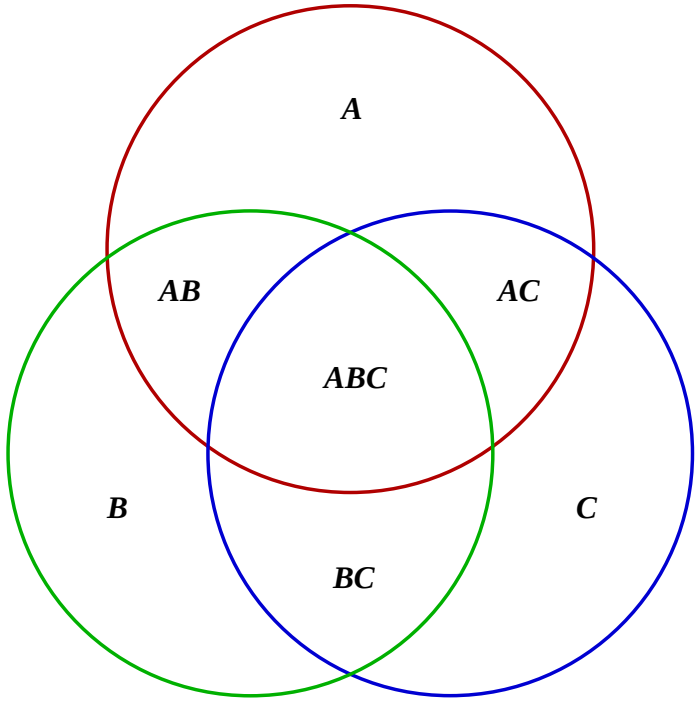
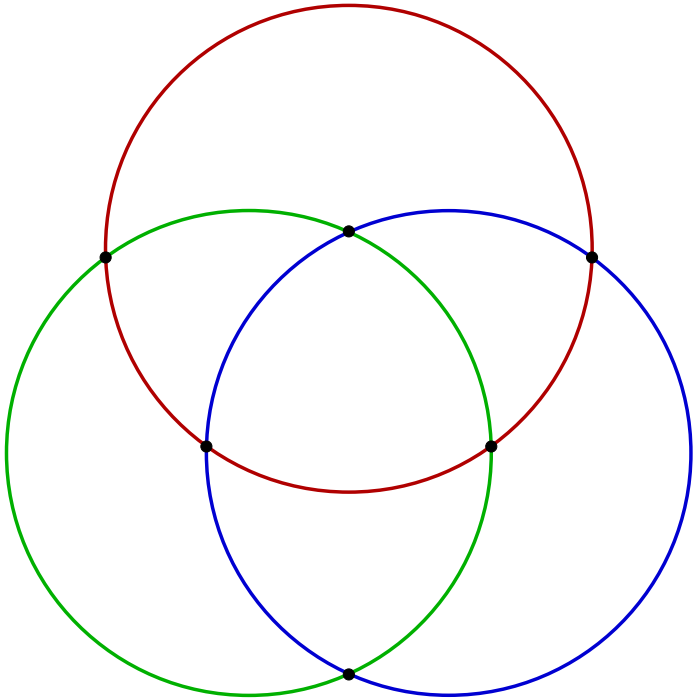


Venn Diagrams : Simple vs non-simple

- Simple : At most 2 curves cross at each intersection point



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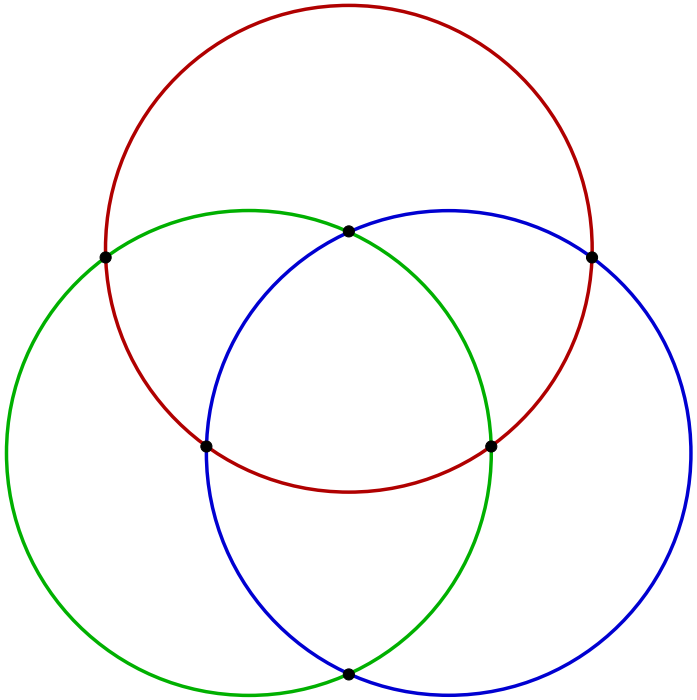

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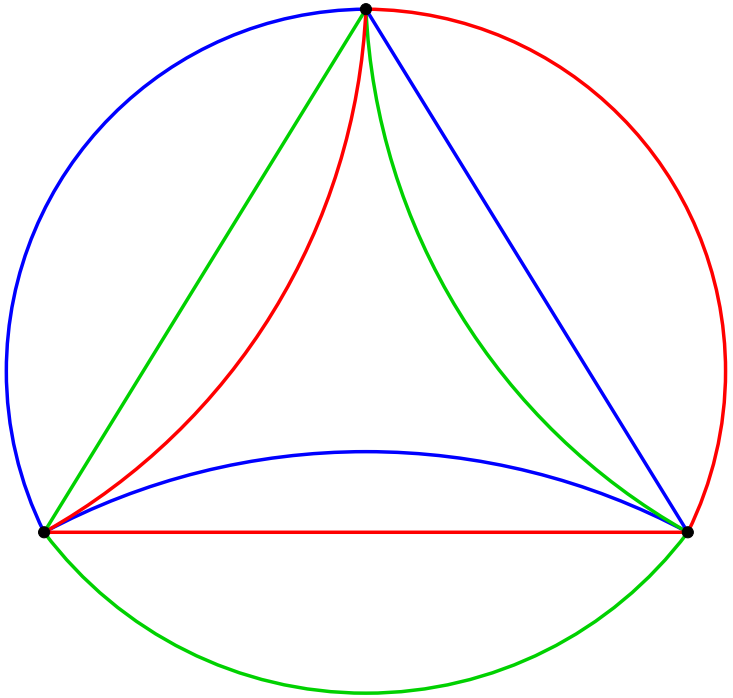

Monotone vs non-monotone Venn Diagrams

- Rank of a region : Number of curves that contain it
- Monotone if every k-region is adjacent to both a $(k-1)$-region and to a $(k+1)$-region
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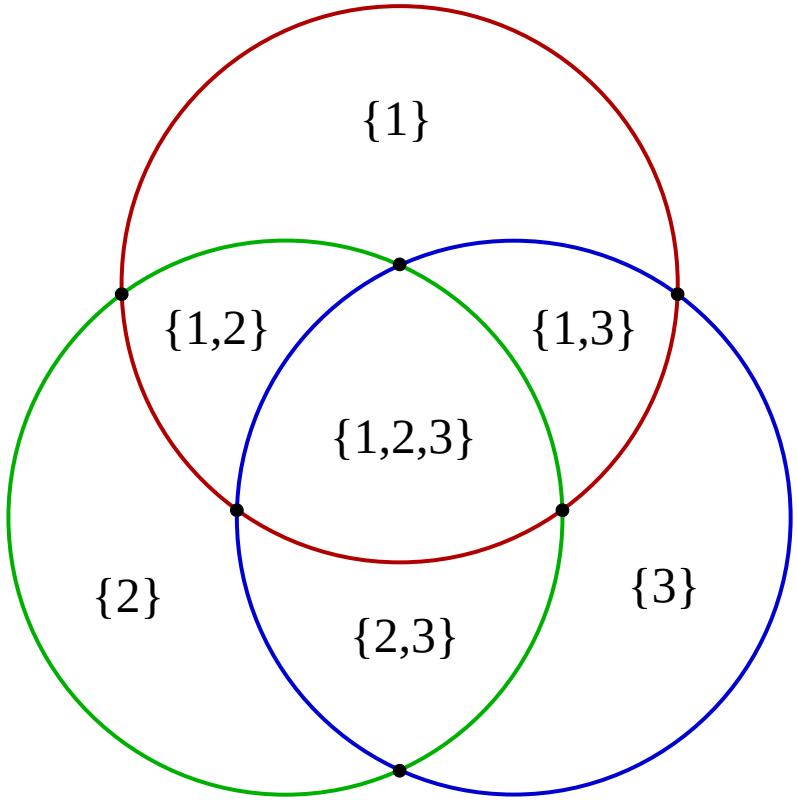

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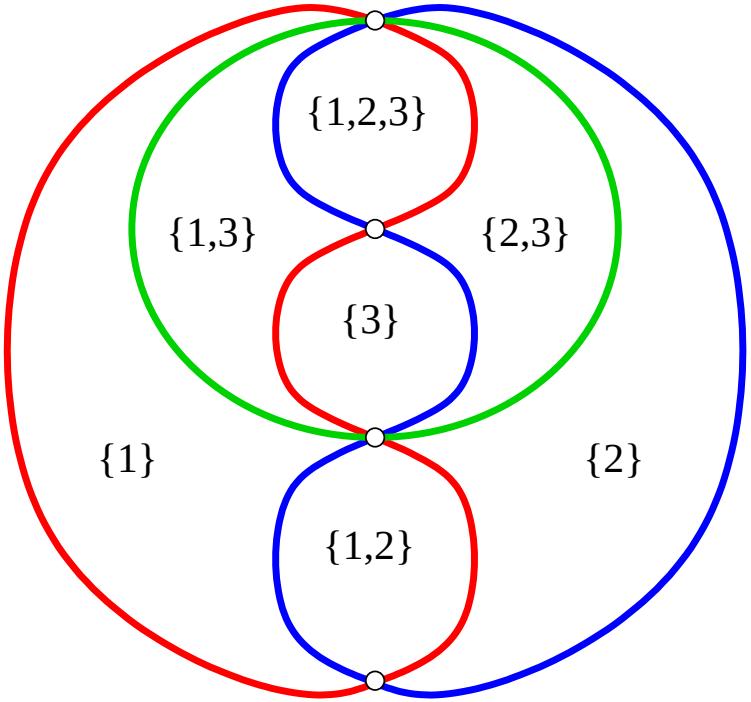

- Number of regions of rank k : $\binom{n}{k}$

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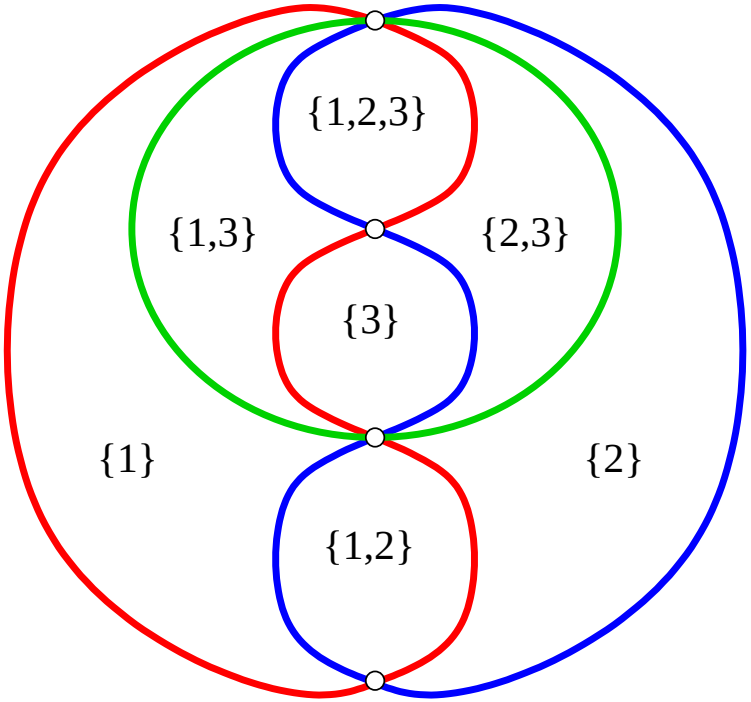

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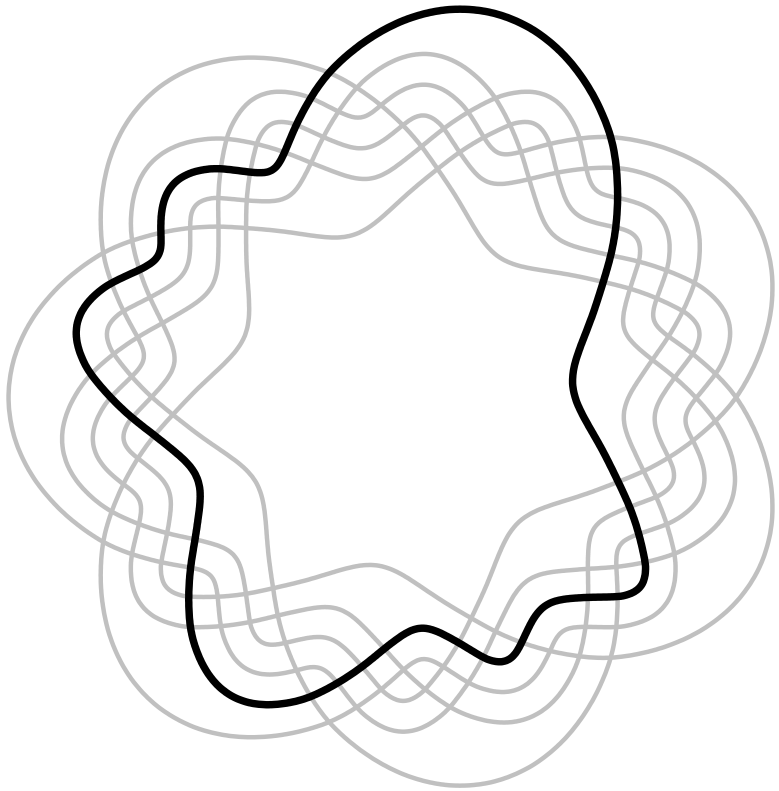
Rotationally-Symmetric Venn Diagrams

The diagram can be constructed by n successive rotations of a single closed curve by $\frac{2\pi}{n}$ radians about a point in the plane.



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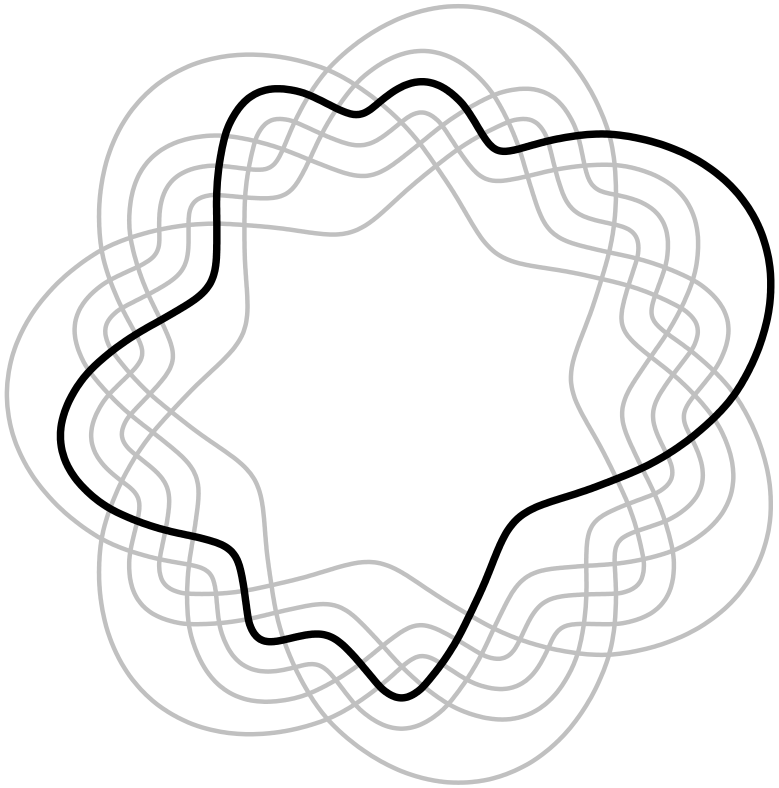

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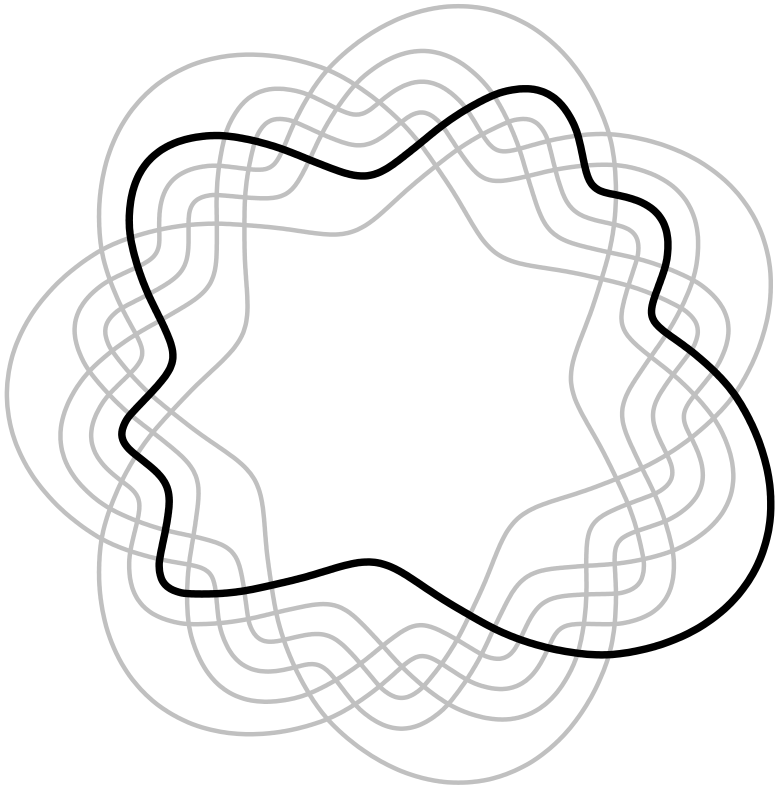

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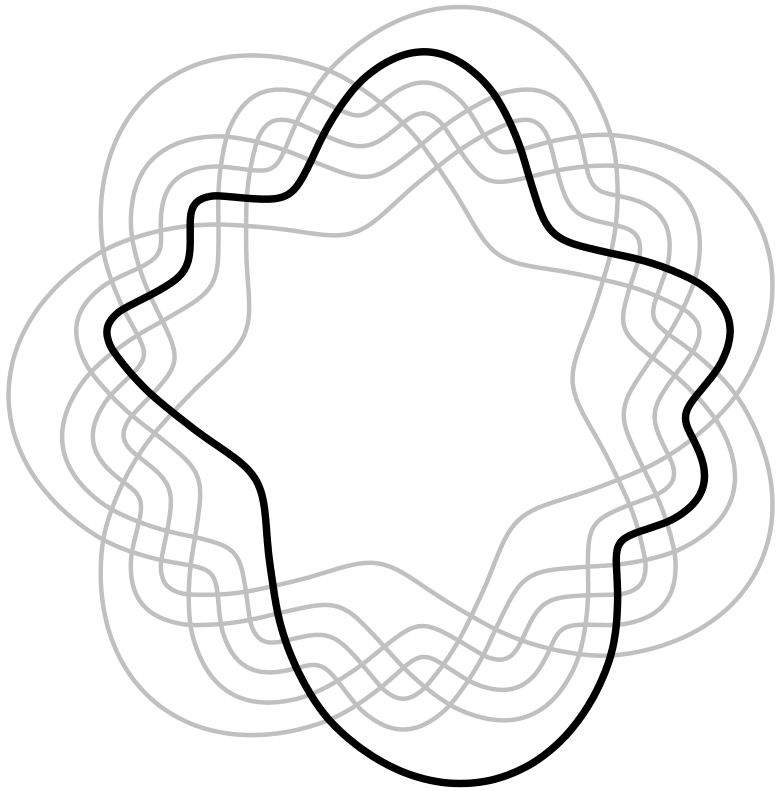

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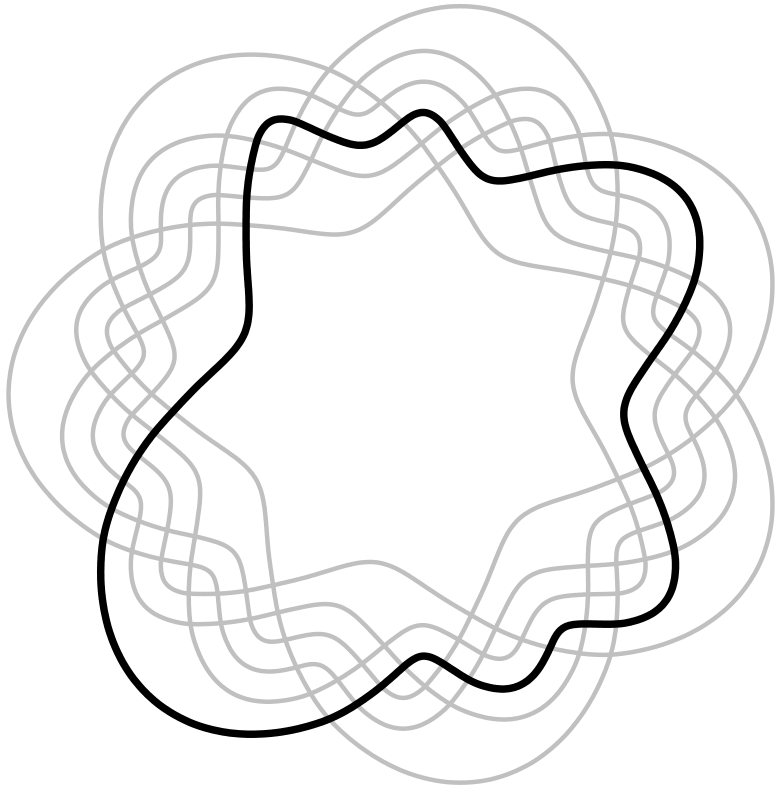

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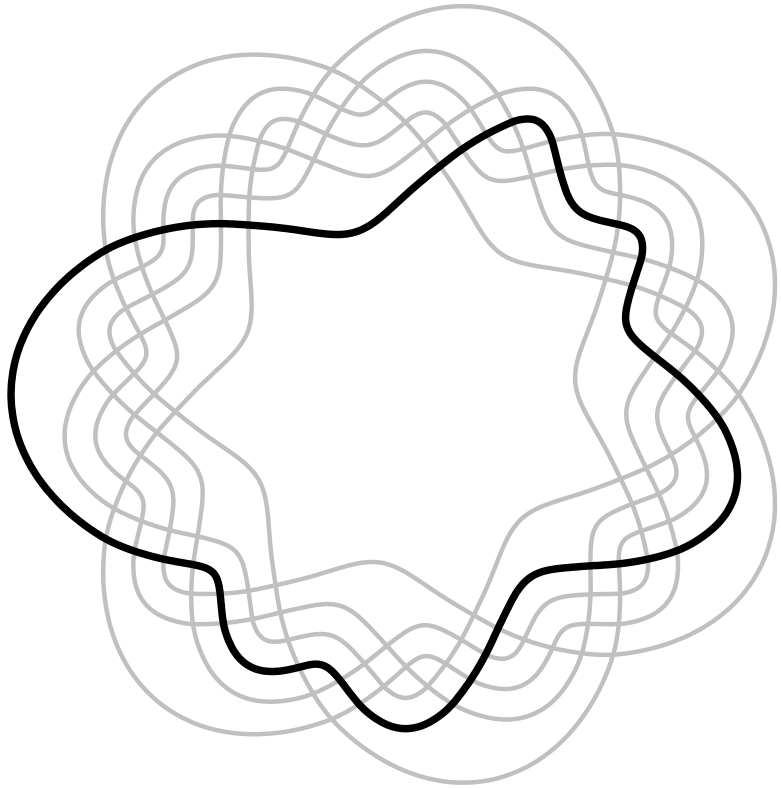

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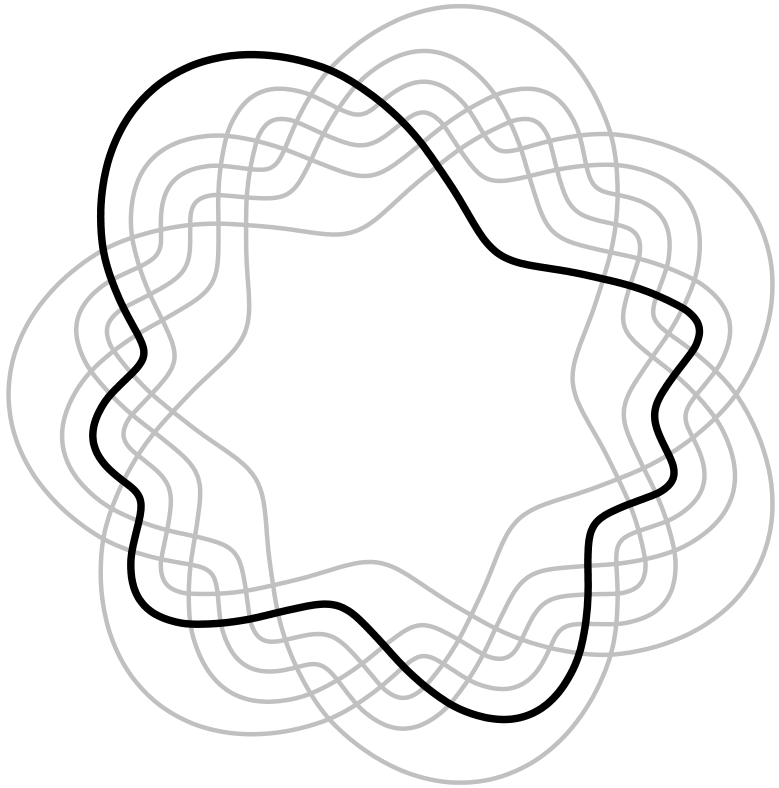

$f = 2^n, \quad e = 2 \times v$

$f + v = e + 2 \quad v = 2^n - 2$

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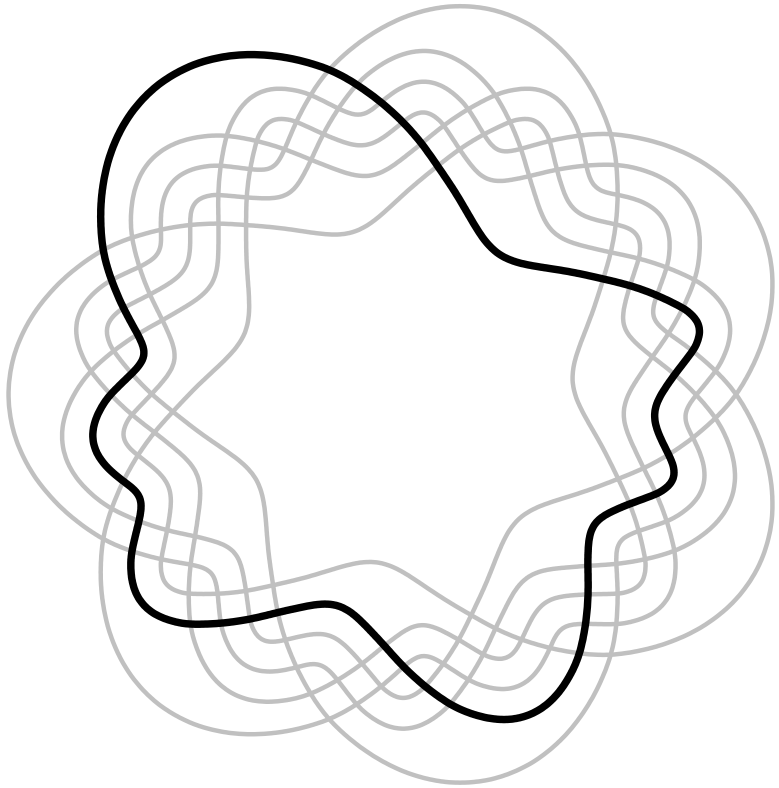

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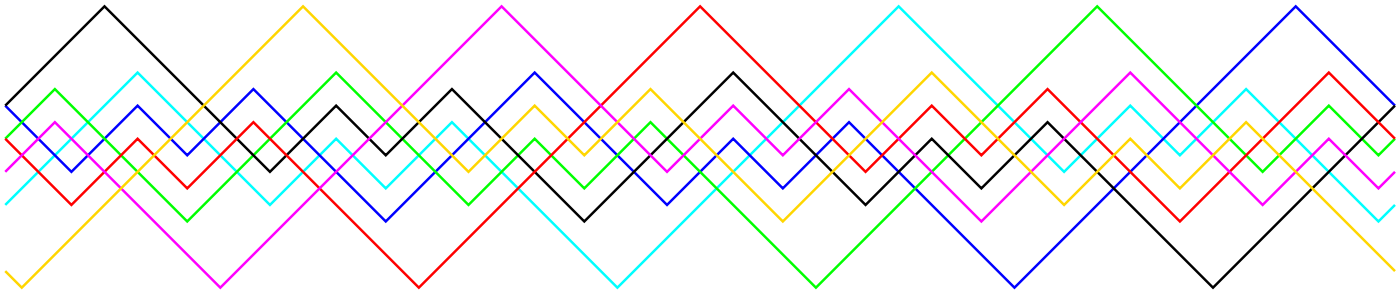
Polar Symmetric Venn diagrams

- Consider diagram as being projected on a cylinder



- Diagram remains invariant by turning cylinder upside-down

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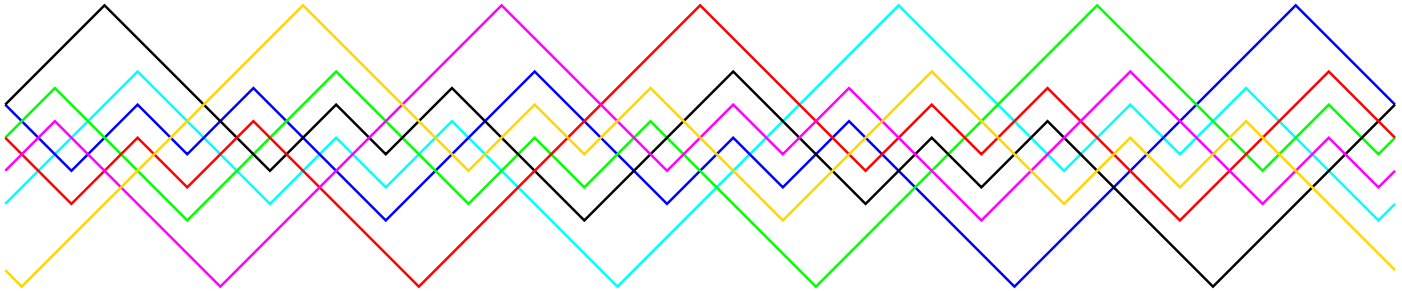

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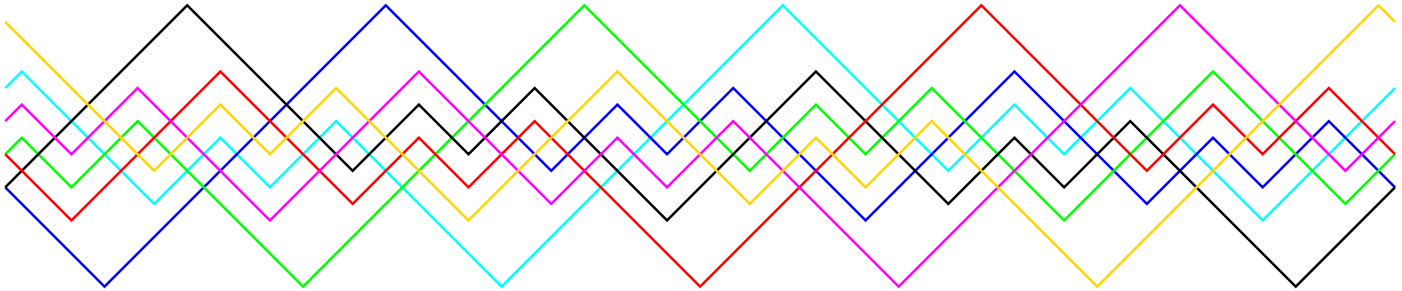

Symmetric Venn Diagrams

- If an n-Venn diagram is rotationally symmetric, then n is prime. [Henderson 1963]
- For any prime number of curves, there is a rotationally symmetric Venn diagram [Griggs, Killian and Savage 2004]

Theorem

Symmetric n-Venn diagrams exist if and only if n is prime.

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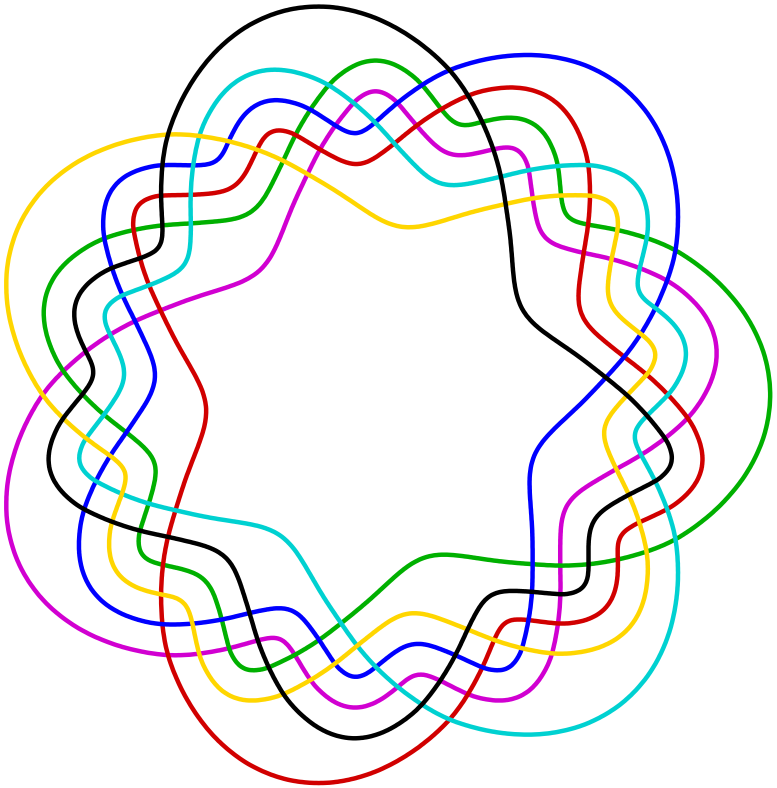
Matrix representation



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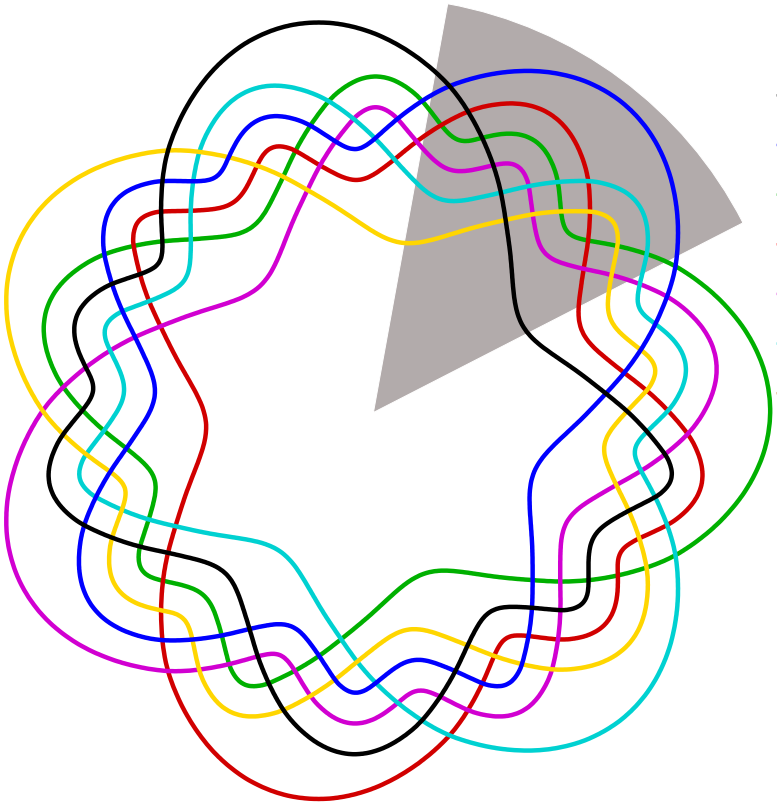

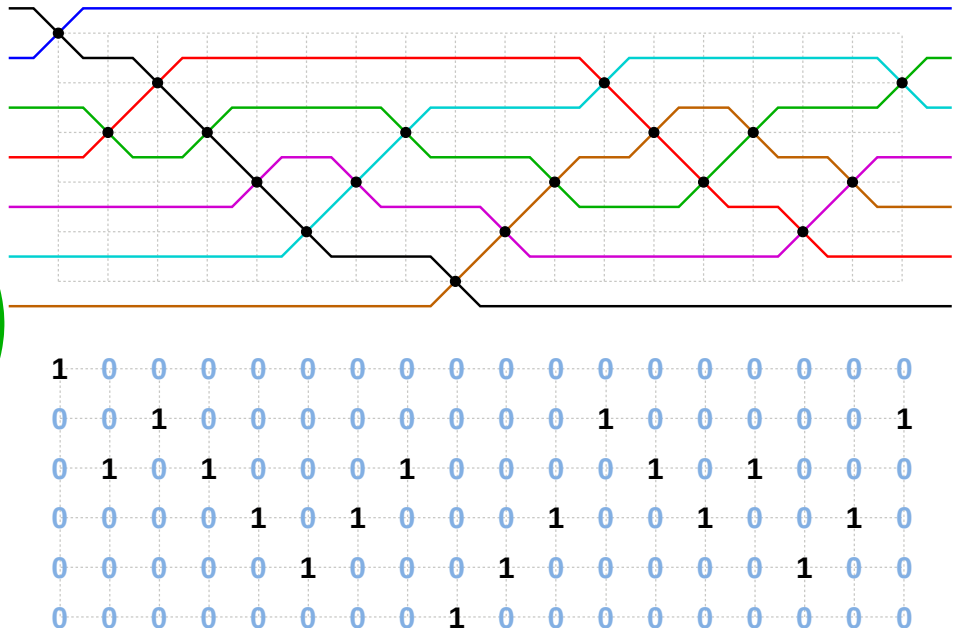

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0	1	0	1	0	0	0	1	0	0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0
0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

- Not all matrices represent Venn diagrams

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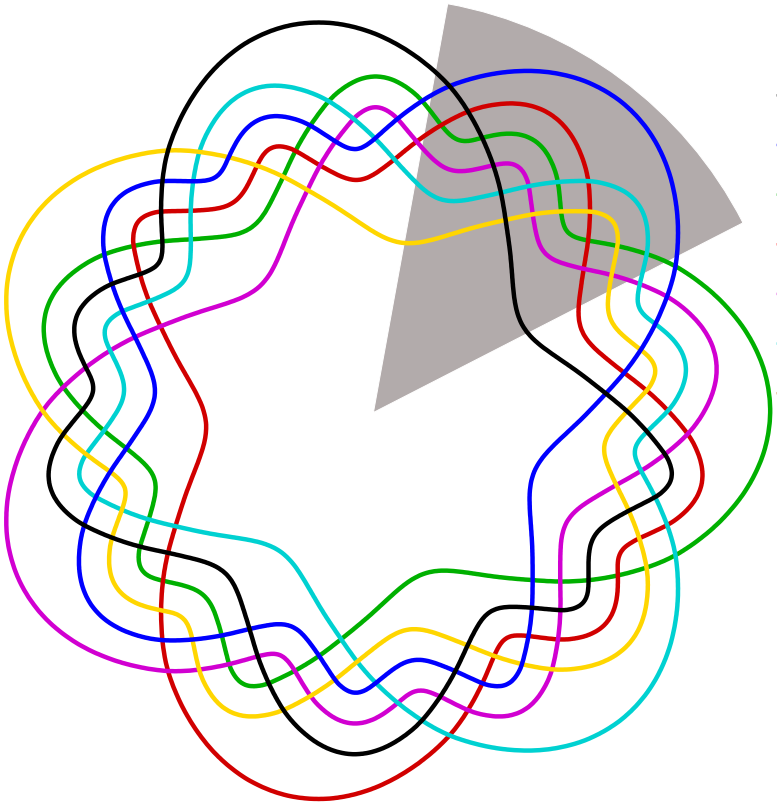

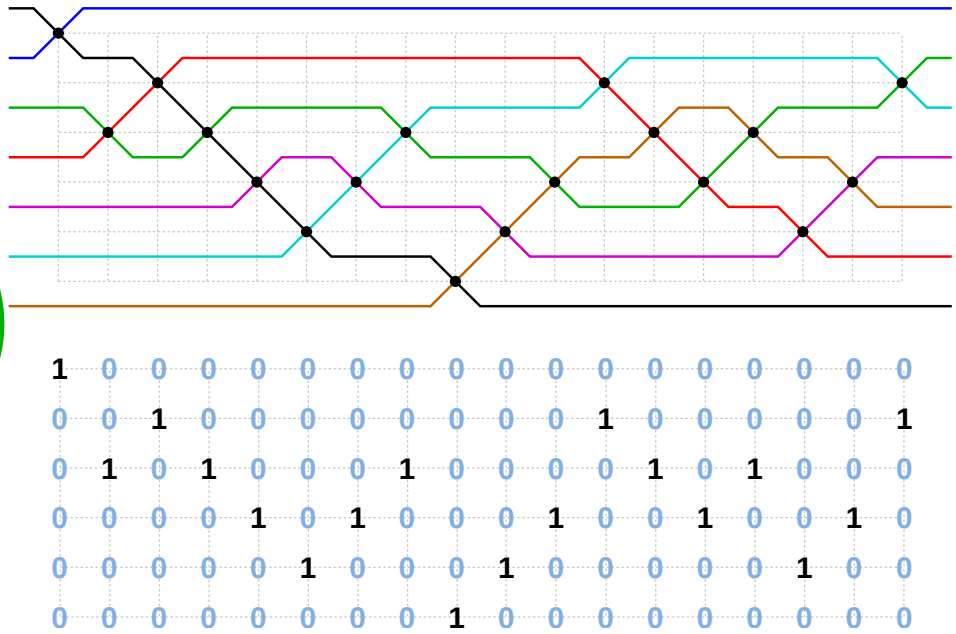

$$\begin{array}{cccccccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

- Not all matrices represent Venn diagrams
- Different matrices may represent isomorphic Venn diagrams

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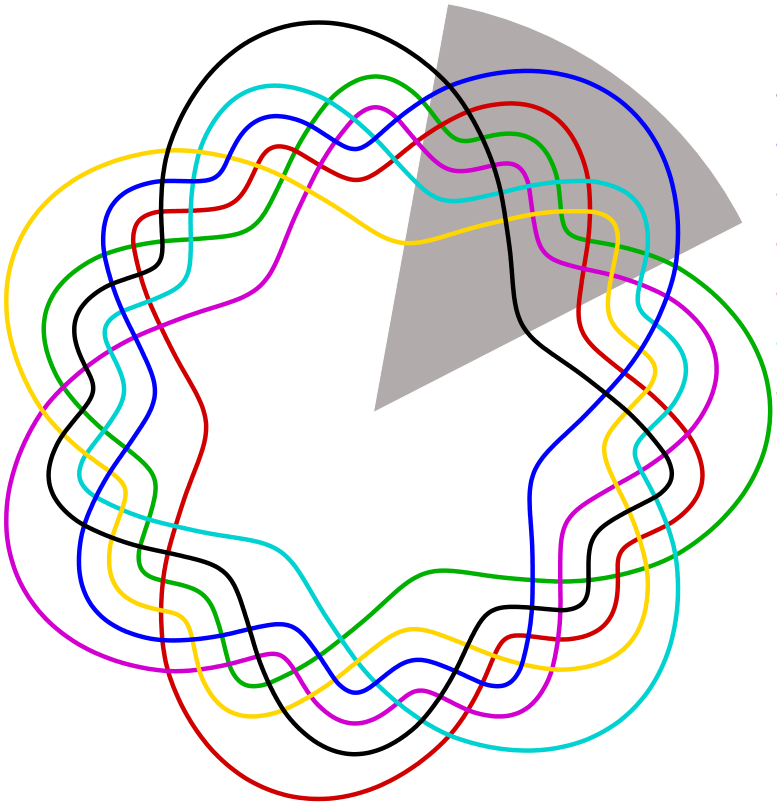

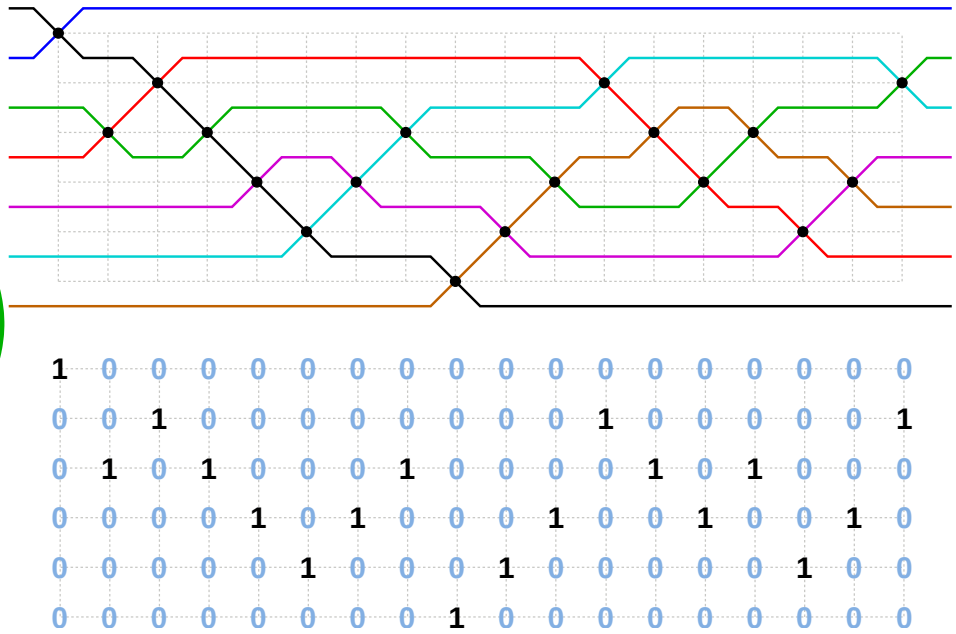

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&1\\
0&1&0&1&0&0&0&1&0&0&0&0&1&0&1&0&0&0\\
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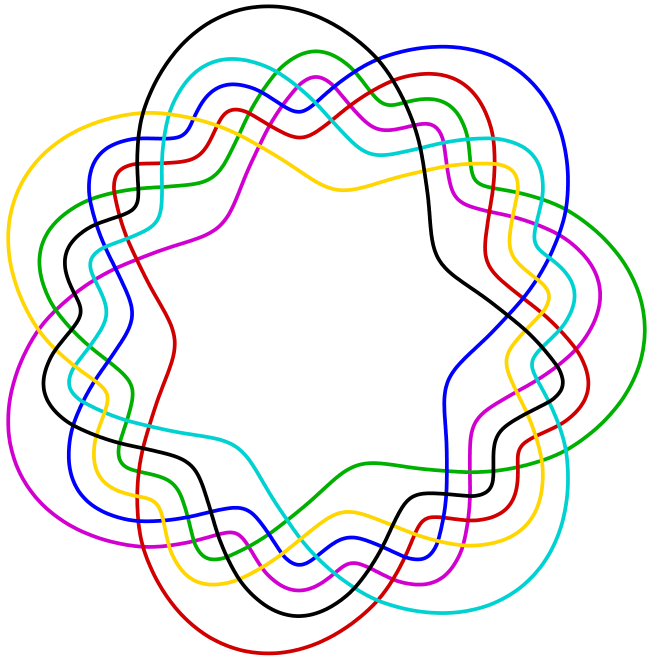
Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



W:

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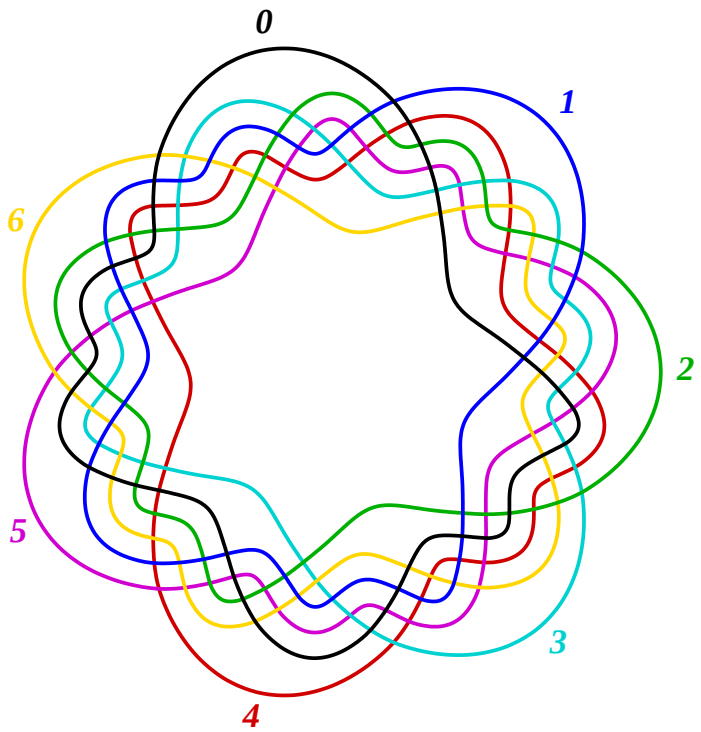

W: 1

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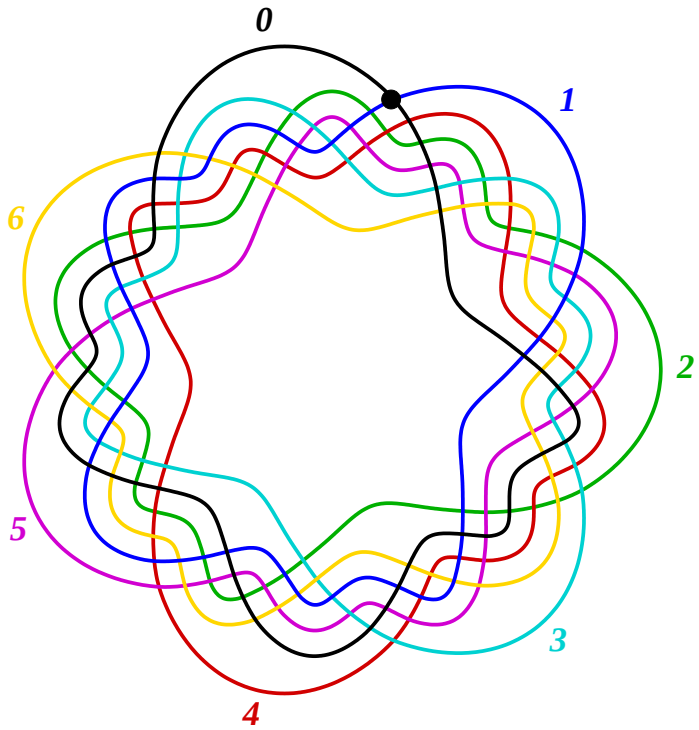

W: 1,4

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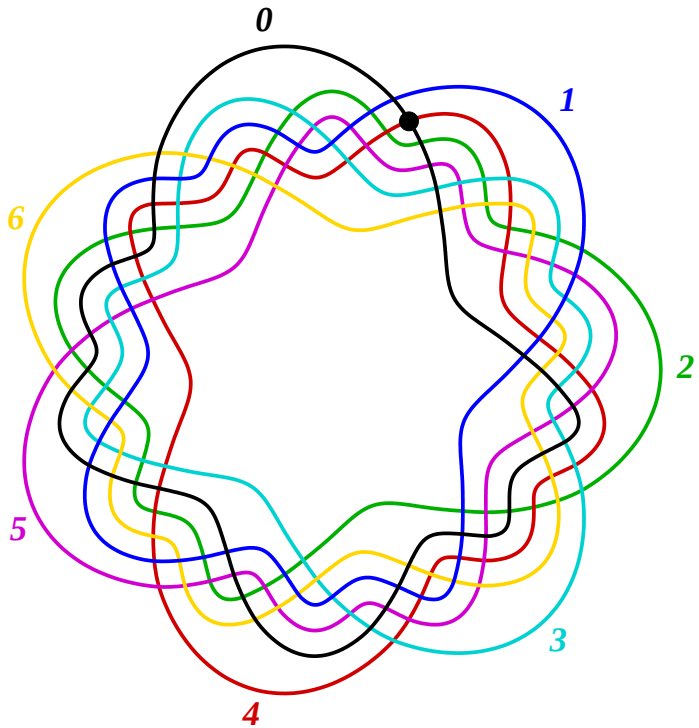

W: 1,4,2

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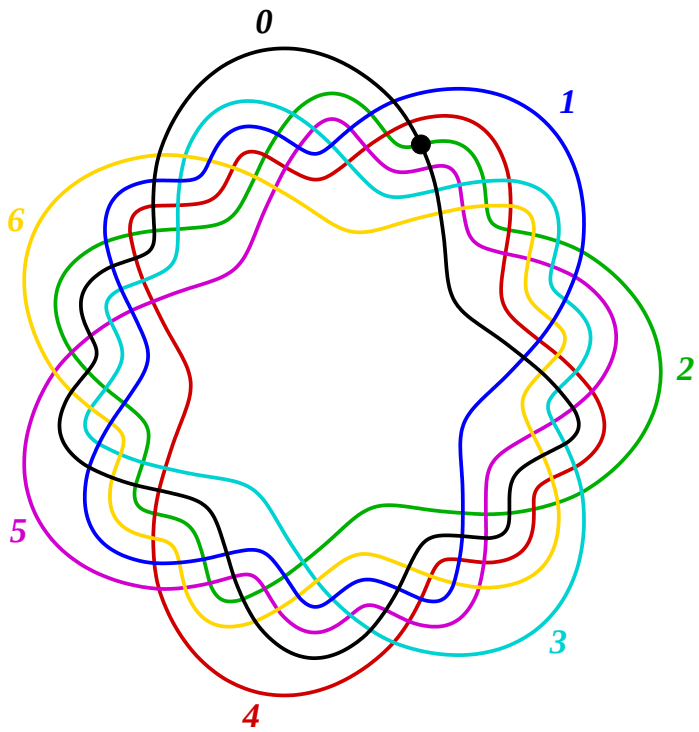

W: 1,4,2,5

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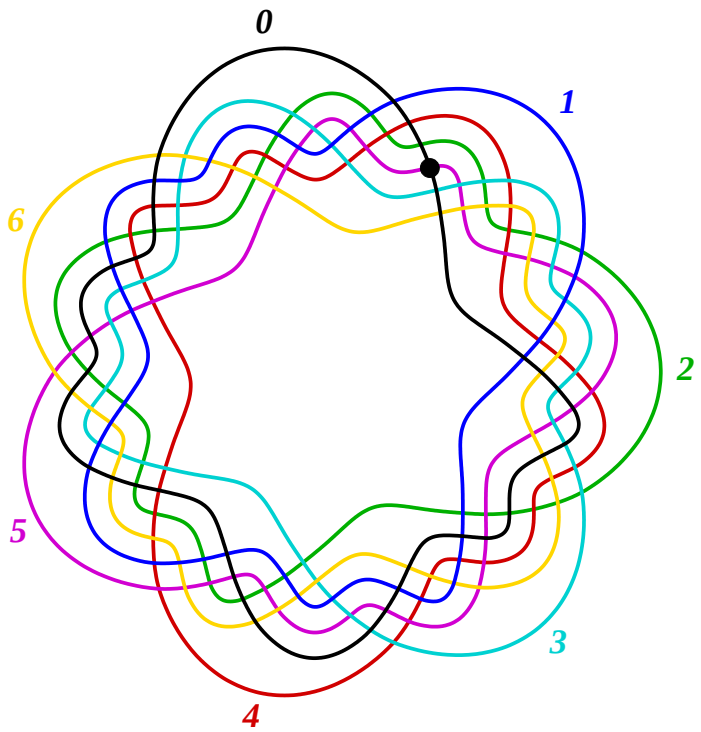

W: 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6

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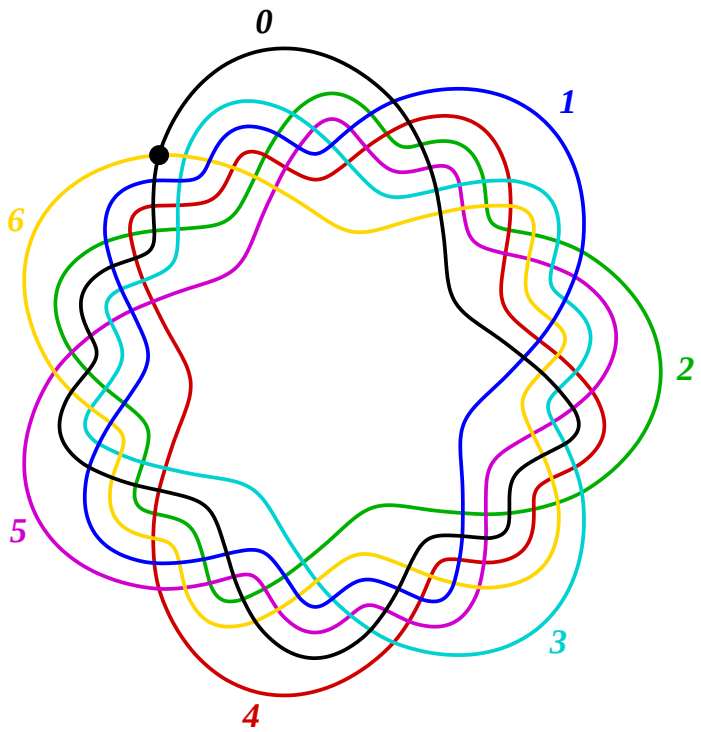

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X :

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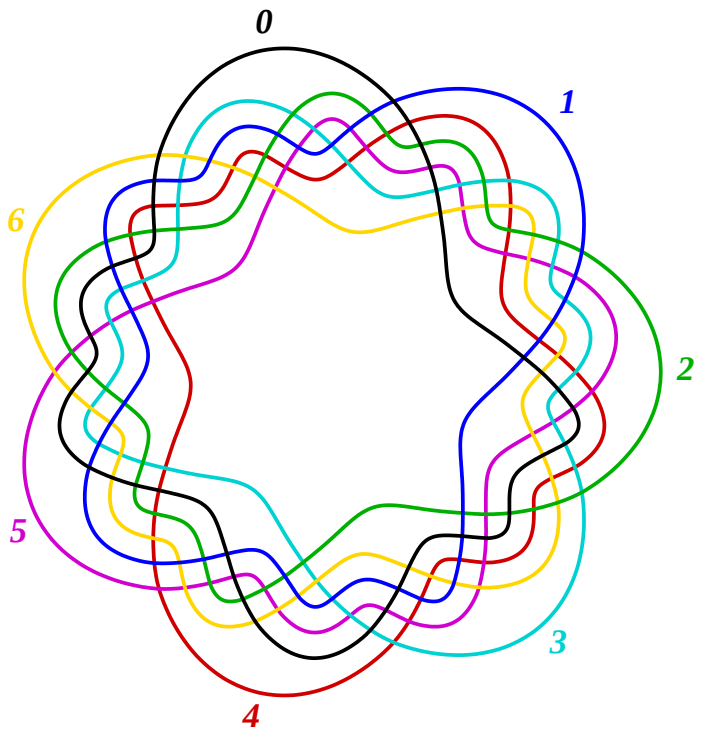

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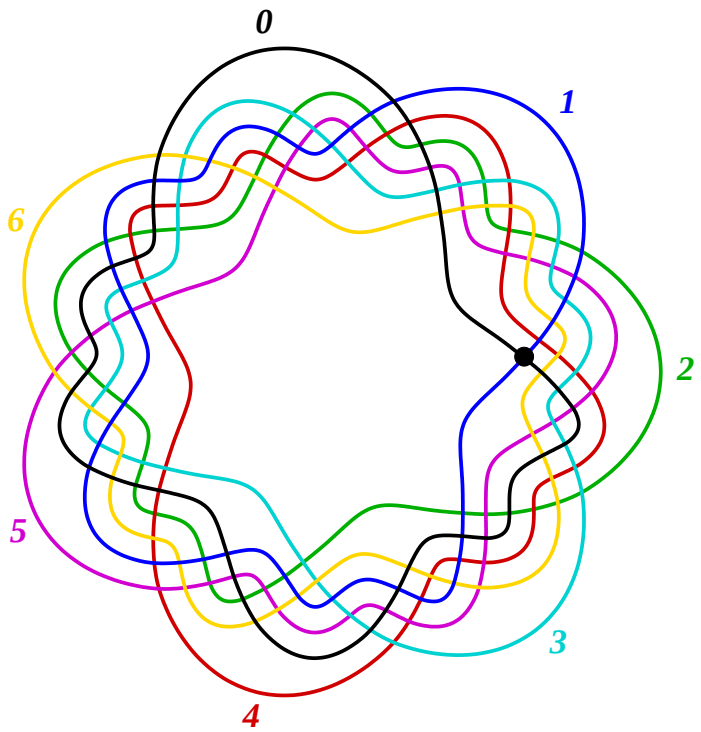

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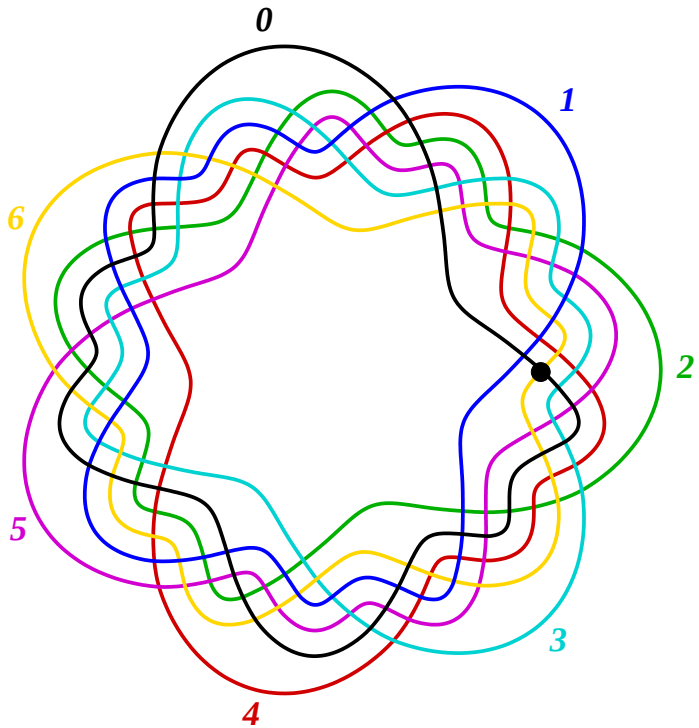

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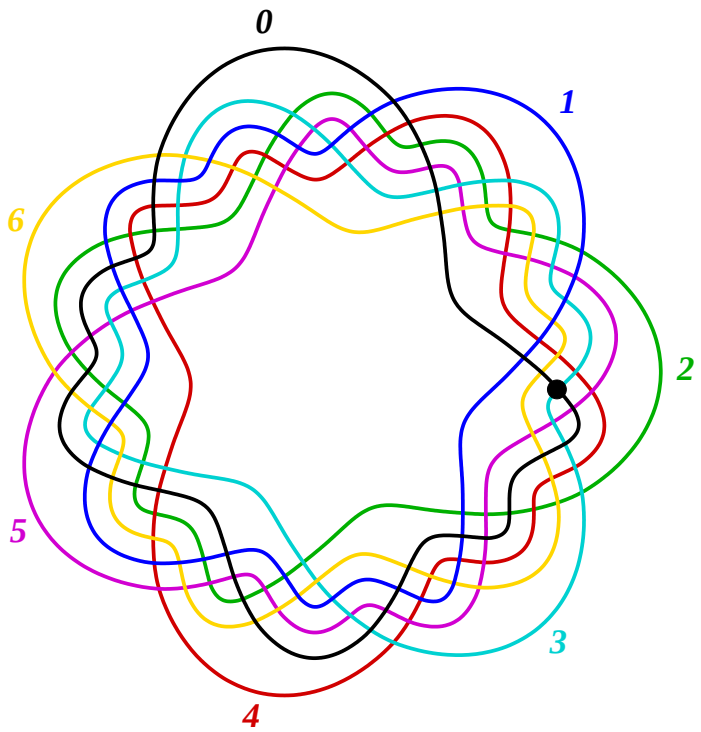

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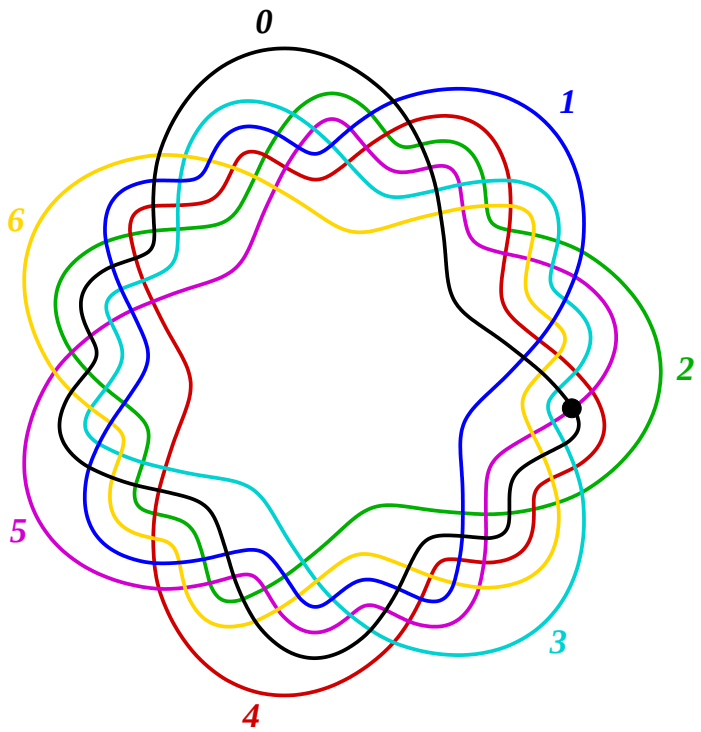
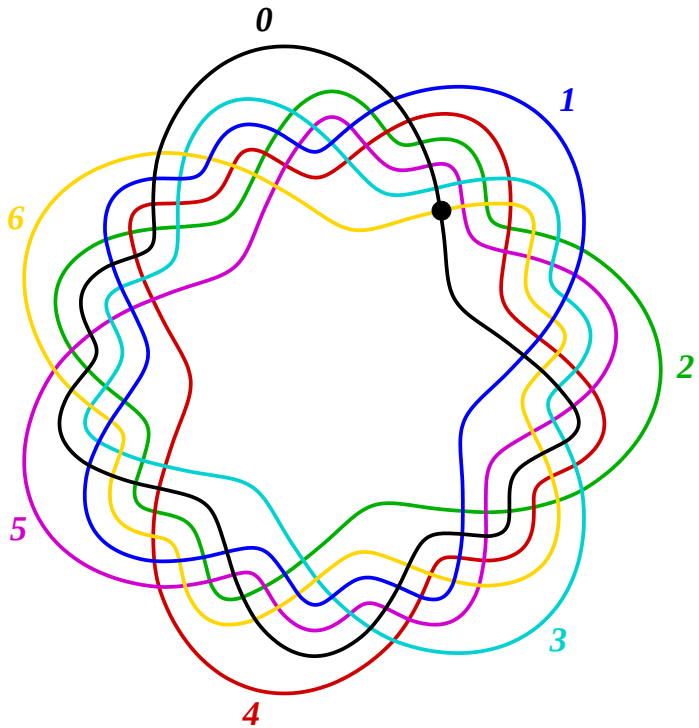


W: 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6

Grünbaum Encoding

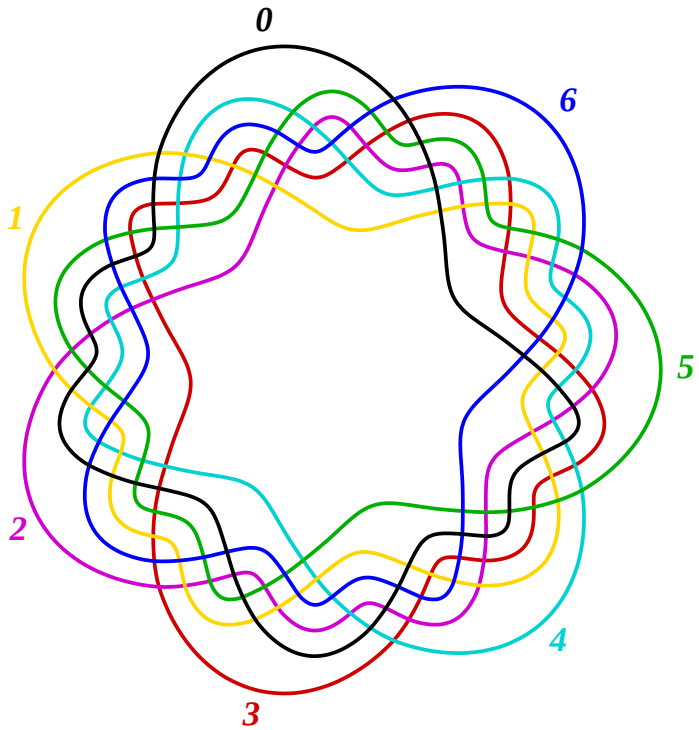
Introduced by Grünbaum to check whether two Venn diagrams are distinct.



W: 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y :

Grünbaum Encoding

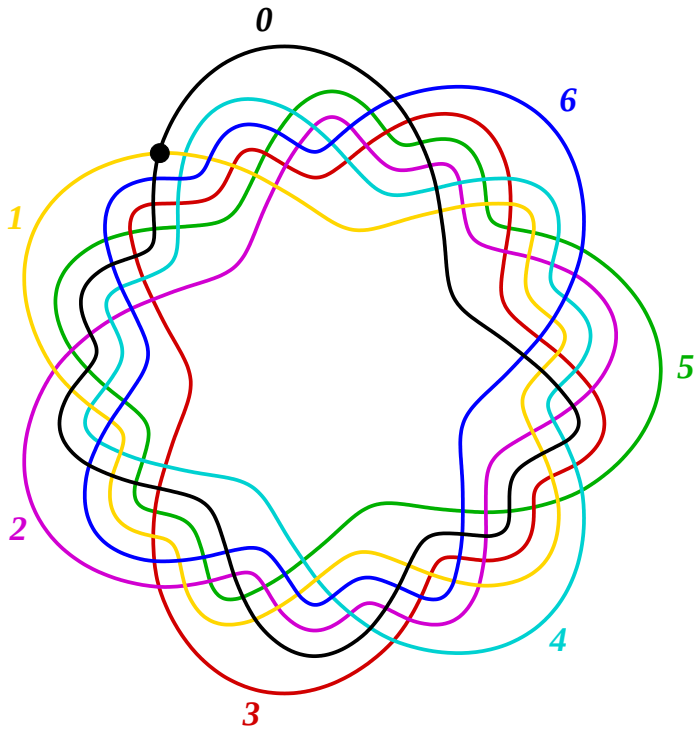
Introduced by Grünbaum to check whether two Venn diagrams are distinct.



W: 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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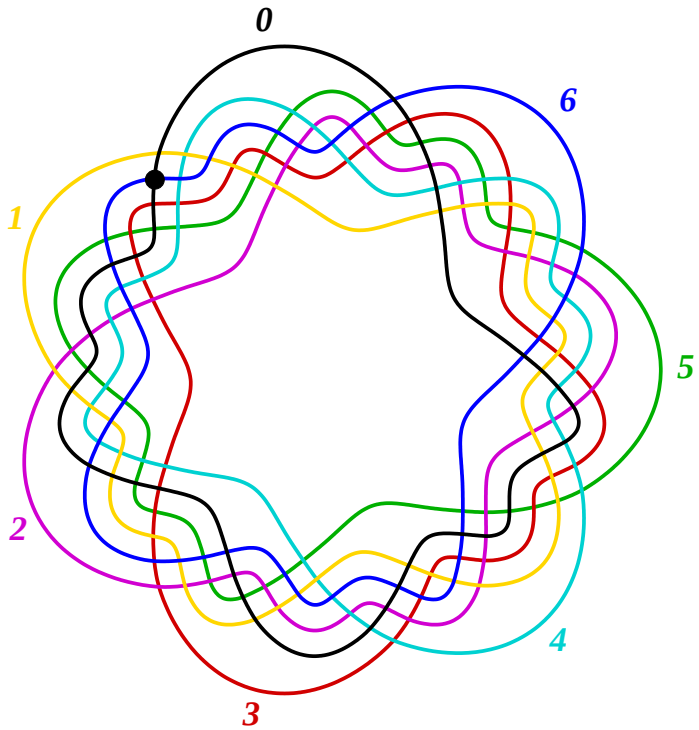
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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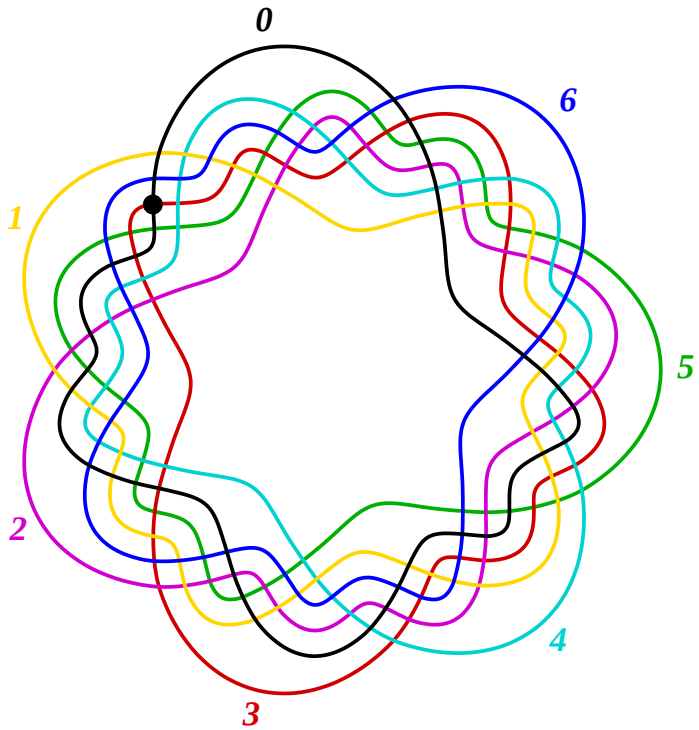
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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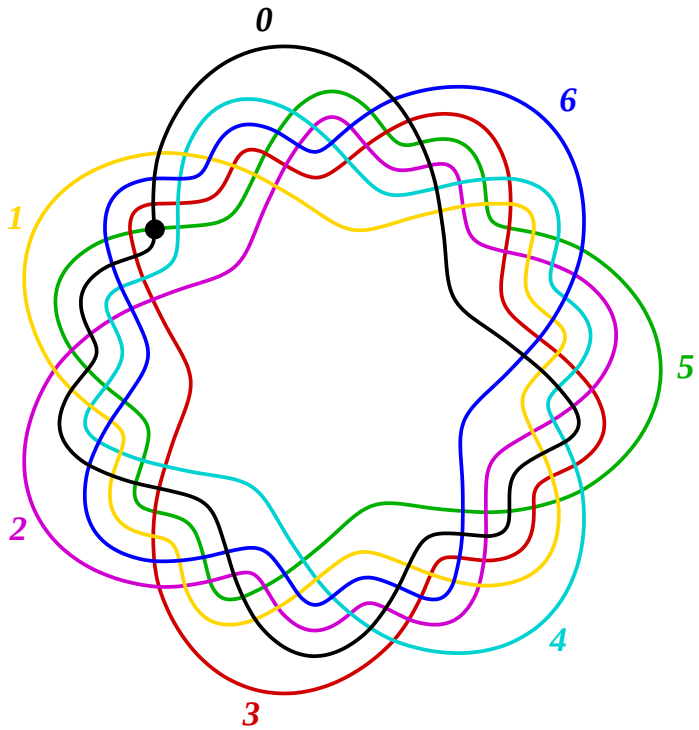
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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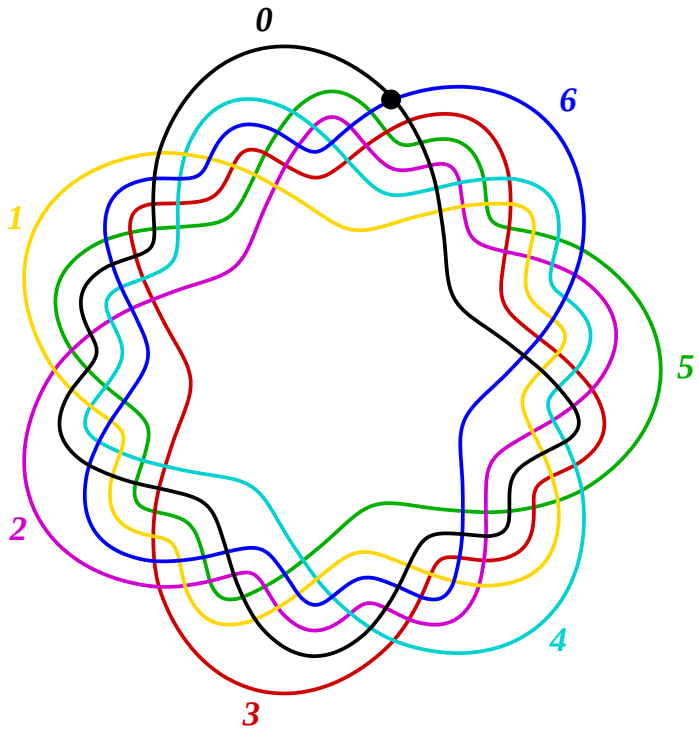
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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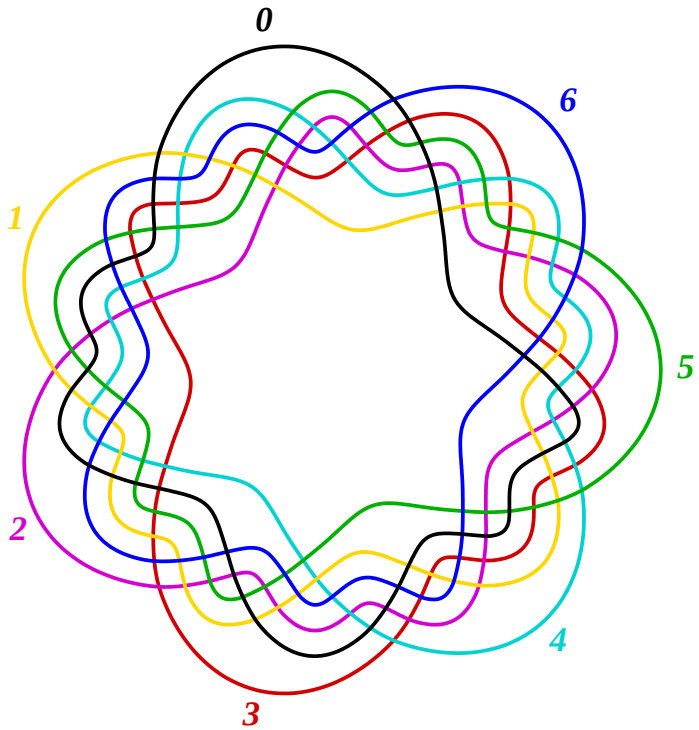
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z :

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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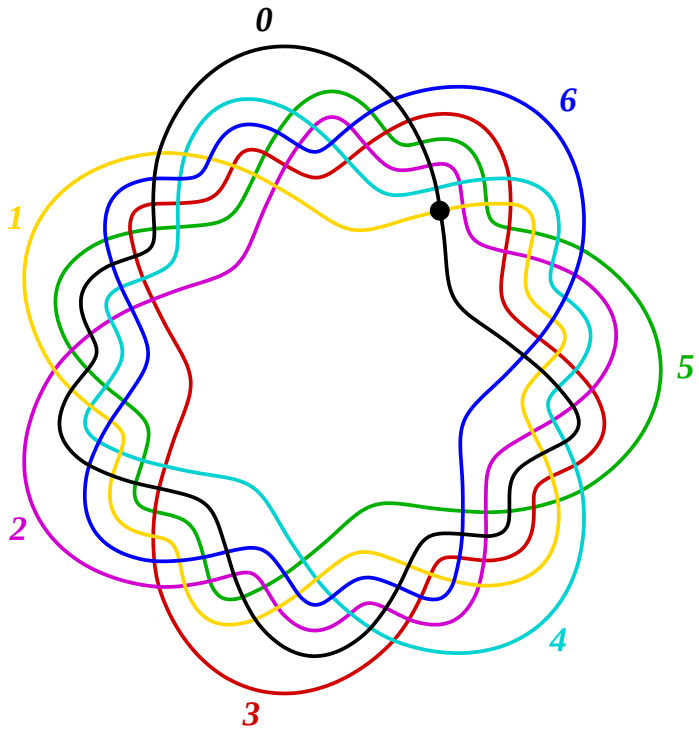
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z : 1

Grünbaum Encoding

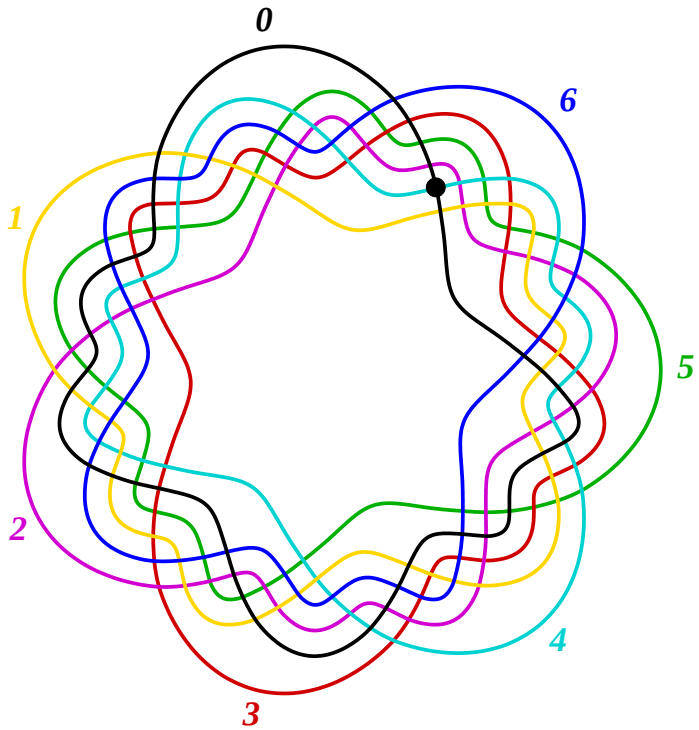
Introduced by Grünbaum to check whether two Venn diagrams are distinct.



W: 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z : 1,4

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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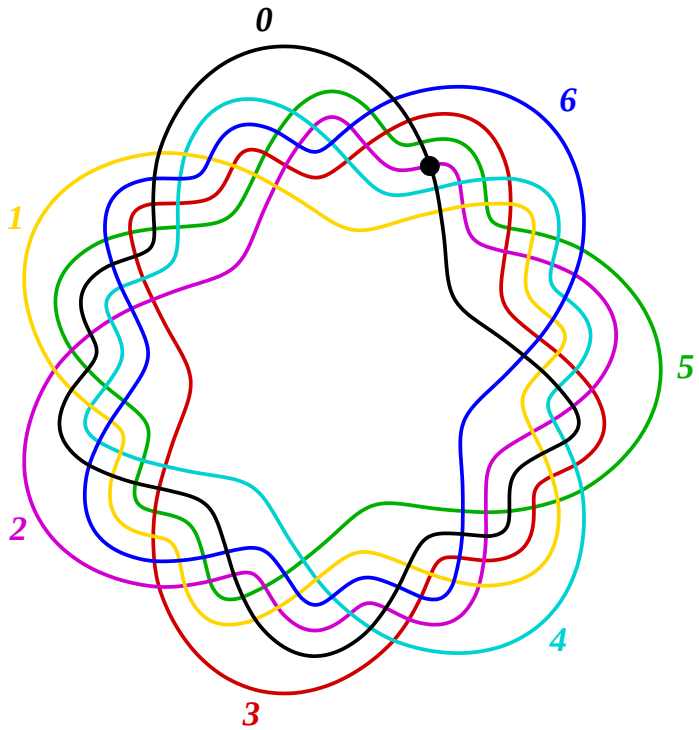
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z : 1,4,2

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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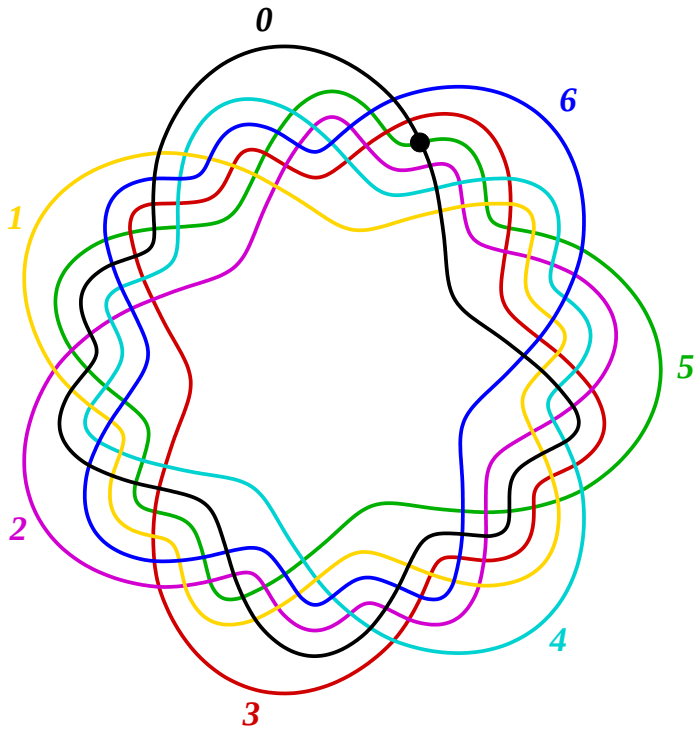
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z : 1,4,2,5

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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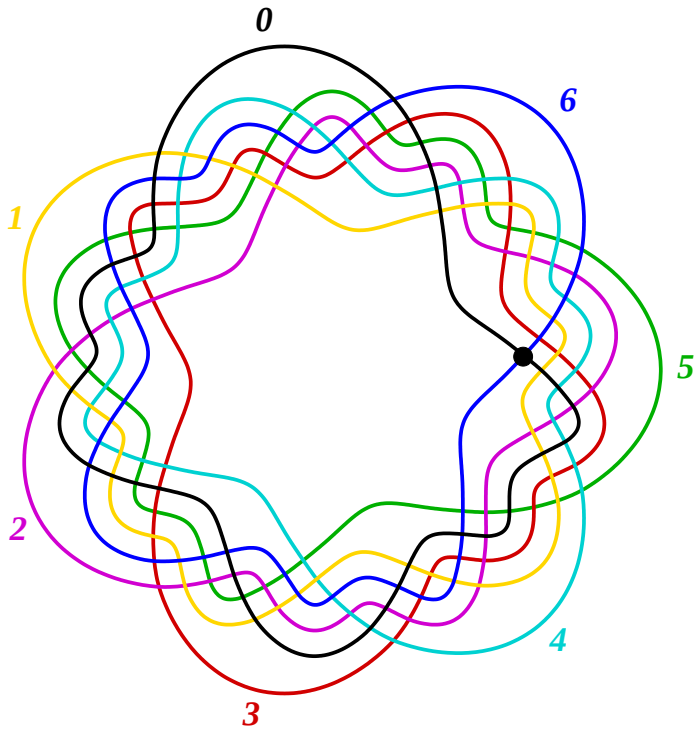
X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z : 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6

Grünbaum Encoding

Introduced by Grünbaum to check whether two Venn diagrams are distinct.



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X : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Y : 1,6,3,5 ,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6,1,4,2,5,3,6
Z : 1,4,2,5 ,3,6,1,6,3,5,3,6,2,5,1,6,1,5,3,6,2,5,1,4,2,6,1,6,2,5,1,4,2,4,1,6

Algorithm

Theorem

Each Grübaum encoding uniquely determines a simple symmetric monotone Venn diagram(up to isomorphism)

1. Generate all candidate matrices.
2. Check validity of matrix.

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1. Generate all candidate matrices.
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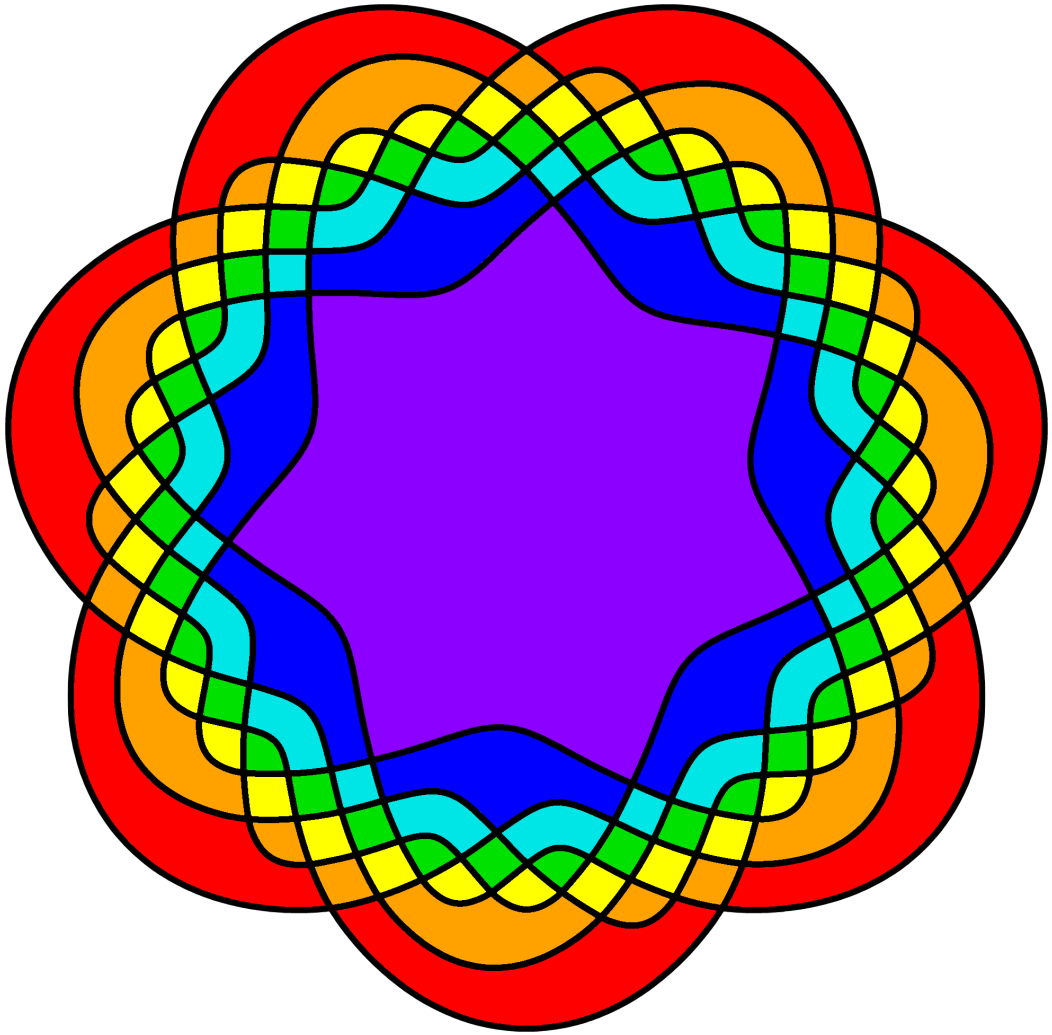
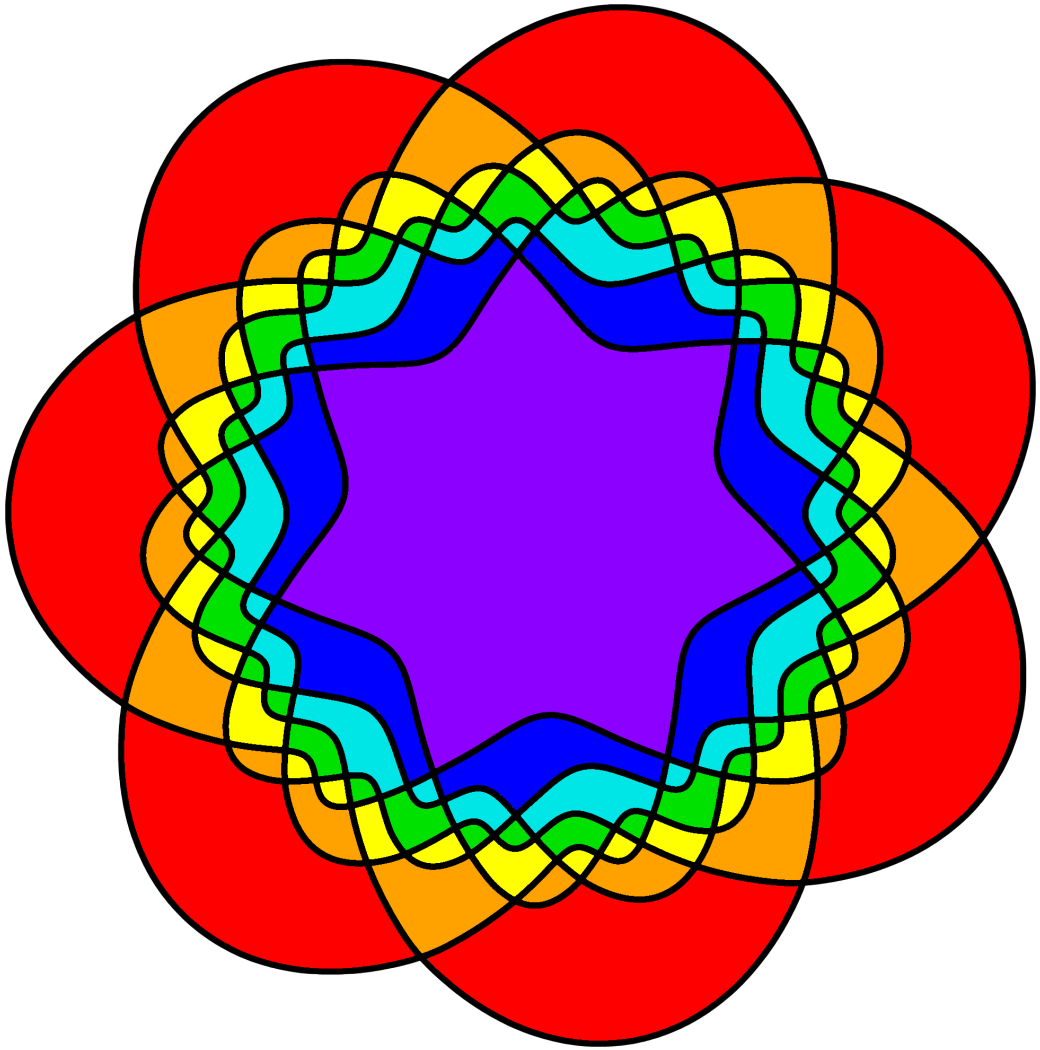
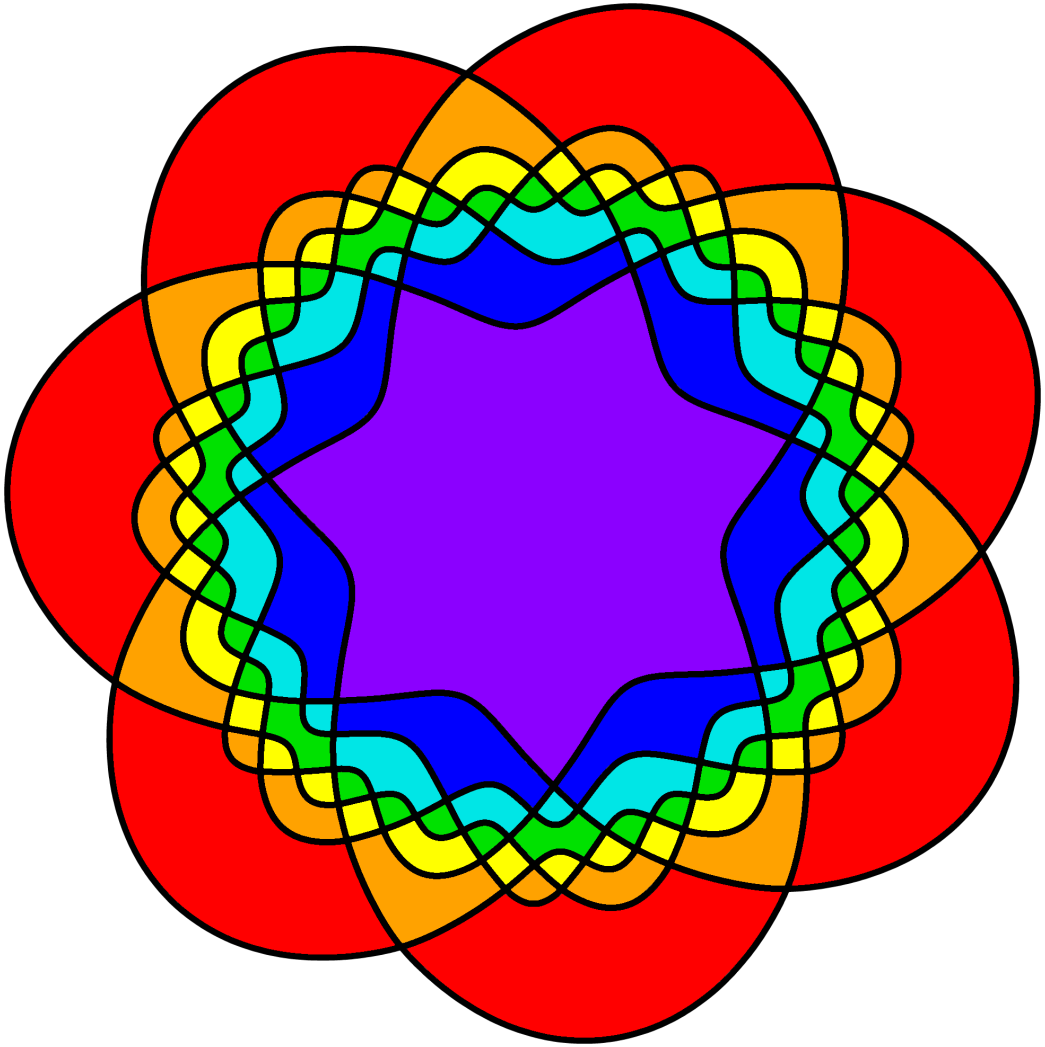
Results



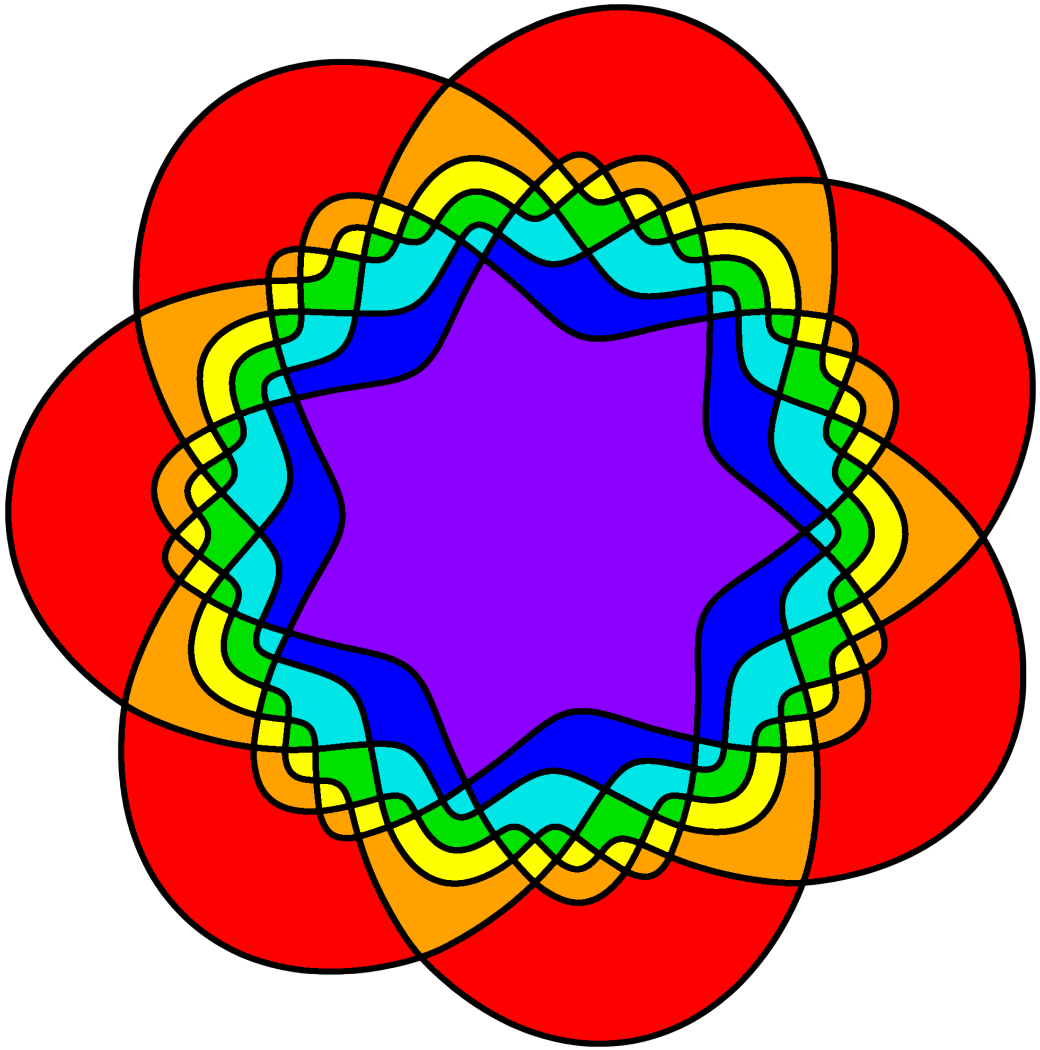
Results



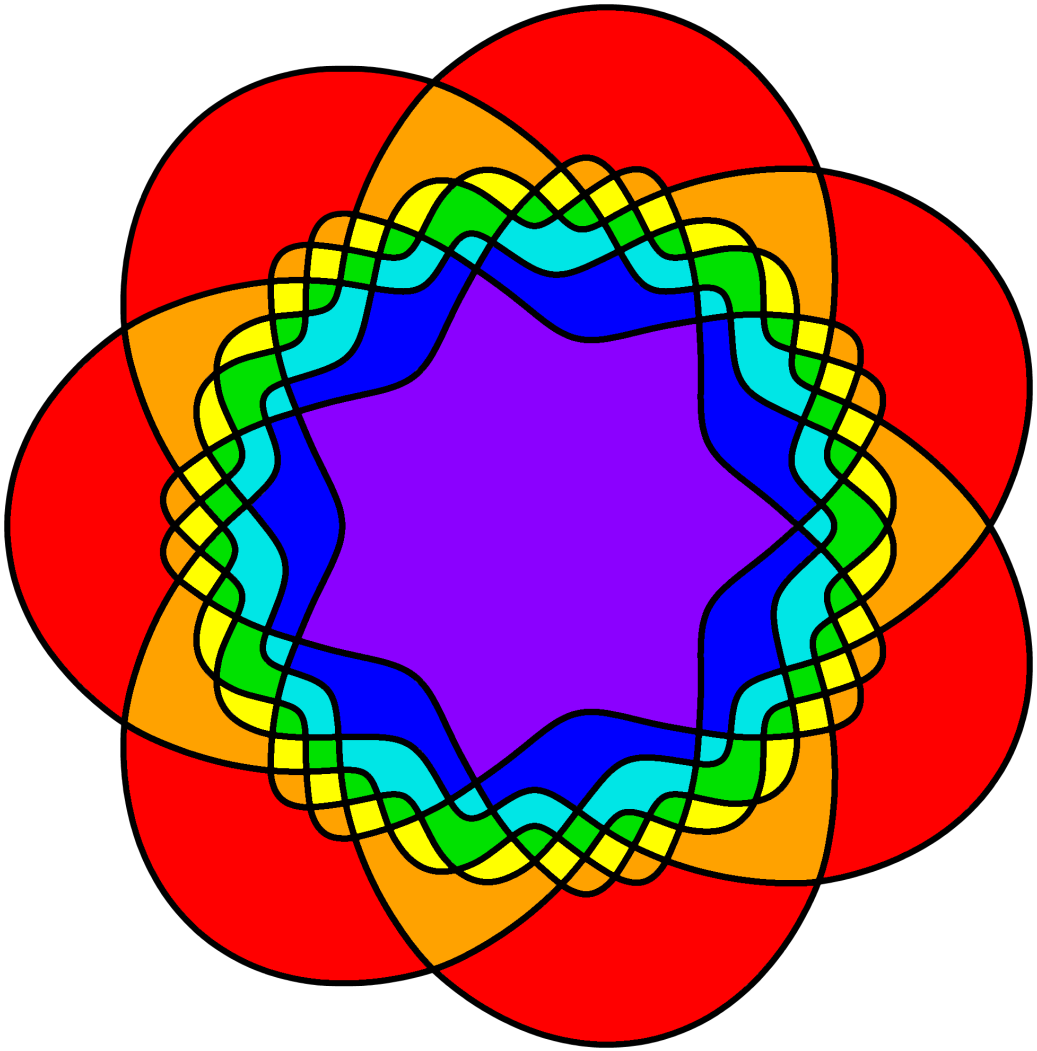
Results



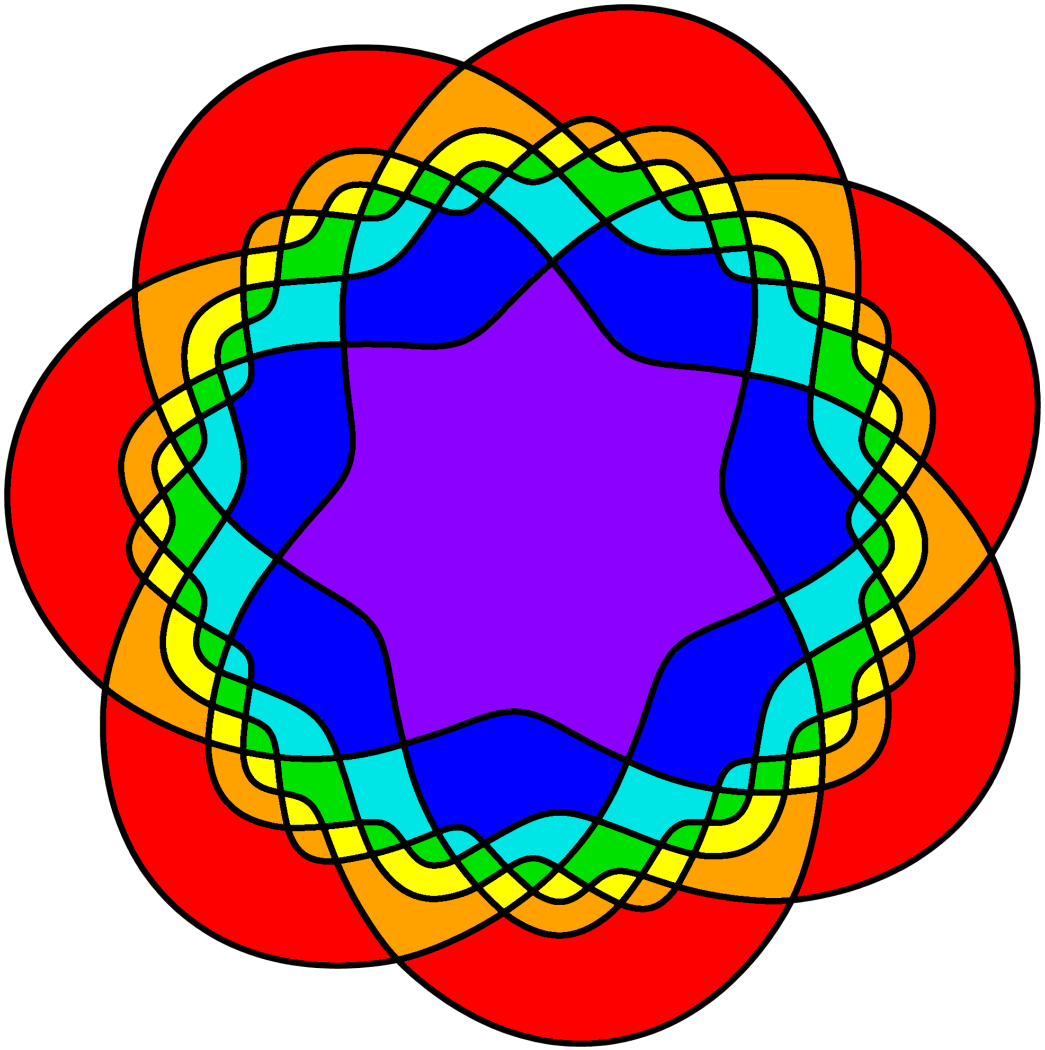
Results



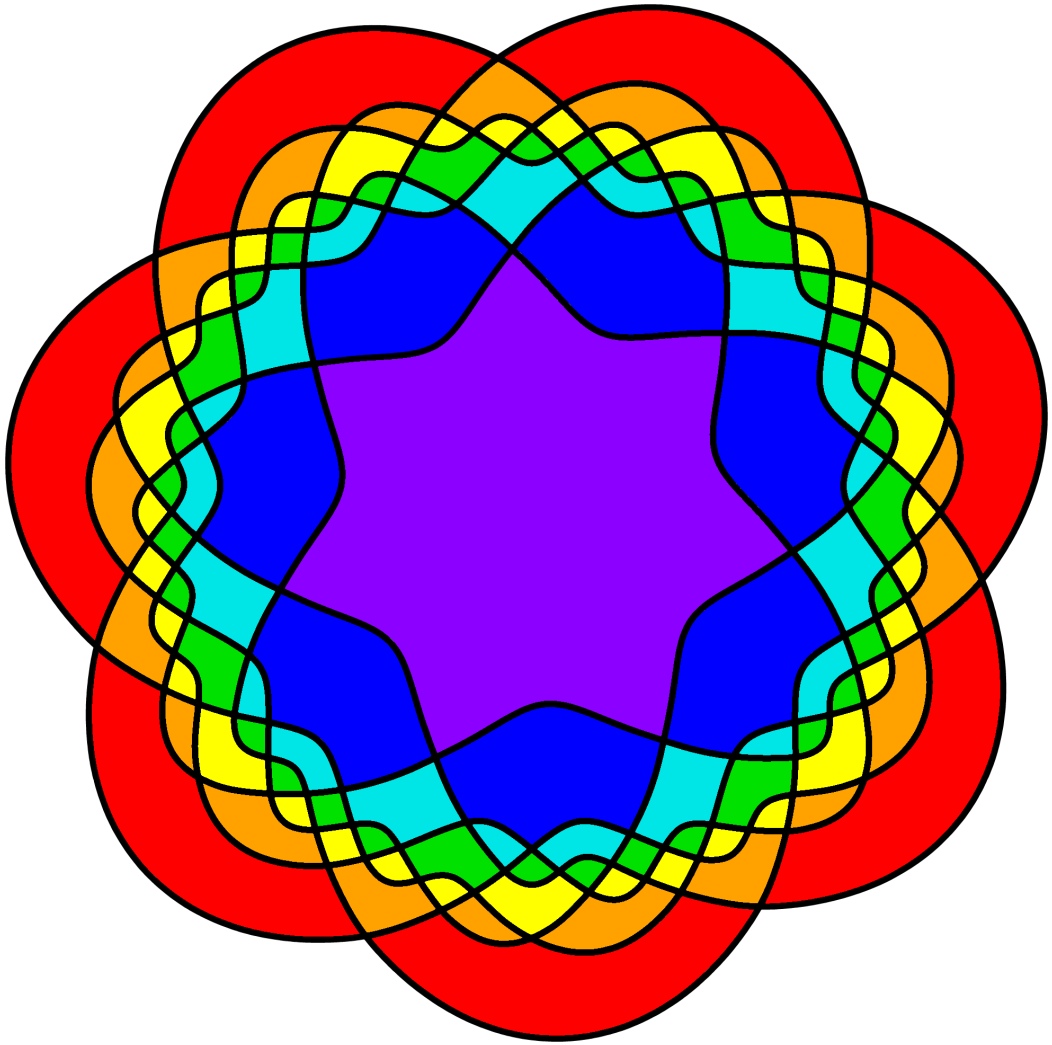
Results



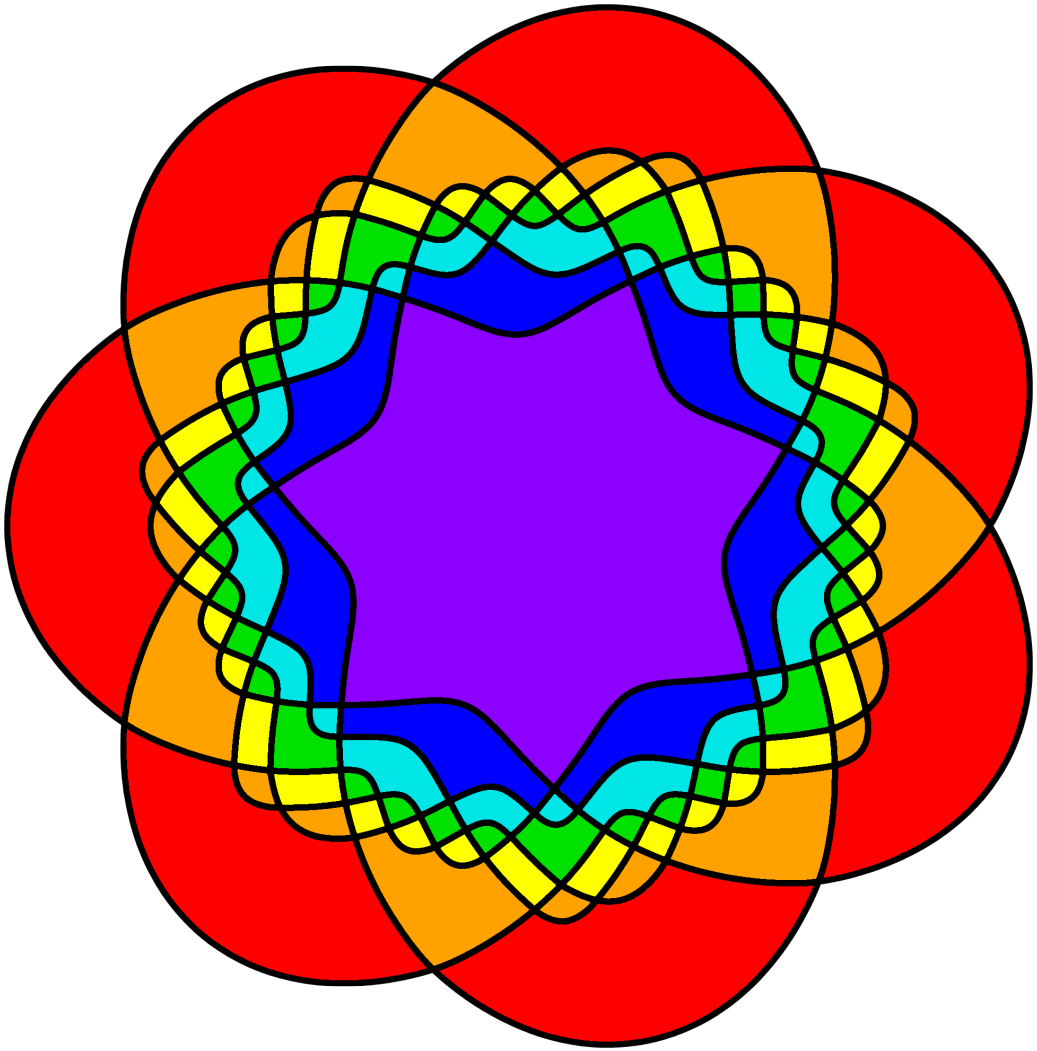
Results



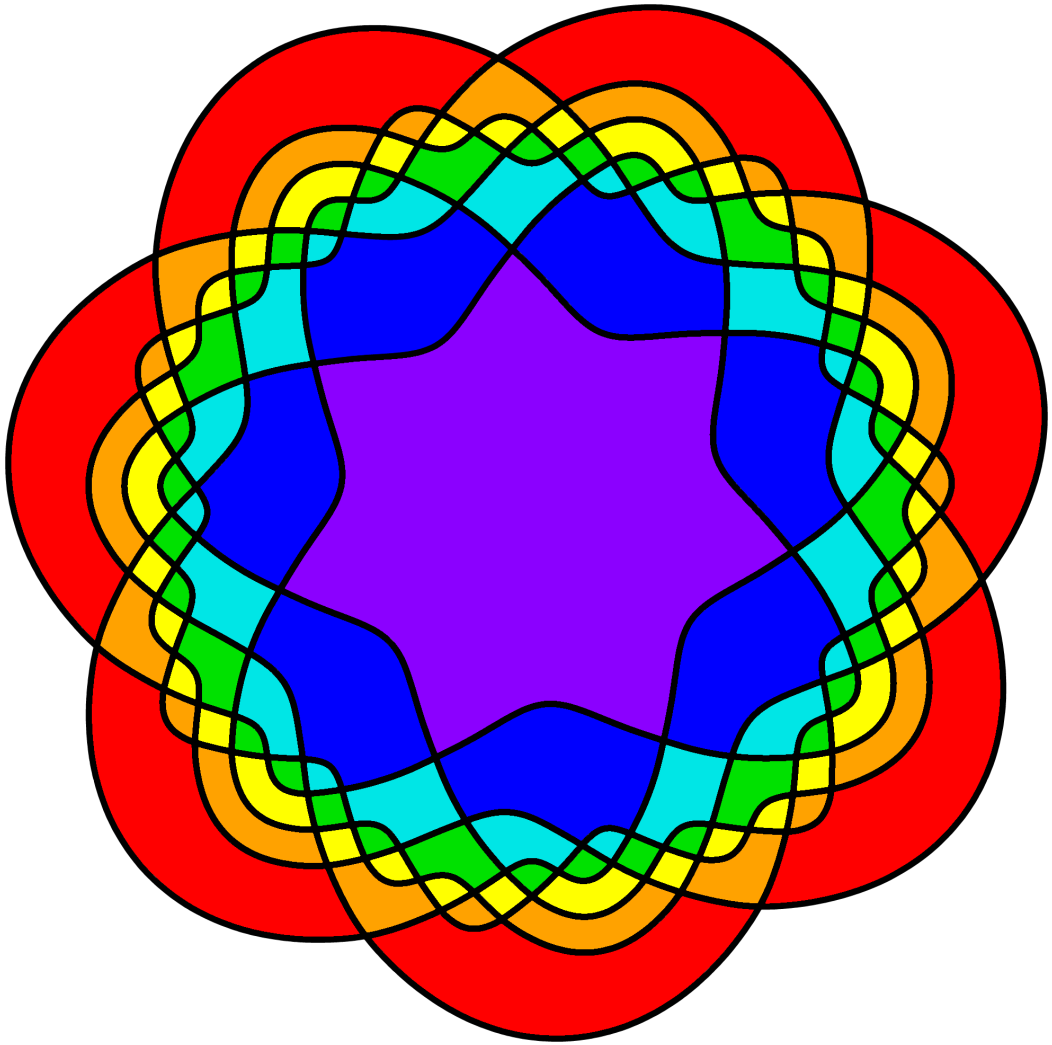
Results



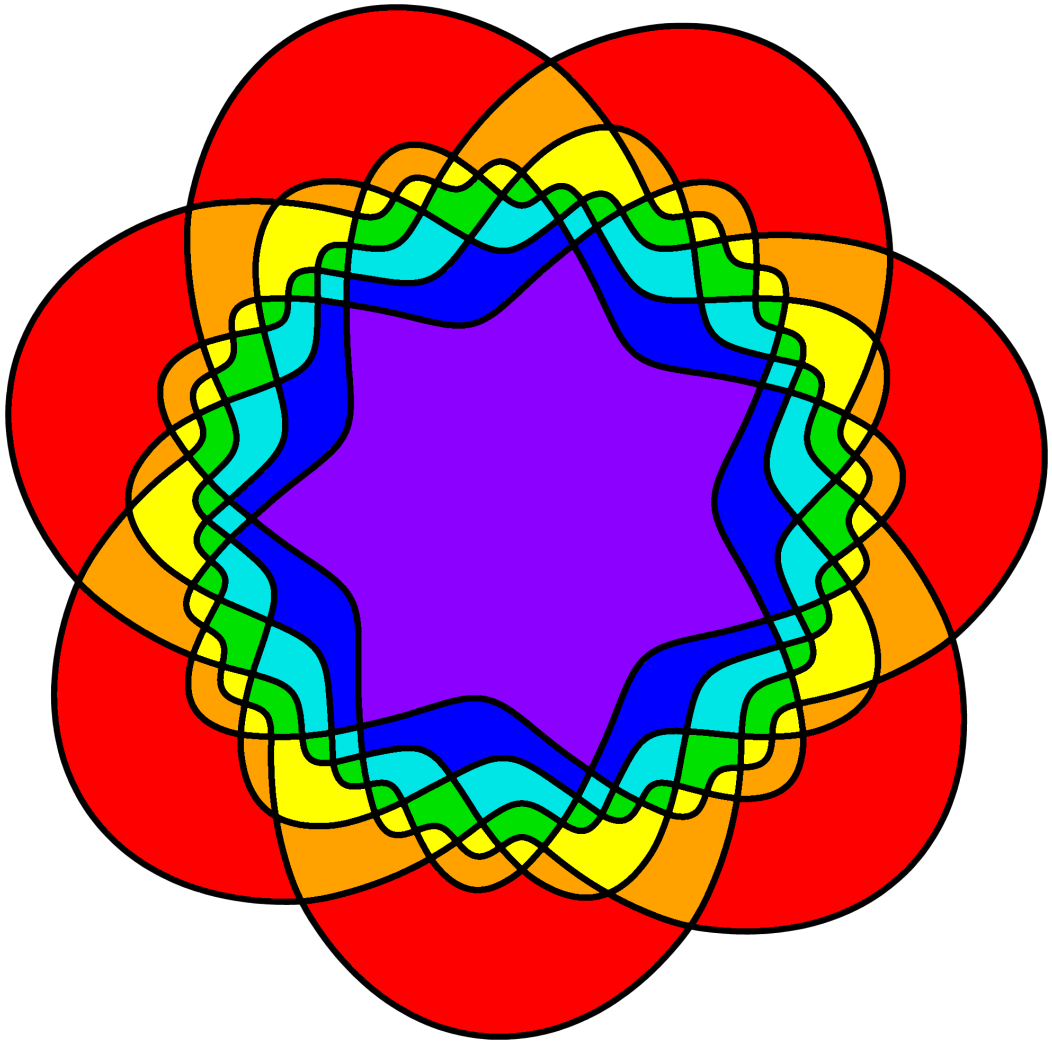
Results



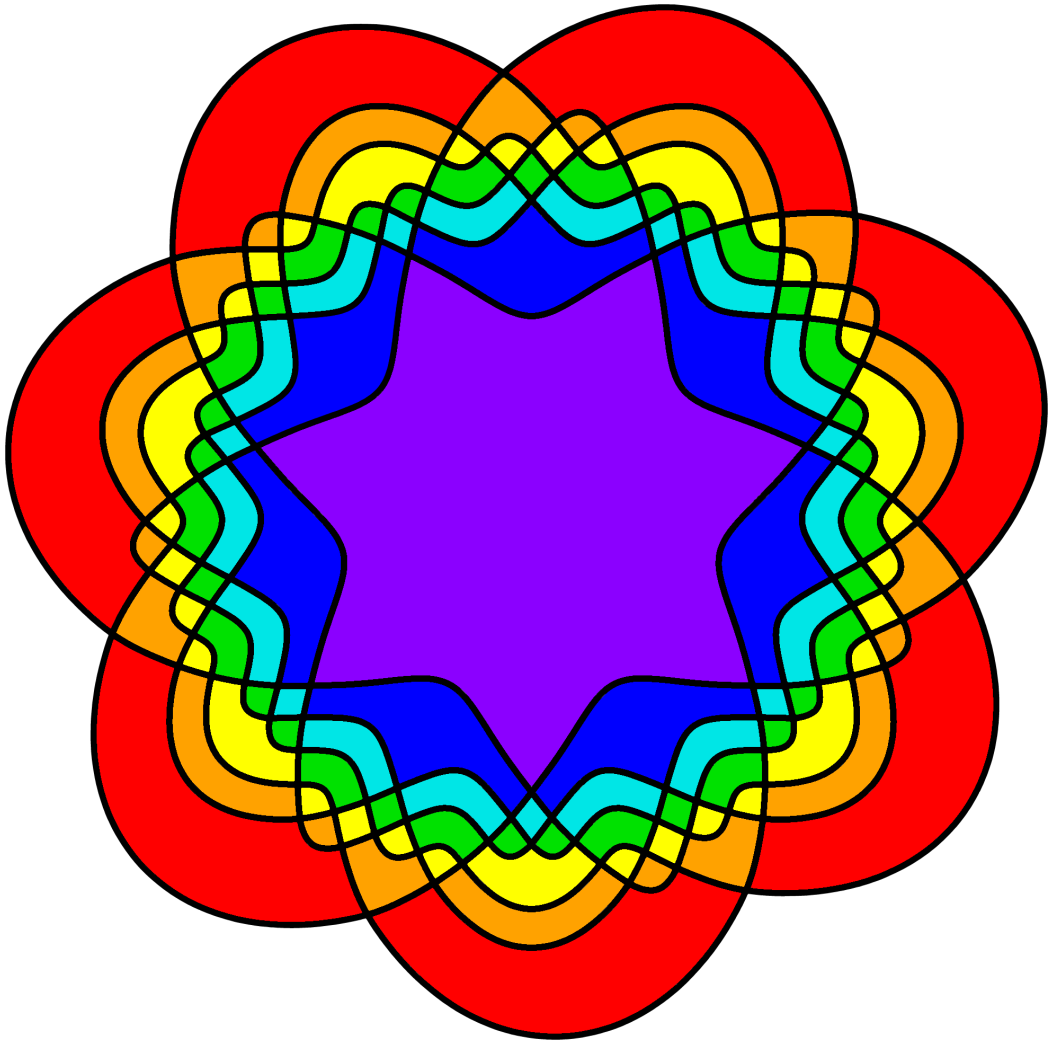
Results



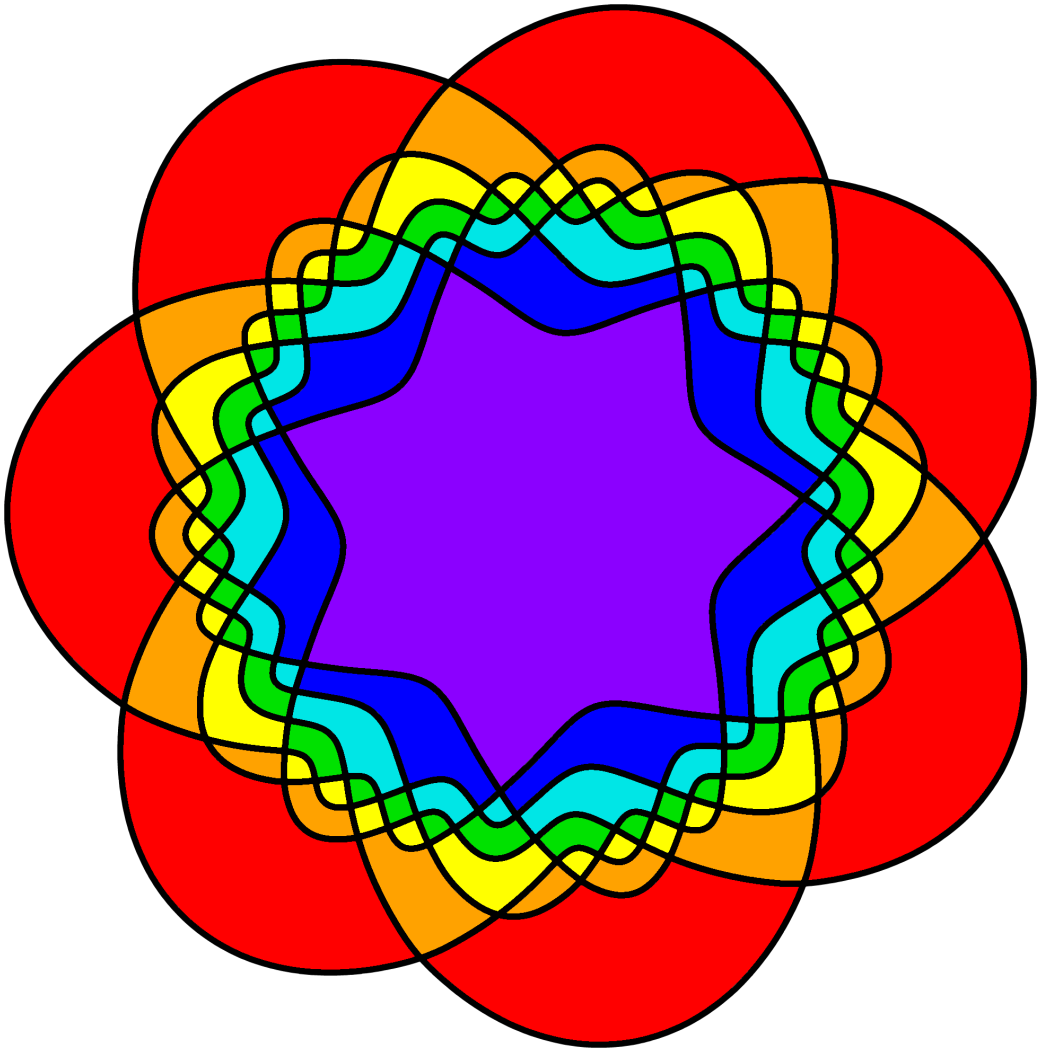
Results



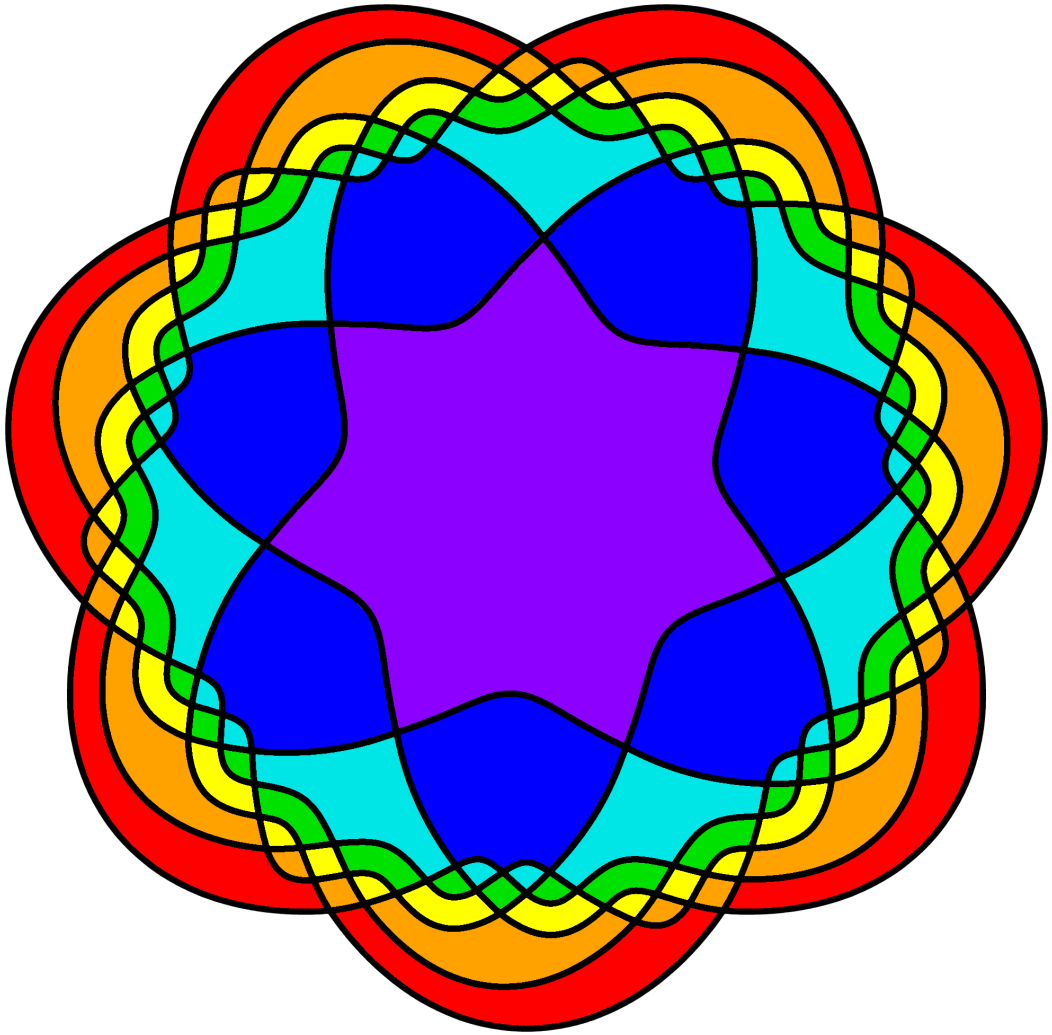
Results



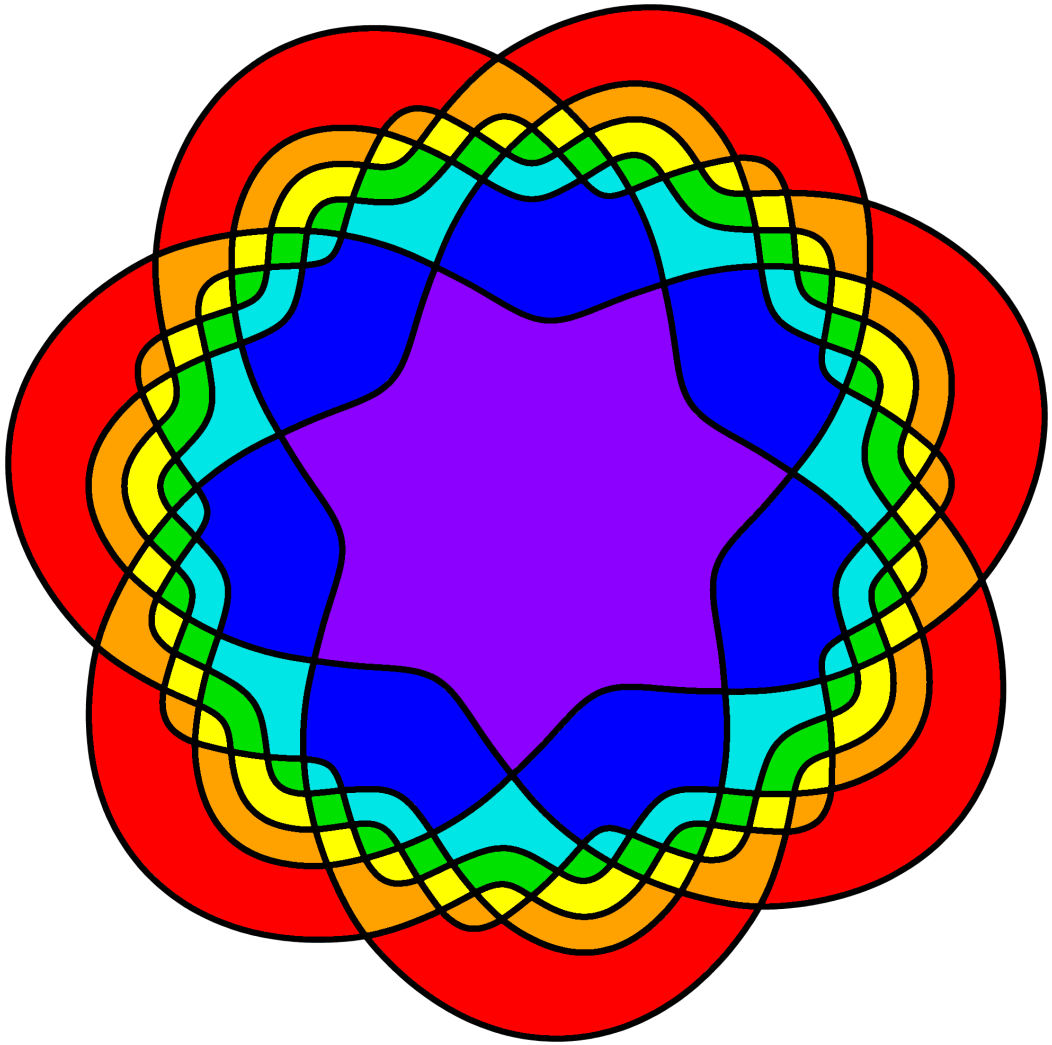
Results



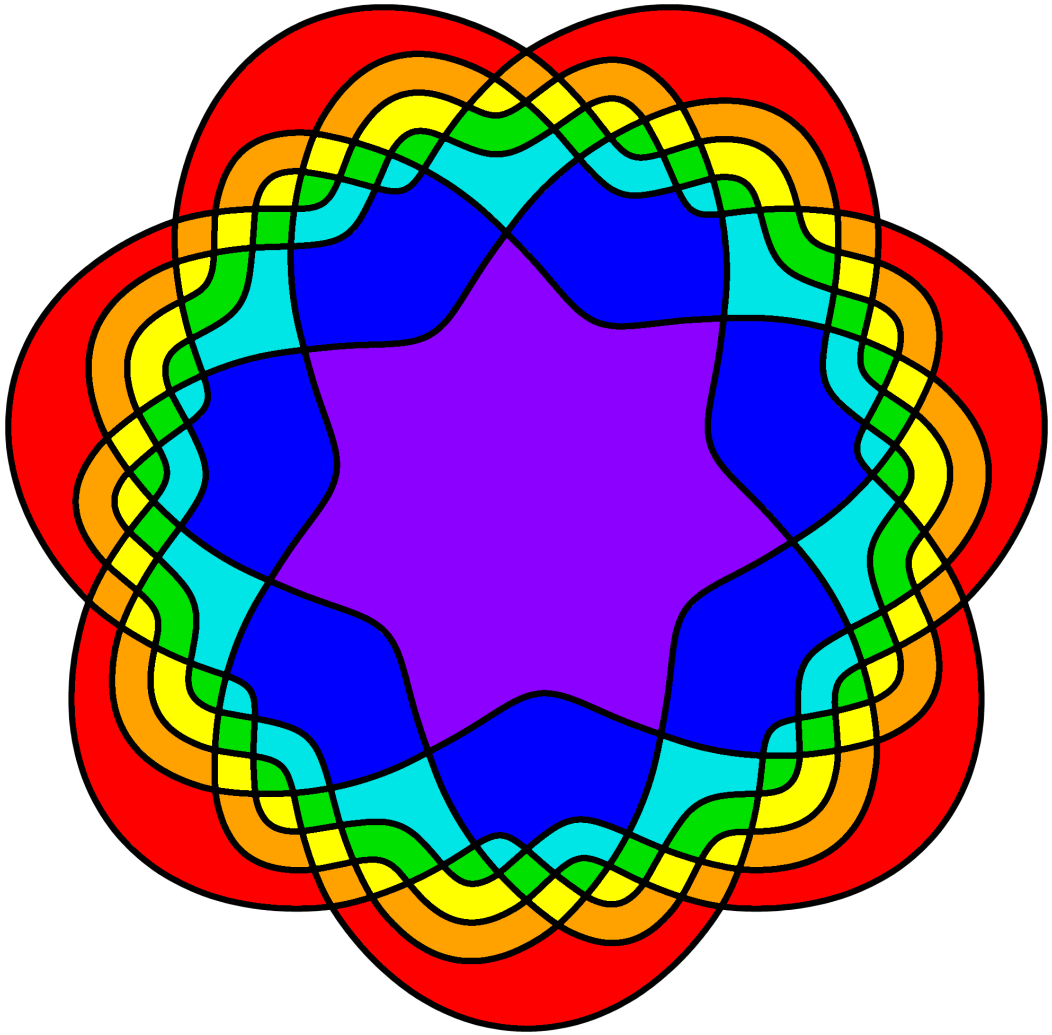
Results



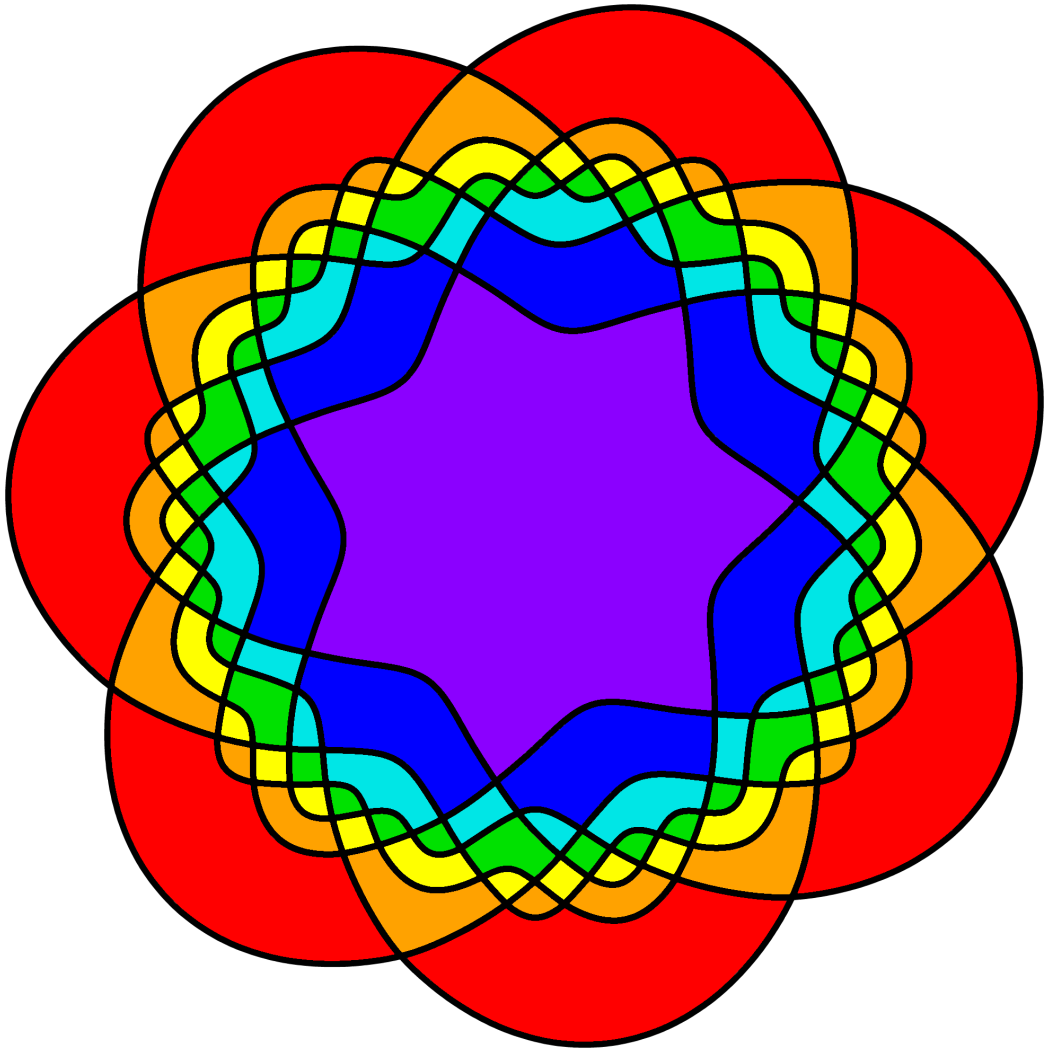
Results



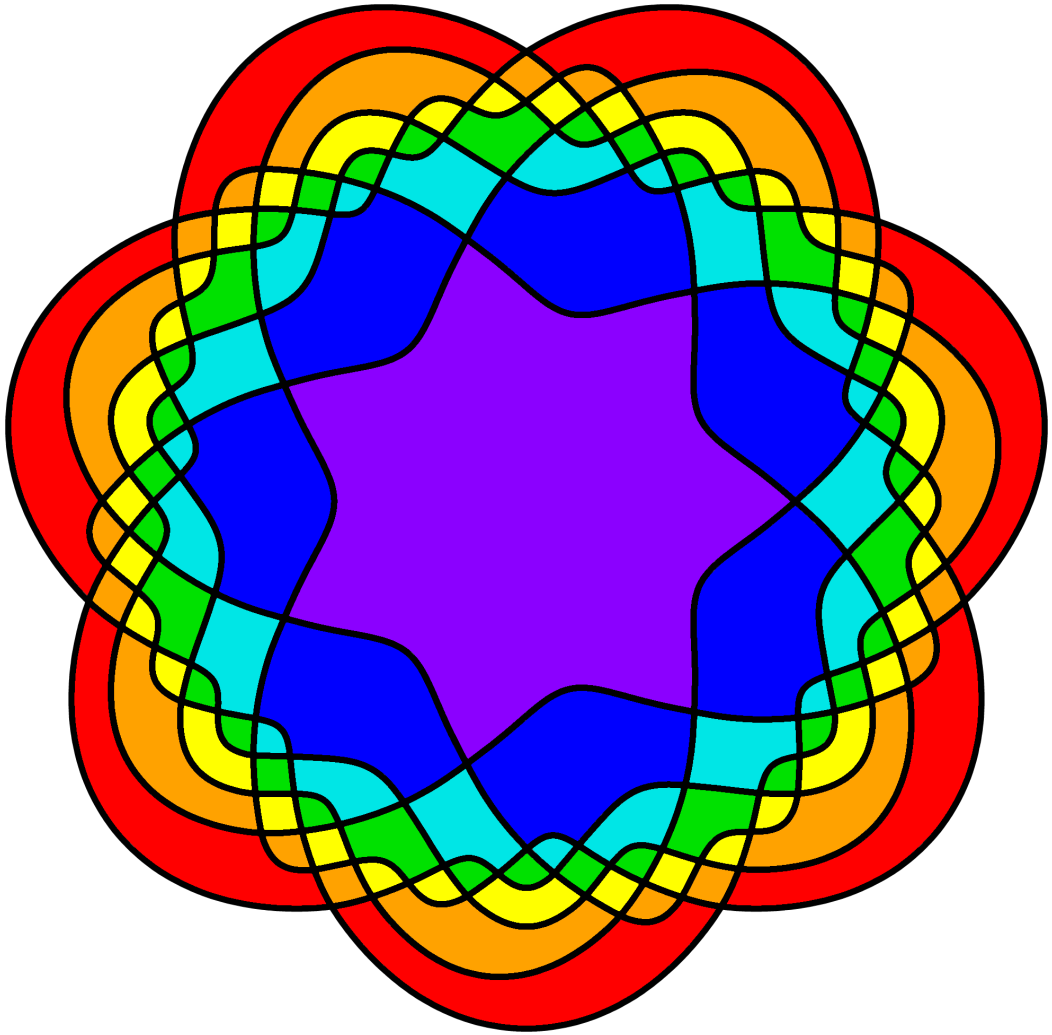
Results



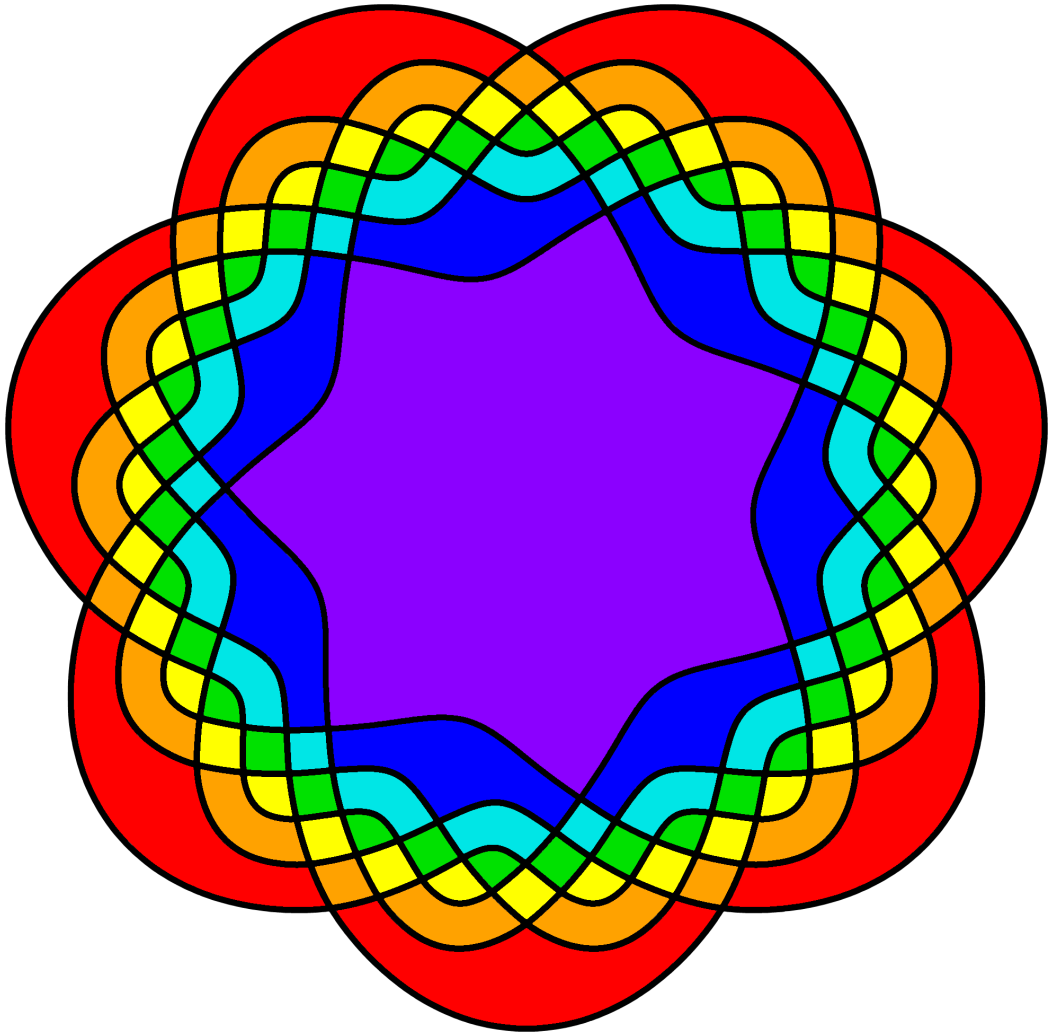
Results



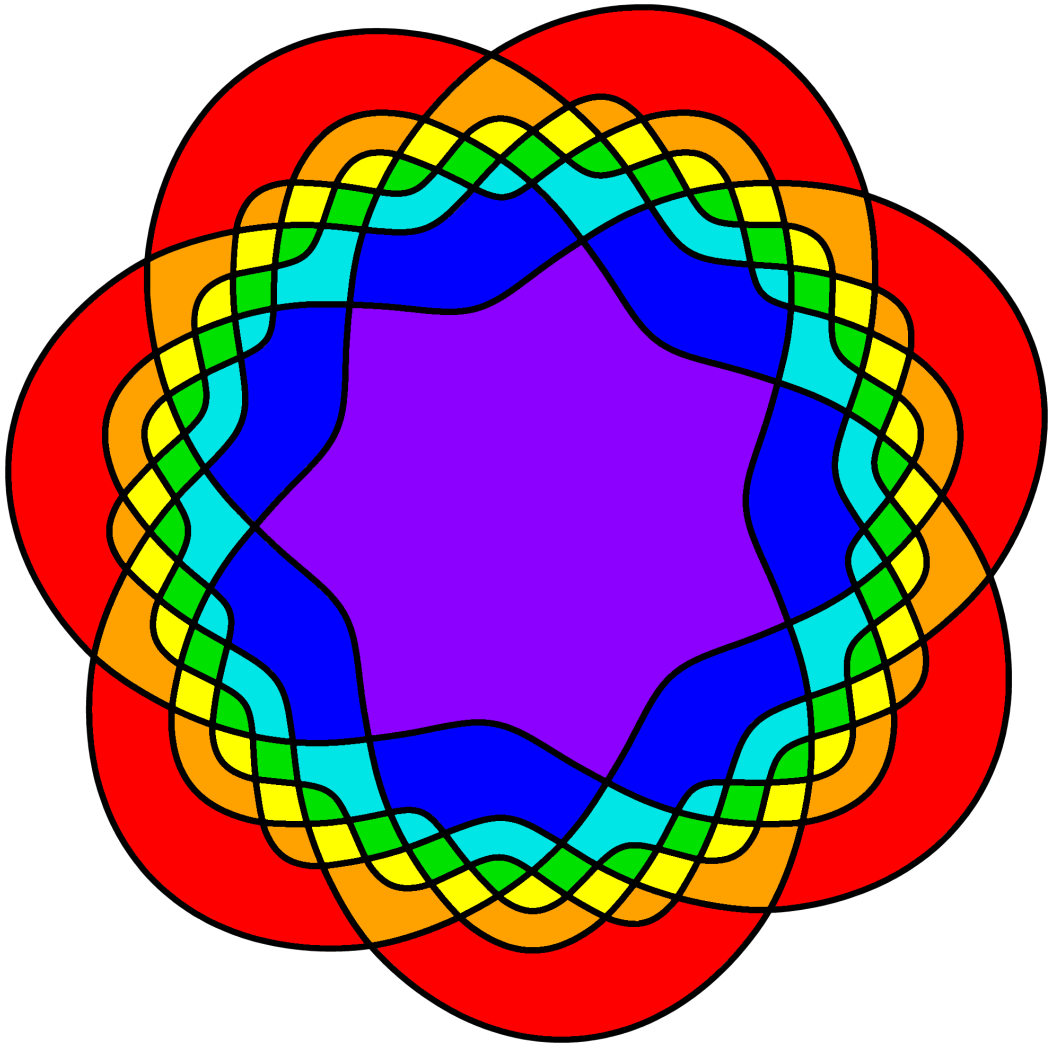
Results



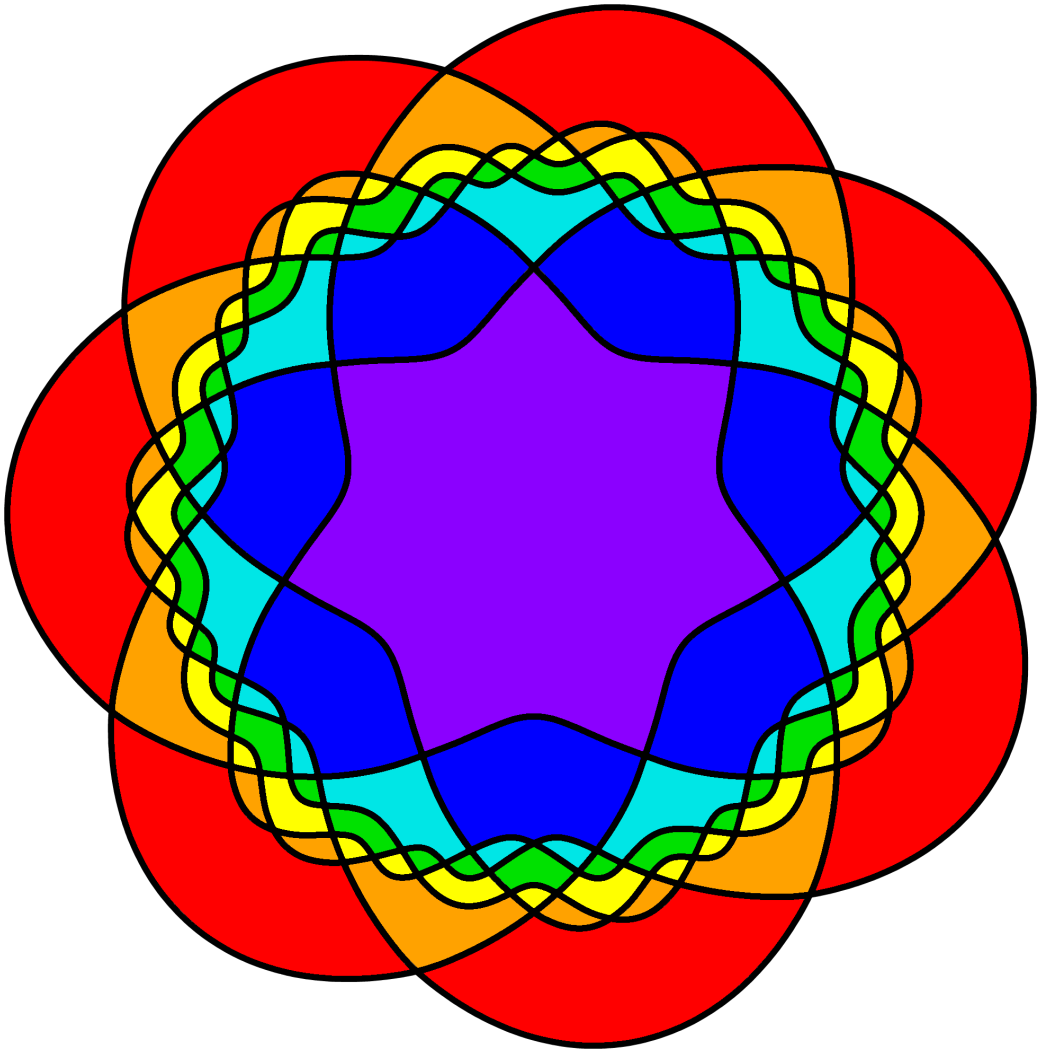
Results : Polar Symmetric



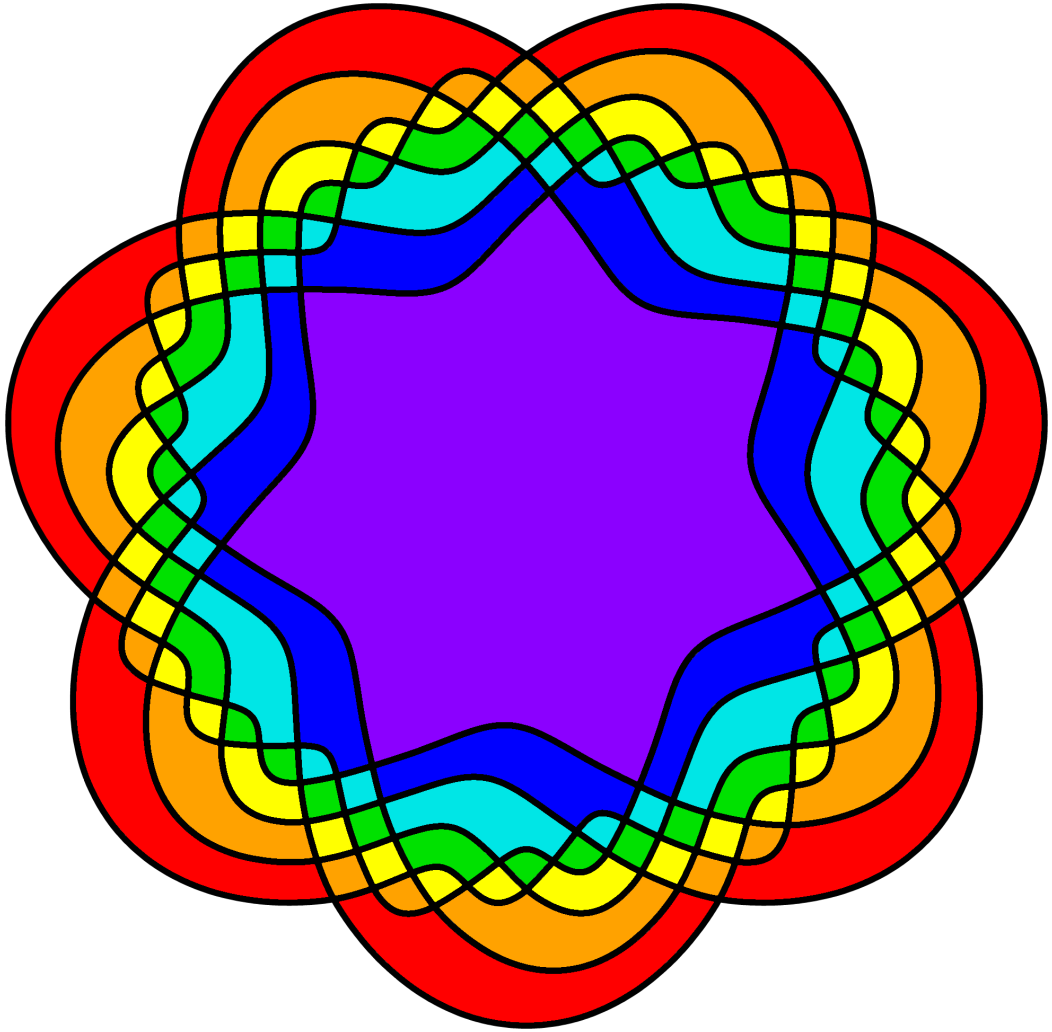
Results : Polar Symmetric



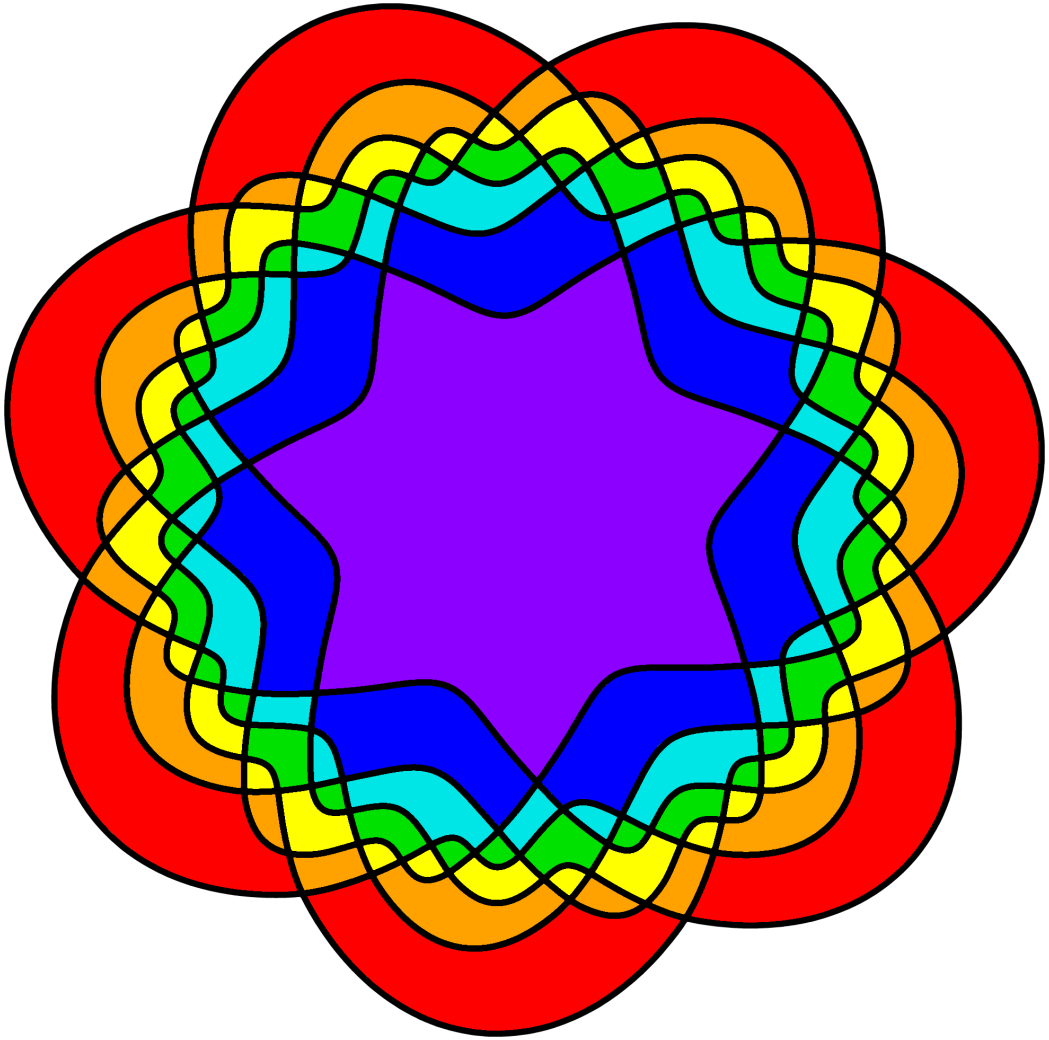
Results : Polar Symmetric



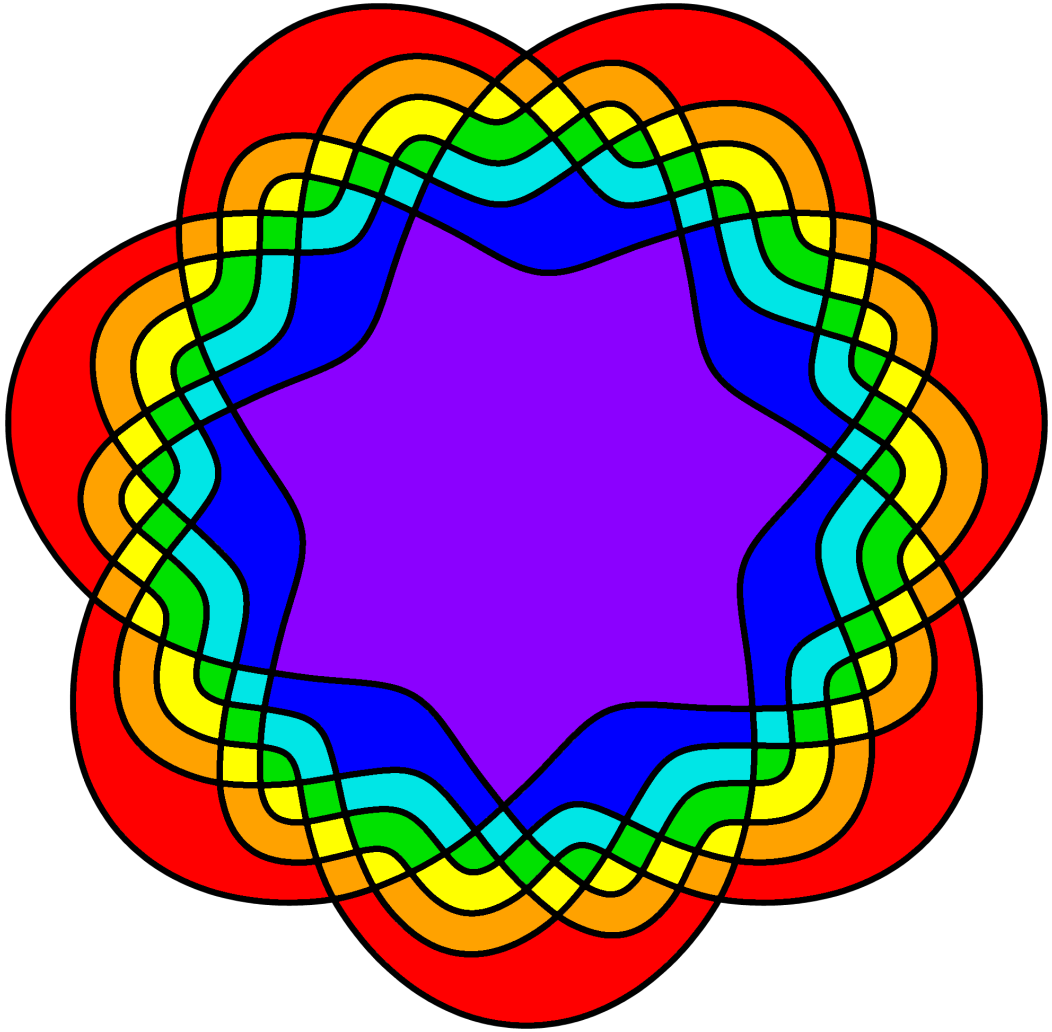
Results : Polar Symmetric



Results : Polar Symmetric



Results : Polar Symmetric



Open problems

- Generate all simple symmetric none-monotone 7-Venn diagrams.
- Is there any simple symmetric Venn diagram for n prime and greater than 7?

Talk summary