CSC 320 Midterm Exam

Oct. 29, 1996

Instructions:

1. Put your name on every page of the exam.
2. No calculators or other aids. Closed book.
3. Read through the entire exam before beginning. You should have six pages including this header page.

Question	Value	Mark
1	40	
2	40	
3	21	
Total	**101**	

Name:

ID Number:

1. For parts (a), (b), (c) and (d) below, you must choose four DIFFERENT languages from the six given here and are required to find a regular expression, a context-free grammar, a DFA, and a PDA for them respectively. Choose carefully to minimize your effort.

   The six languages to choose from:

   $L_1 = \{a^p : p \text{ is prime }\}$.

   $L_2 = \{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ is equal to the number of } b\text{'s in } w\}$.

   $L_3 = \{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ is congruent to the number of } b\text{'s in } w \text{ modulo } 2\}$.

   $L_4 = \{w \in \{a, b, c\}^* : w = a^n b^n c^n, n \geq 0\}$.

   $L_5 = \{w \in \{a, b\}^* : w \text{ contains } aaba \text{ and } ababb\}$.

   $L_6 = \{u \in \{a, b\}^* : w = w^R\}$.

   Fill in your choices for each part:

Part	Requirement	Language chosen
(a)	Regular Expression	
(b)	Context-free Grammar	
(c)	Deterministic Finite Automaton	
(d)	Pushdown Automaton	

(a) [10 marks] Give a regular expression for one of the languages.

(b) [10 marks] Give a context-free grammar for one of the languages.

[Question #1, continued]

(c) [10 marks] Draw the transition diagram of a DFA for one of the languages (include comments).

(d) [10 marks] Describe a PDA for one of the languages (include comments).

2.(a) [10 marks] State the pumping lemma for regular languages (as presented in class, the ‘‘beginning of the string’’ pumping lemma).

(b) [10 marks] Let $w = a^k b\, a^{k^2}$. Describe all possible ways of choosing x, y, z such that $w = xyz$, and $y \neq \varepsilon$.

(c) [10 marks] Apply the pumping lemma to $w = a^k\, b\, a^{k^2}$ to prove that $L = \{a^n\, b\, a^m : n^2 \le m \le n^3\}$ is not accepted by a DFA with $k^2 + k + 1$ states.

(d) [10 marks] A more judicious choice for w would have made the argument for (c) much simpler. Suggest a better choice for w. How does this simplify the argument you gave for (c)?

3. Circle **True** or **False** and justify your answer. **No marks will be given unless there is a correct justification.**

(a) [7 marks] $L = \{ a \}^*$ is countable.

True False

(b) [7 marks] Every subset of a regular language is regular.

True False

(c) [7 marks] If $x \notin L_1$ and $y \notin L_2$ then $xy \notin L_1 \cdot L_2$.

True False