Consider this recurrence which is only defined for values of $n = 2^k$ for some integer $k \geq 1$:

$T(2) = 8$
$T(n) = 7 * n + T(n/2)$

1. Solve this recurrence using repeated substitution.

2. Try proving that your answer is correct by induction to see if you obtained a correct solution or not.

To make the math easier, use 2^k in place of n.

To evaluate how you are doing, grade your paper.

The grades are only for feedback.

You will get full participation credit just for participating.

Step 0: $T(n) = 7 * n + T(n/2)$

But to make the math easier, use $n = 2^k$:

Step 0: $T(2^k) = 7 * 2^k + T(2^{k-1})$ [5 marks]

But $T(2^{k-1}) = 7 * 2^{k-1} + T(2^{k-2})$

Therefore:

Step 1: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + T(2^{k-2})$ [5 marks

But $T(2^{k-2}) = 7 * 2^{k-2} + T(2^{k-3})$

Therefore:

[5 marks]

Step 2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + T(2^{k-3})$

The general pattern: [25 marks]

Step i: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2}$

$+ \ldots + 7 * 2^{k-i} + T(2^{k-i-1})$

Step 0: $T(2^k) = 7 * 2^{k-0} + T(2^{k-1})$

Step 1: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + T(2^{k-2})$

Step 2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + T(2^{k-3})$

The general pattern:

Step i: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^{k-i} + T(2^{k-i-1})$

Check your pattern with Steps 0, 1, 2 to make sure it is correct.

Step 0: $T(2^k) = 7 * 2^{k-0} + T(2^{k-1})$

Step 1: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + T(2^{k-2})$

Step 2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + T(2^{k-3})$

The general pattern:

Step i: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^{k-i} + T(2^{k-i-1})$

Check your pattern with Steps 0, 1, 2 to make sure it is correct.

The base case is that $T(2) = 8$.

At which step i is $T(2)$ on the RHS of the equation?

The general pattern:

Step i: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2}$

$+ \ldots + 7 * 2^{k-i} + T(2^{k-i-1})$

We need that $T(2) = T(2^1) = T(2^{k-i-1})$

Solve $1 = k - i - 1$ for i.

Solution: $i = k-2$.

[10 marks]

The base case is that T(2)= 8.

It occurs at step k-2.

Use the pattern for step i to write down step k-2:

Step i: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^{k-i} + T(2^{k-i-1})$

Step k-2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^{k-(k-2)} + T(2^{k-(k-2)-1})$

Simplify:

Step k-2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^2 + T(2^1)$

[20 marks]

Step k-2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^2 + T(2^1)$

Plug in T(2)= 8. [10 marks]

Step k-2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2} + \ldots + 7 * 2^2 + 8$

This is almost one of my favorite formulas:

$2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^k = 2^{k+1} - 1$

Step k-2: $T(2^k) = 7 * 2^k + 7 * 2^{k-1} + 7 * 2^{k-2}$
$+ \ldots + 7 * 2^2 + 8$

$T(2^k) = 7 * [2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^k] + 8$

$- 7 * [2^0 + 2^1]$

$= 7 * (2^{k+1} - 1) + 8 - 7 - 14$

$= 7 * 2^{k+1} - 20$ [20 marks] $= 14n - 20.$

Check that T(2)= 8: $T(2^1) = 7 * 2^{1+1} - 20 = 28 - 20 = 8$