CSC 320 Midterm Exam #1 Summer 2002

1. For parts (a), (b), and (c) below, you must choose three DIFFERENT languages from the five given here and are required to find a regular expression, a context-free grammar, and a DFA for them respectively. Choose carefully to minimize your effort.

The five languages to choose from:

$L_1 = \{a^{n^2}\}$.
$L_2 = \{b\, a^n\, b\, a^{2n} b\}$
$L_3 = \{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ is odd and the number of b's is odd }\}$.
$L_4 = \{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ is equal to the number of } b\text{'s in } w\ \}$.
$L_5 = \{w \in \{a, b\}^* : w \text{ contains } aaba \text{ or } ababb\}$
Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Context-free Grammar	
(c)	Deterministic Finite Automaton	

(a) [10 marks] Give a regular expression for one of the languages.

(b) [10 marks] Give a context-free grammar for one of the languages.

(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments).

2. [20 marks] Given that $M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1)$ is a FA that accepts a language L_1 and $M_2 = (K_2, \Sigma, \Delta_2, s_2, F_2)$ is a FA that accepts a language L_2, describe how to construct a finite automata $M = (K, \Sigma, \Delta, s, F)$ that accepts $L_1\ o\ L_2$.

It is fine to draw a visual aid to help in determining your solution. However for full marks, you must precisely describe how to create M.

3. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

(a) [5 marks] $\phi^* = \phi$

True False

(b) [5 marks] If you intersect a regular language with a context-free language, then the resulting language must be regular.

True False

(c) [5 marks] If $x \notin L_1$ and $y \notin L_2$ then $x\,y \notin L_1 \cdot L_2$.

True False

(d) [5 marks] The set containing all context-free languages over the alphabet $\{a, b\}$ is countable.

True False

4.(a) [10 marks] State precisely the pumping lemma for regular languages.

(b) [10 marks] Describe all ways of factoring $w = a^r\, b^s\, c^{r+s}$ as $x\, y\, z$ where y is not equal to the empty string.

(c) [10 marks] Apply the pumping lemma to $w = a^r\, b^s\, c^{r+s}$. to prove that $L = \{a^n\, b^m\, c^p : n + m \leq p\}$ is not accepted by a DFA with $k = 2(r + s)$ states.