

1 CSC 528 Assignment #3

1. Prove that

$$\nu x = x - \left\lfloor \frac{x}{2} \right\rfloor - \left\lfloor \frac{x}{4} \right\rfloor - \left\lfloor \frac{x}{8} \right\rfloor - \dots$$

2. Determine the structure of an infinite binary tree $\mathcal{T}$ with the following property: the first n nodes of $\mathcal{T}$ are a binary tree that maximizes the number of leaves at the lowest level in the tree (over all trees with n nodes). The tree should have some similarities to the infinite sideways heap. [It's not required, but you may want to think about how hard it would be to navigate in this tree.]
3. In his lecture, Knuth describes an application of the `nca` problem to the “minimum in range problem” (or something like that). Illustrate the binary search tree construction with the input sequence 27182818284 (another well-known sequence). Now do exercise 150 in 7.1.3.
4. Prove the following equation from Knuth's lecture, where $i \leq j$:

$$\max\{\rho(i), \rho(i+1), \dots, \rho(j)\} = \lambda(j \ \& \ -i).$$

If you are stuck, see exercise 138 in 7.1.3.