Name: Put your name if you want your attendance credit.

E-mail: Please put your e-mail as well.

Please mark each square for which you CANNOT attend (with an X, or preferably put your class number in the box to aid in error detection).

Time/Day	Mon.	Tues.	Wed.	Thurs.	Fri.
10:00					
10:30		CSC 225	CSC 225		CSC 225
11:00		CSC 225	CSC 225		CSC 225
11:30		CSC 320	CSC 320		CSC 320
12:00		CSC 320	CSC 320		CSC 320
12:30					
1:00					
1:30		320 Tut.	320 Tut.		
2:00		320 Tut.	320 Tut.		
2:30					CAG
3:00					CAG
3:30					
4:00					
4:30					

CSC 320 is done at 12:30 so I did NOT fill in the box for 12:30 for our class.

CSC 320: Proof of the Day

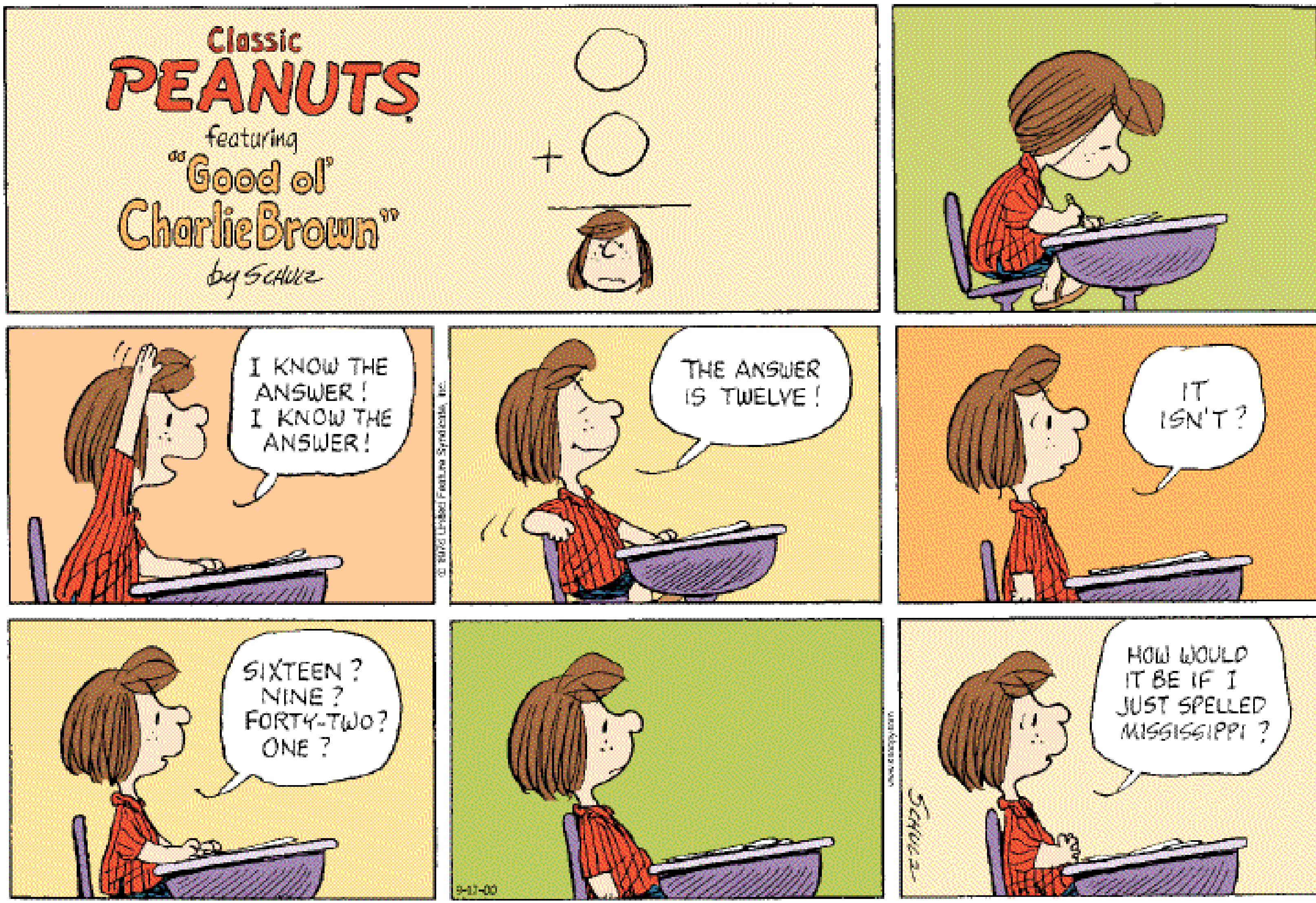
Definition: for functions f and g which map the natural numbers to real values, the function $f(n)$ is in $O(g(n))$ if there exists constants $c > 0$ and $n_o \geq 0$ such that for all

$n \geq n_0, \quad f(n) \leq c \cdot g(n)$.

Use this definition to prove that

$3 + 4n + 5n^2 + 2n^3$ is in $O(n^3)$.

You will learn a lot more if you try the problems and get them wrong than if you do not try. Also, it helps me know where the class is in terms of understanding.



If you are on my connex roster, you should have received an e-mail I sent out Monday reminding you to bring your schedule to class today. Please make sure you can access connex.

Start work now on assignment #1. Please ask me questions if any of the instructions are not clear to you.

Your first tutorial is this Tuesday or Wednesday.

Note: you don't have to wait until the tutorial if you have questions. Ask me or send me e-mail.

$$\sum_{i=c}^{n} f(i)$$

Meaning of notation (translation to code):

```
sum= 0;
for (i=c; i ≤ n; i++)
      sum= sum + f(i);
```

The value of the expression is sum at the termination of this loop.

Natural Numbers

$\mathbb{N} = \{ 0, 1, 2, 3, 4, \ldots \}$

Inductive Definition:

[Basis] 0 is in the set $\mathbb{N}$

[Inductive step]:

If k is in $\mathbb{N}$

then k+1 is in $\mathbb{N}$

The idea behind the simplest form of an induction proof:

Suppose we are given a statement $S(n)$ and we want to show that $S(n)$ is true for all integers $n \geq 0$.

If we prove:

1. that $S(0)$ is a true statement, and
2. if $S(n)$ is a true statement then so is $S(n+1)$

then this is sufficient to prove that $S(n)$ is true for all integers $n \geq 0$.

Theorem S(k): $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Proof:

[Basis] When k=0, $\Sigma_{i=0 \text{ to } k} 2^i = 2^0 = 1$ and

$2^{k+1} - 1 = 2^{0+1} - 1 = 1$ so this formula is correct for k=0.

Note: the induction hypothesis which I will use for this proof is the statement (hypothesis since we have not finished proving it yet) S(k):

$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1.$ We are trying to prove S(k+1).

S(k+1): $\sum_{i=0}^{k+1} 2^i = 2^{(k+1)+1} - 1$

[Induction step] Assume S(k) is true.

We want to prove S(k+1). Separating the sum into two parts gives

$$\sum_{i=0}^{k+1} 2^i =$$

$$\sum_{i=0}^{k} 2^i + 2^{k+1}.$$

IMPORTANT: always indicate to your reader where you apply the induction hypothesis.

By the induction hypothesis, the red term above is equal to $2^{k+1} - 1$. Therefore,

$$\sum_{i=0}^{k+1} 2^i =$$

$$2^{k+1} - 1 + 2^{k+1} = 2 * 2^{k+1} - 1 = 2^{k+2} - 1 = 2^{(k+1)+1} - 1$$

as required.

Common problem in solutions submitted:

Many students applied the induction hypothesis without explaining to the reader what you were doing. You will lose marks on the assignment if you do not explain where you are applying the induction hypothesis.

A proof is intended for someone to read. It will be easier for someone to understand and believe in your proof if you explain what you are doing algebraically at every step.

Your proof will be more elegant if you don't change the variable names (for this problem, stick to k instead of switching to j or n or m).

Some students wrote: assume $k=k+1$.

But this is invalid since it would imply $0=1$.

Standard approach to an induction proof that for all $n \geq 0$, $f(n)= g(n)$:

[Basis] Prove that $f(0)= g(0)$.

[Induction step]

Assume that $f(n)= g(n)$. We want to prove that $f(n+1)= g(n+1)$.

The induction hypothesis is the mathematical statement that $f(n)=g(n)$.

1. Rewrite $f(n+1)$ in terms of some terms involving $f(n)$ and some other terms by applying standard mathematics.

2. Replace $f(n)$ by $g(n)$. This is called applying the induction hypothesis. On your assignments, I want you to explain what you are doing at this step by writing:

By induction, $f(n) = g(n)$ and therefore, ….

They may not teach this in Math 122, but a good mathematician explains what they are doing at every step of a proof.

3. Simplify algebraically until you have shown that $f(n+1) = g(n+1)$.

Model Solution

Theorem: $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Proof:

[Basis] When k=0, $\Sigma_{i=0 \text{ to } k} 2^i = 2^0 = 1$ and

$2^{k+1} - 1 = 2^{0+1} - 1 = 1$ so this formula is correct for k=0.

Assume that

$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1.$ We want to prove that

$$\sum_{i=0}^{k+1} 2^i = 2^{(k+1)+1} - 1 = 2^{k+2} - 1.$$

[Induction step] Separating the sum into two parts gives

$$\sum_{i=0}^{k+1} 2^i =$$

$$\sum_{i=0}^{k} 2^i + 2^{k+1}.$$

By induction,

$\sum_{i=0}^{k} 2^i$ is equal to $2^{k+1} - 1$. Therefore,

$$\sum_{i=0}^{k+1} 2^i =$$

$$2^{k+1} - 1 + 2^{k+1} = 2 * 2^{k+1} - 1 = 2^{k+2} - 1$$

as required.

What is wrong with my induction proof?

In a drunken haze I decided that the solution to the recurrence $T(1)=1$, $T(n)= 1 + T(n-1)$ is

$1 + 2 + 3 + \ldots + n$.

Theorem: The solution to the recurrence is $n(n+1)/2$.

Proof. [Basis] $T(1)=1$ and $1 *(1+1)/2 = 1$ as required.

[Induction step] Assume that $1 + 2 + \ldots + n-1 + n = n(n+1)/2$.

We want to prove that $1 + 2 + \ldots + n-1 + n + (n+1) = (n+1)(n+2)/2 = (n^2 +3n +2)/2$.

By induction, $1 + 2 + \ldots + n= n(n+1)/2$.

So $1 + 2 + .. + n + (n+1)= n(n+1)/2 + (n+1)$.

Simplifying: $(n^2 + n + 2n + 2)/2= (n^2 +3n +2)/2$ as required.