Give recurrences T(n) and S(n) for the time and space complexity of:

```
public static void get_space(int level, int [] A)
{
        int [] B;     int i, n;
        n= A.length;
        if (n==1) return;

        for (i=1; i <= n; i++)
        {
             B= new int[n-1];
             get_space(level+1, B);
        }
}
```

CSC 225 Announcements:

Last day for withdrawing from first-term courses with-out penalty of failure: Thursday Oct. 31.

Grading Scheme:

5 written assignments: 3% each.
3 programming assignments: 5% each.
Students must have an average of at least 50% on the assignments in order to write the final exam.

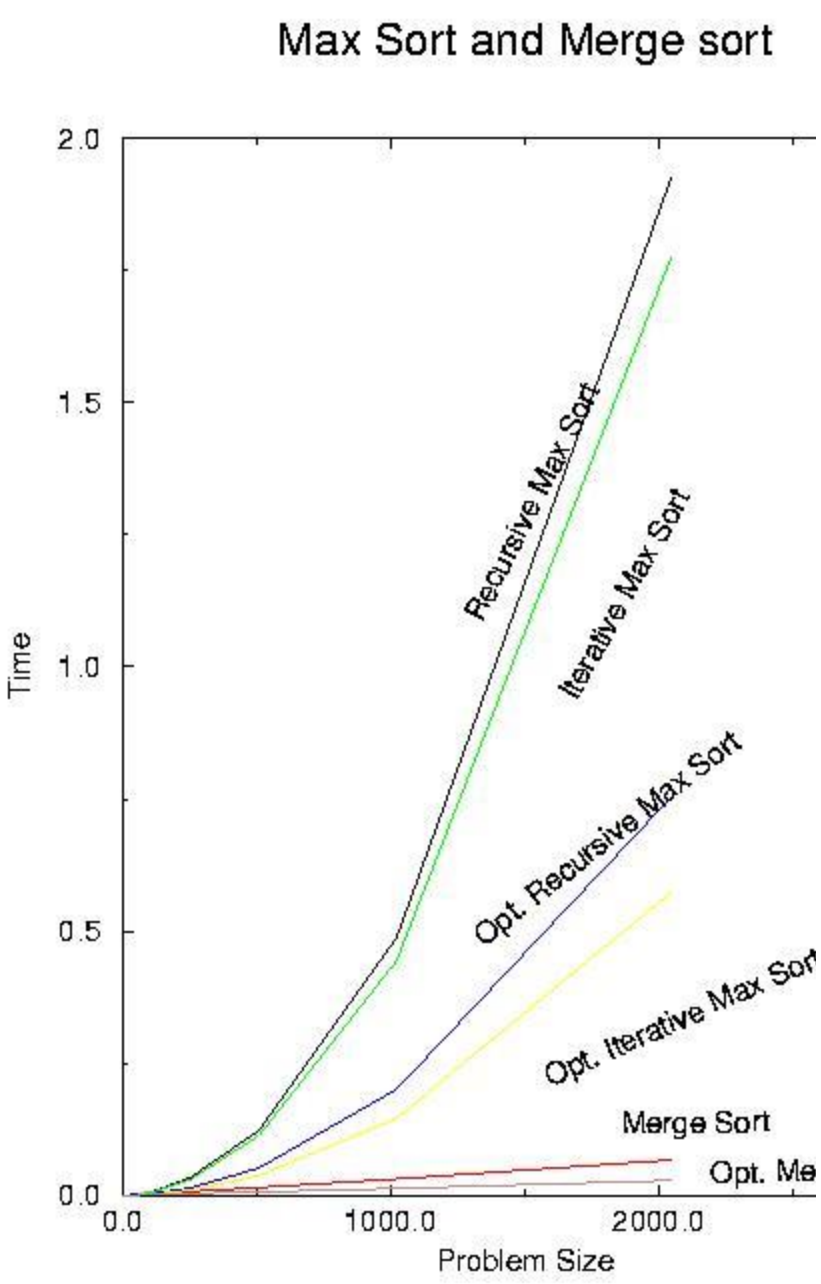
I will compute the average 2 ways:
1. Each of 8 assignments weighted equally.
2. Programs: 50% and Written: 50%.

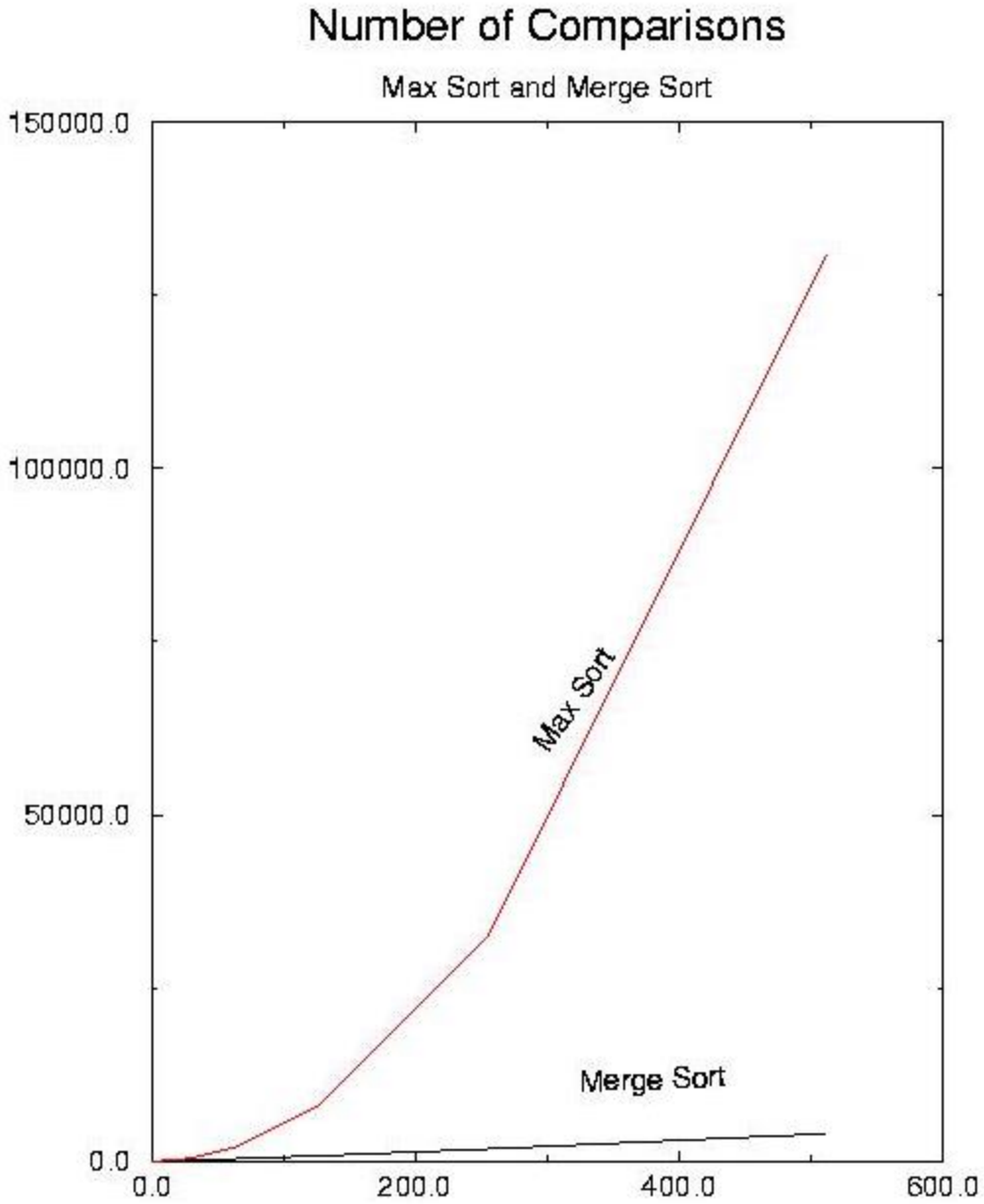
If either score is ≥ 50% then you have met this condition.

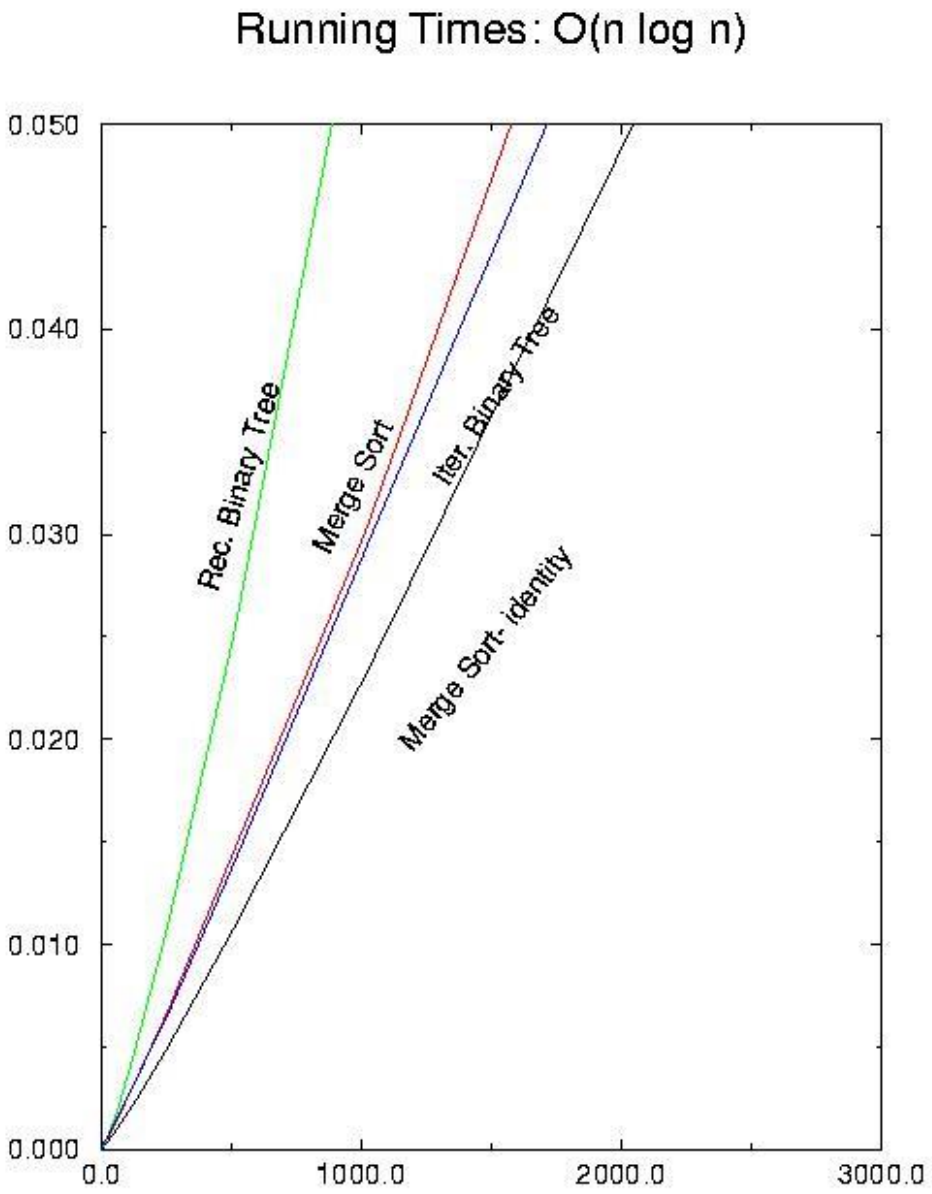
Actual Running Times of Some Sorting Algorithms

In 1999, the CSC 225 students programmed various sorting algorithms in C and timed them on various inputs.

This is where the following plots came from.

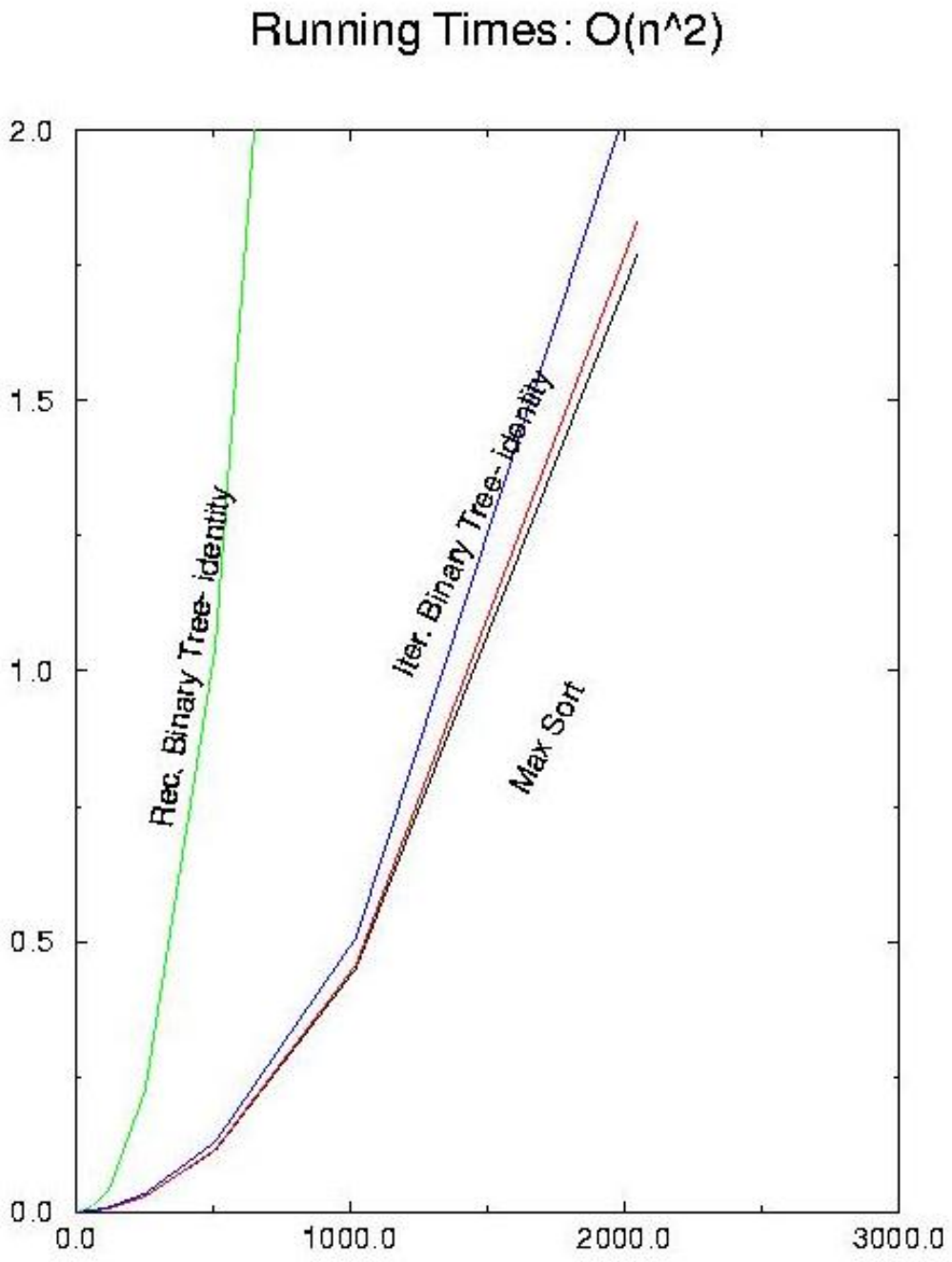

Max Sort and Merge sort
2.0
1.5
1.0
0.5
0.0
Time
0.0
1000.0
2000.0
Problem Size
Recursive Max Sort
Iterative Max Sort
Opt. Recursive Max Sort
Opt. Iterative Max Sor
Merge Sort
Opt. Me


Number of Comparisons
Max Sort and Merge Sort
150000.0
100000.0
50000.0
0.0
0.0
200.0
400.0
600.0
Max Sort
Merge Sort

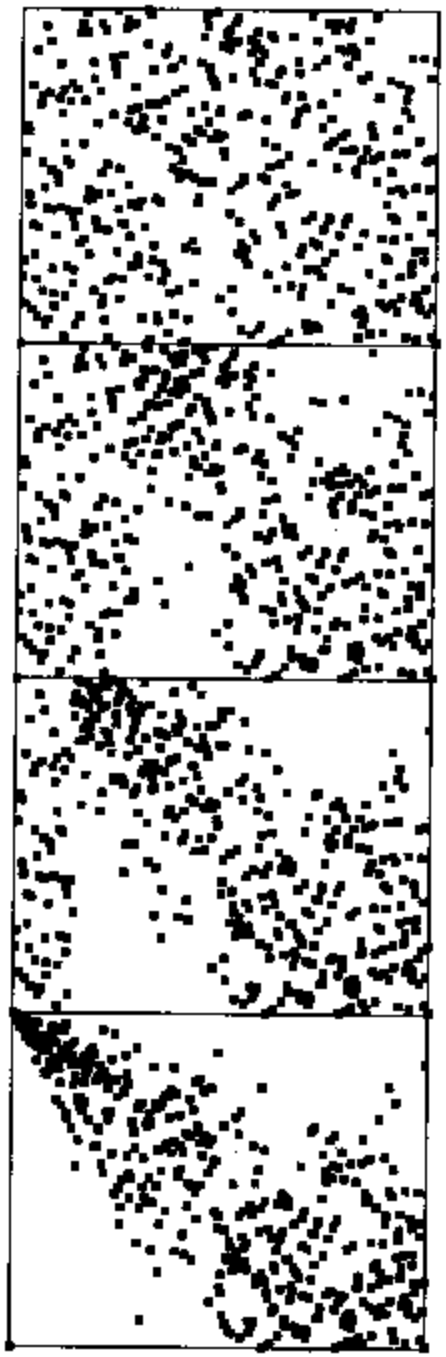


Binary Tree Sort

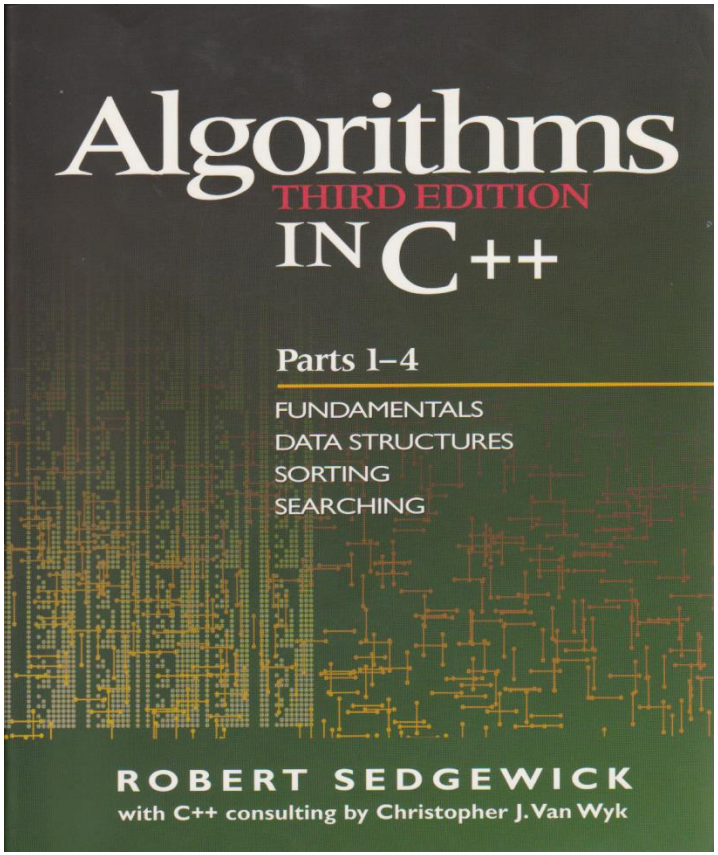
Random Inputs



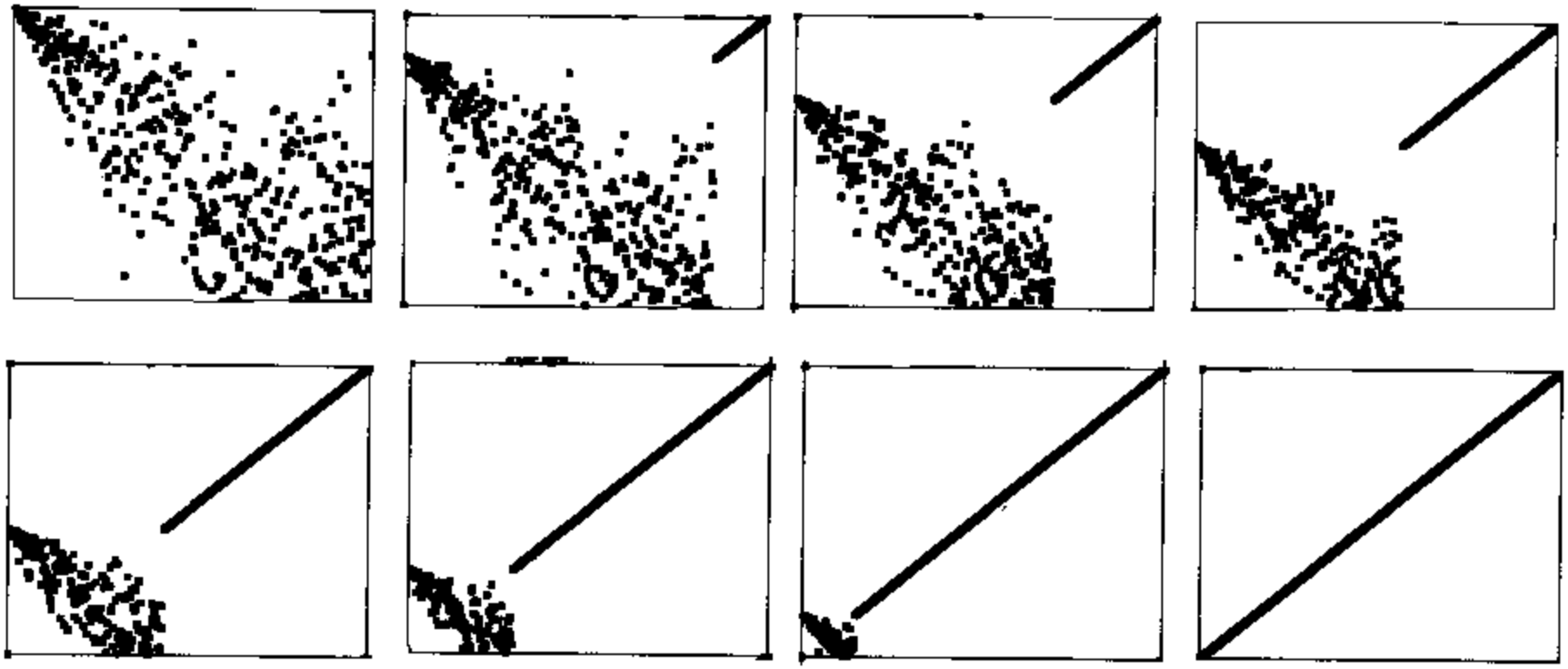
Sorted Inputs



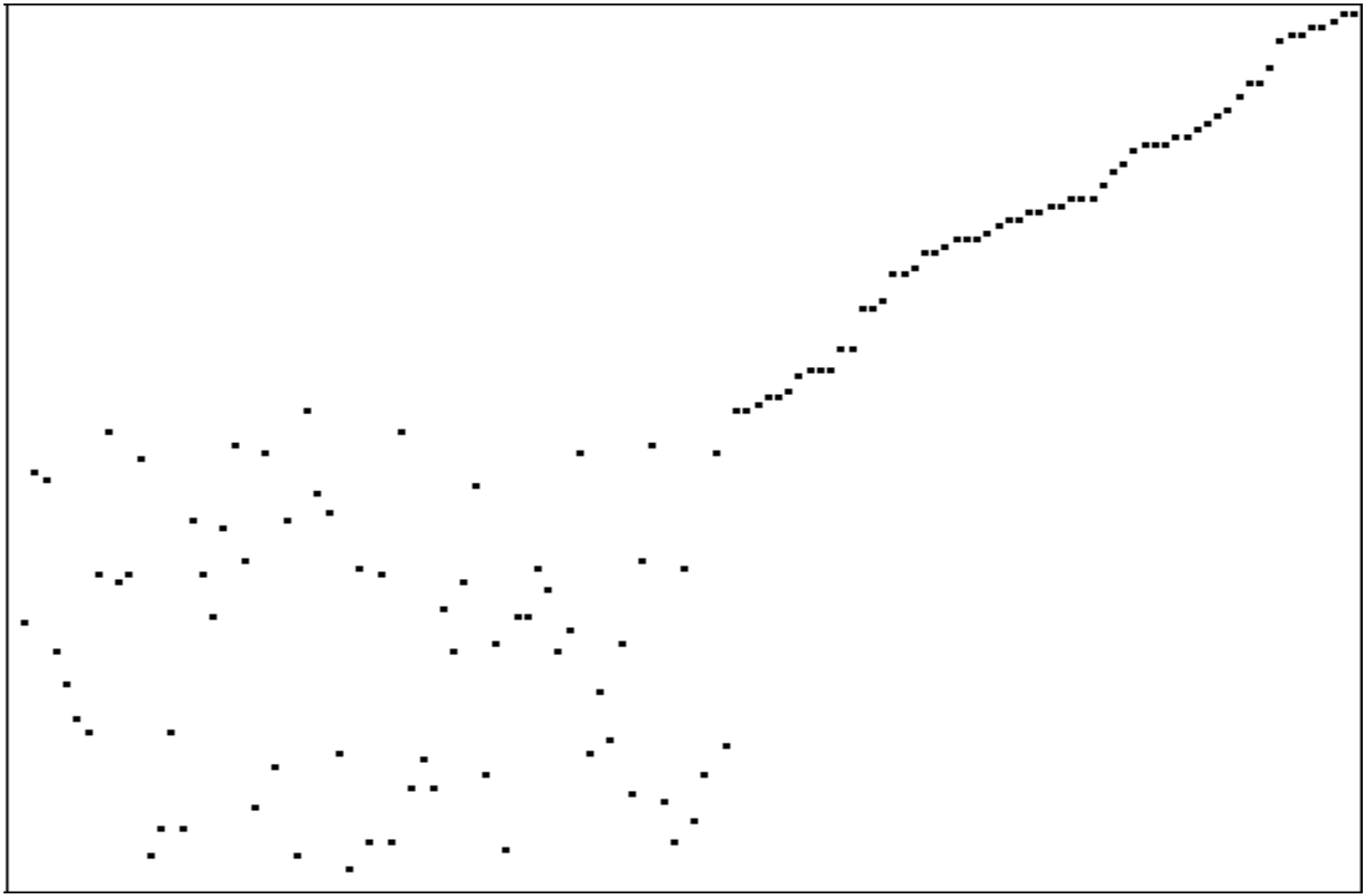
Building the heap- which algorithm is this?



Dynamic Performance of Heapsort

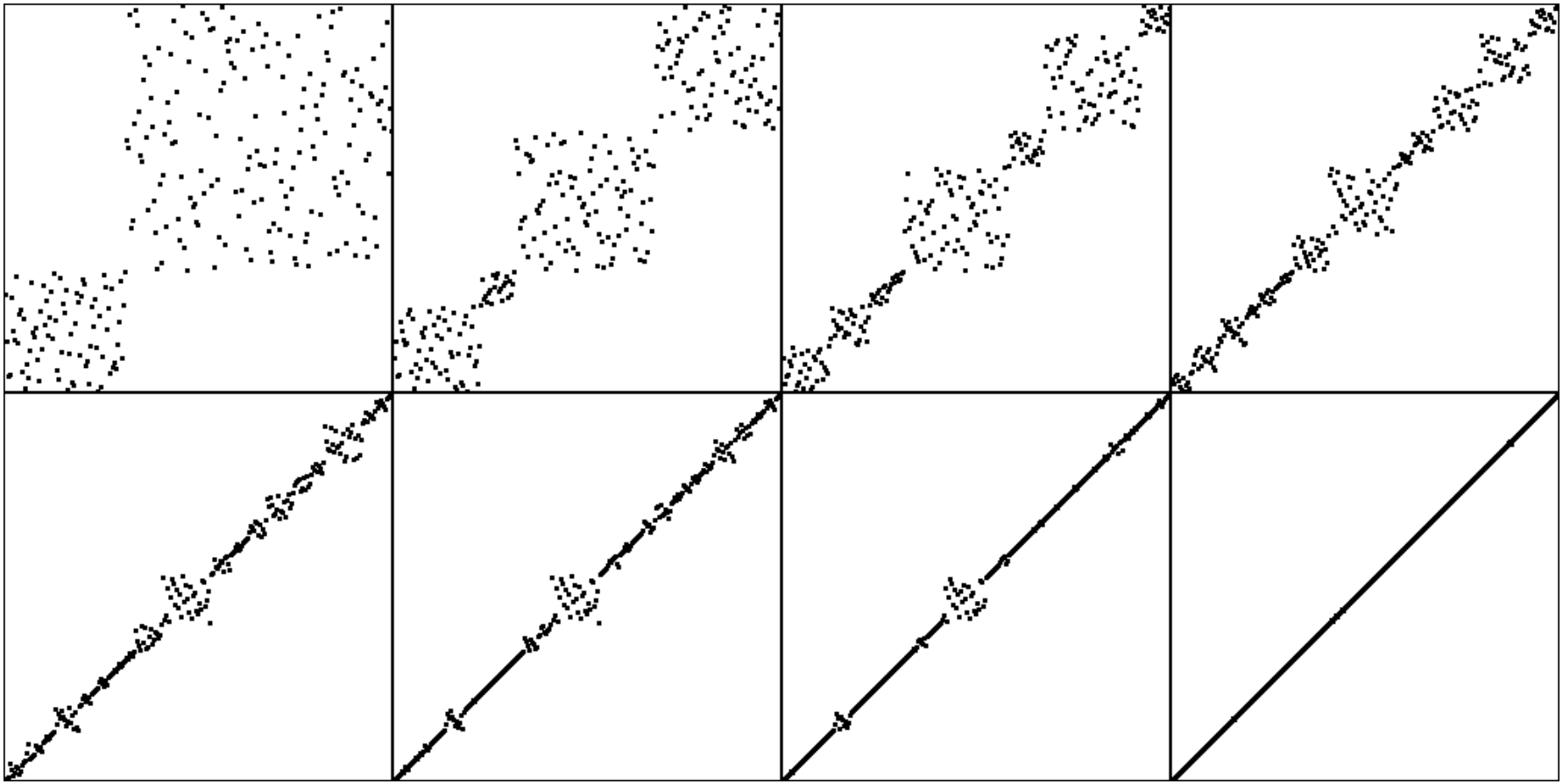


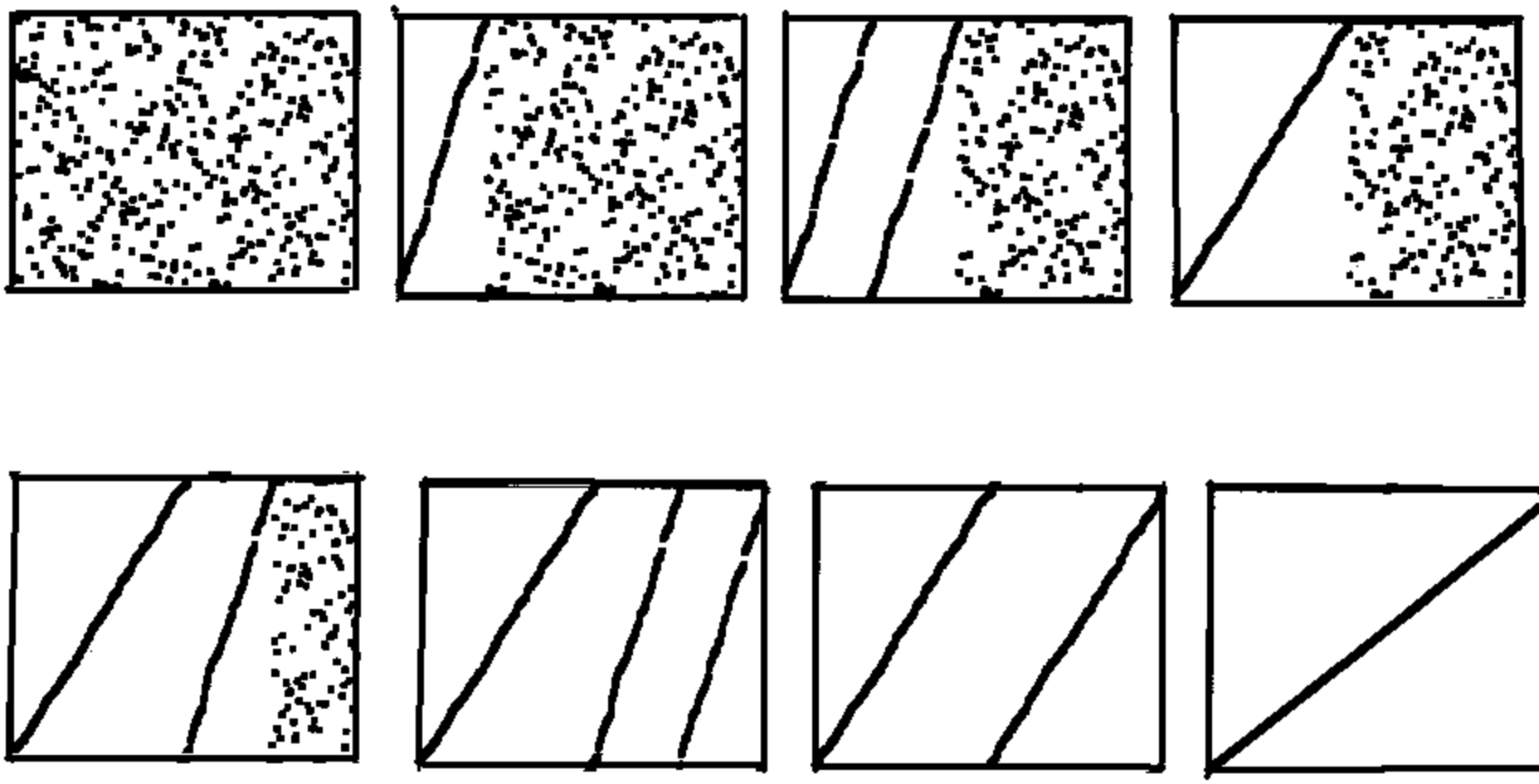
MaxSort



From: [LW95] Kenneth Lambert and Thomas Whaley, *An Invitation to Computer Science Laboratory Manual*, West Publishing Company, 1995. Conference, 12:5 (1997) 57—70.

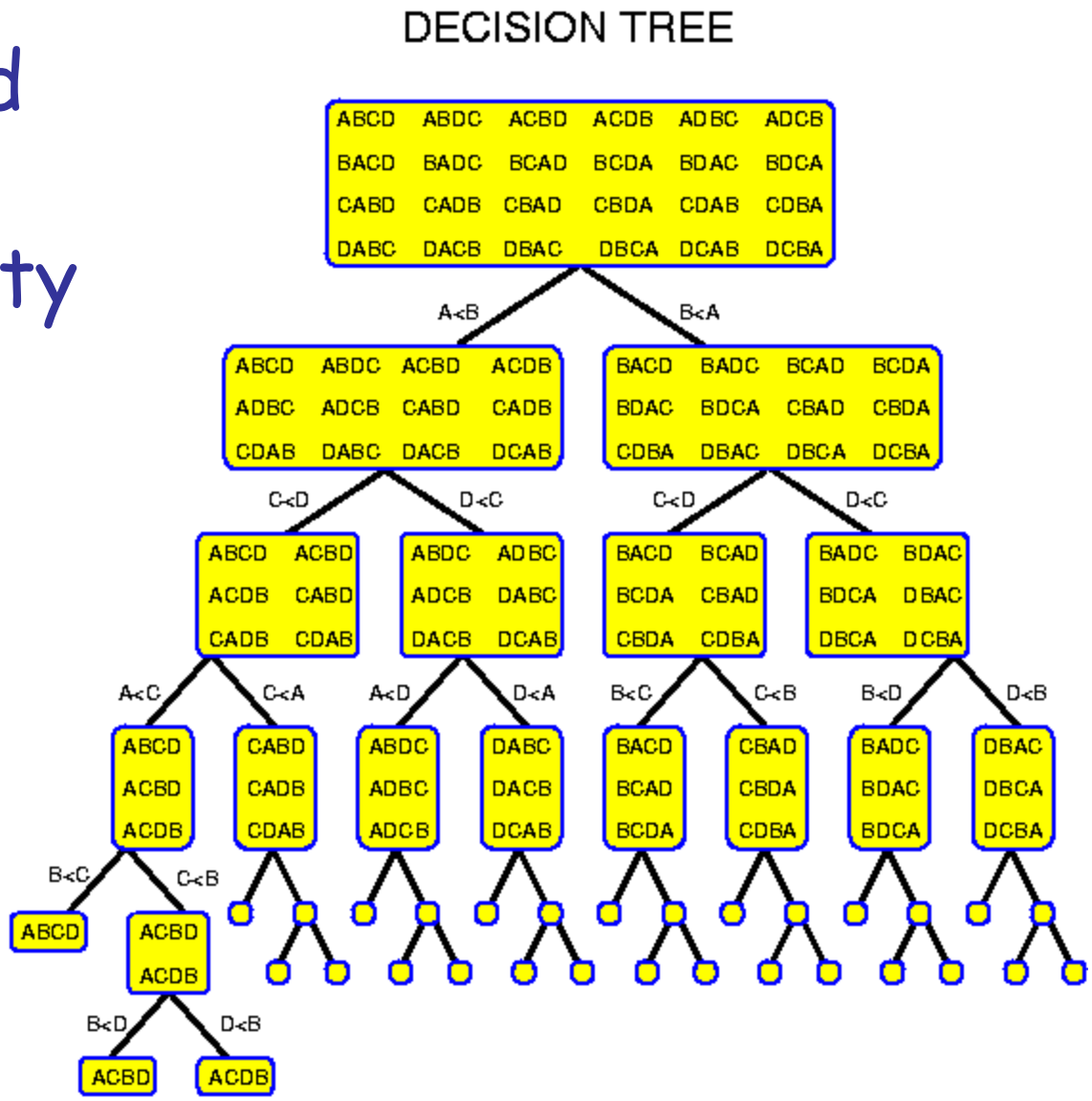
Quicksort





Mergesort

A Lower Bound on the Worst Case Complexity for Sorting



http://users.informatik.uni-halle.de/~jopsi/dinf204/notes_full.shtml

The Comparison Model:

The problem: Sort n integers.

Operations permitted on the data: comparisons and swaps.

It's very hard to prove good lower bounds for algorithm time complexities.

An easy lower bound for sorting is that any algorithm must take time which is $\Omega(n)$ because if the algorithm does not examine all the data items, then an adversary can change the value of an unexamined data item and make the answer wrong.

We can do better:

Theorem:

For the comparison model, any sorting algorithm requires at least $\Omega(n \log n)$ time in the worst case.

This theorem cannot be beat in the Big Oh sense because we have algorithms which take time in $O(n \log n)$ in the worst case which means it is a tight lower bound.

1. Sort these words in lexicographic order:

eat

either

earn

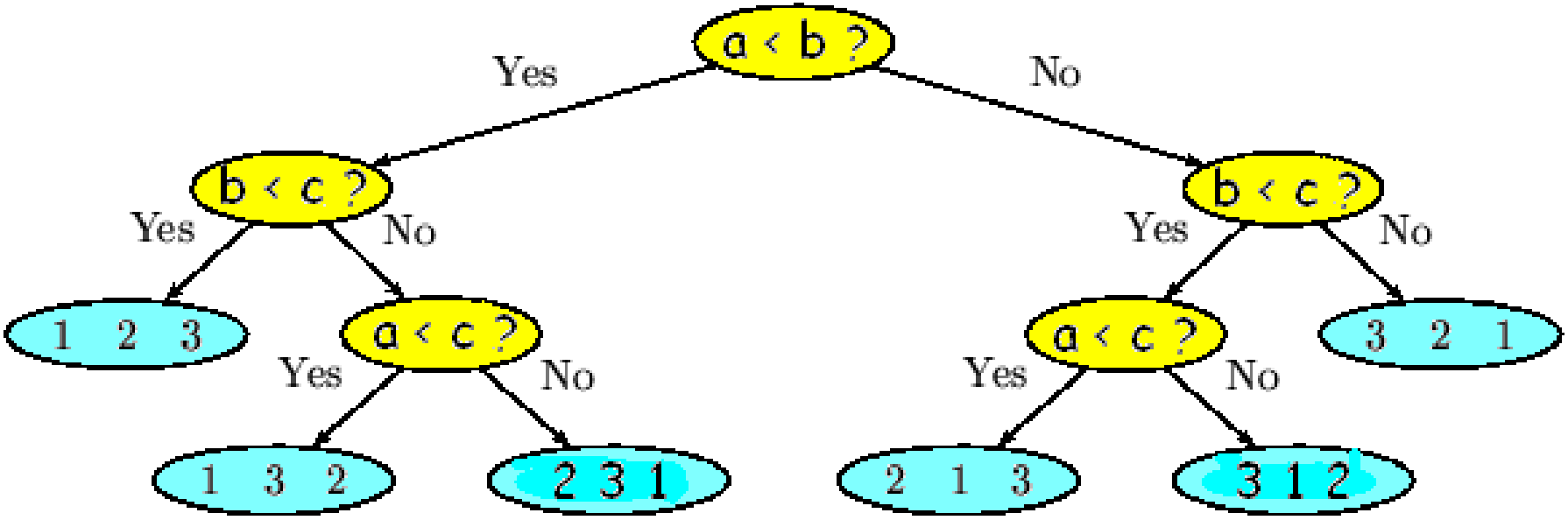
eaten

2. Write down a definition of lexicographic order.

The permutations on 4 symbols listed in lexicographic order (by columns):

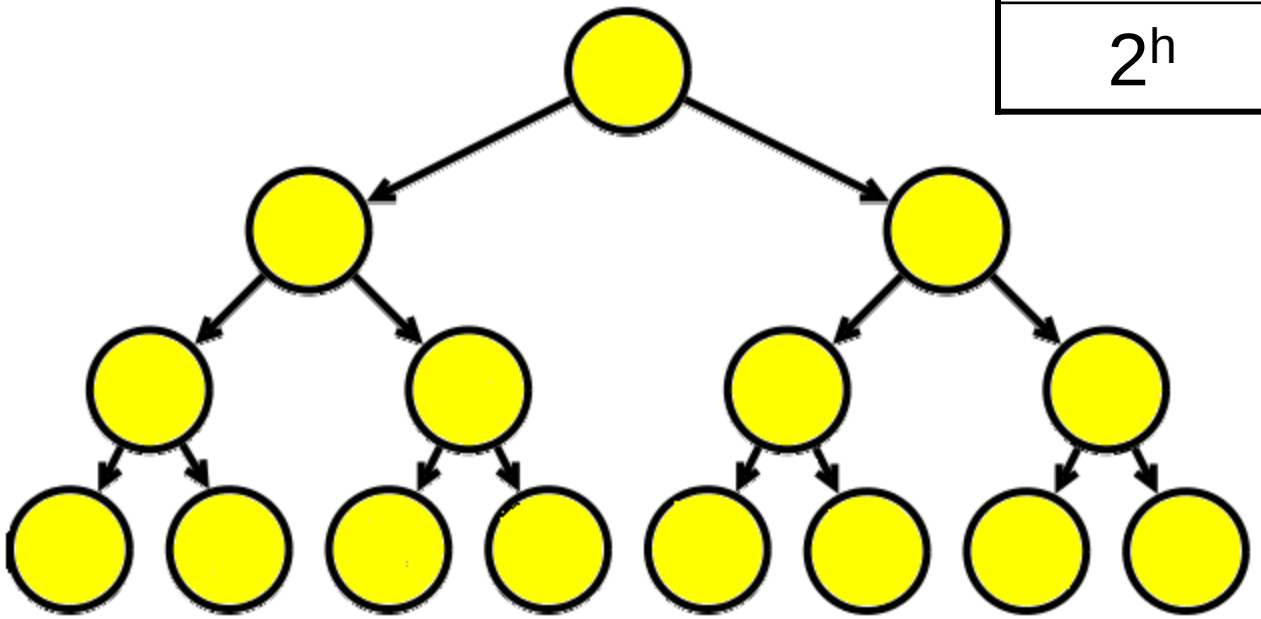
1 2 3 4	2 1 3 4	3 1 2 4	4 1 2 3
1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
1 3 2 4	2 3 1 4	3 2 1 4	4 2 1 3
1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
1 4 2 3	2 4 1 3	3 4 1 2	4 3 1 2
1 4 3 2	2 4 3 1	3 4 2 1	4 3 2 1

A Decision Tree: Input is a, b, c



Note that a complete binary tree which has r leaves has height $\Theta(\log_2 r)$:

Leaves	Nodes	Height
?	1	0
2	3	1
4	7	2
8	15	3
	…	…
2^h	$2^{h+1} - 1$	h



We can use our tactics for lower and upper bounding to prove that:

$$\log_2(\, n! \,) \in \Theta(n \log_2 n)$$

Which sorting algorithms have optimal time complexities for the comparison model (in a Big Oh sense)?

These $\Theta(n \log_2 n)$ in the worst case:

Heapsort, Mergesort, Mediansort

Not optimal since worst case is $\Theta(n^2)$:

Quicksort, Maxsort, Binary Tree Sort