

Instructions: Closed book and notes. Answer all questions.

1. [5 marks] Below is the Farey series  $\mathcal{F}_6$ . Show where to insert the missing numbers to get  $\mathcal{F}_7$ .

$$\frac{0}{1} \quad \frac{1}{6} \quad \frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{2}{5} \quad \frac{1}{2} \quad \frac{3}{5} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{5}{6} \quad \frac{1}{1}$$

2. [5 marks] Give a simplified expression for the binomial coefficient

$$\binom{-2}{k}$$

3. [6 marks] In the set  $\{66j \bmod 100 : j = 0, 1, \dots, 99\}$  how many times does the number 14 occur? What about the number 15?

4. [9 marks] Let  $\sigma(n)$  be the sum of the divisors of  $n$ . For example,  $\sigma(6) = 1+2+3+6 = 12$ . Simplify (and explain why your simplification works)

$$\sum_{d|m} \mu(d)\sigma(m/d).$$