Problems of the day:

1. Let P= {(b, acbb), (aac, a), (b, ca)}.

   Prove that P has a match.

2. How many ways can aab be factored as

   x y z such that $|y| \geq 1$?

Write down all possibilities:

x	y	z
ε	a	ab

....

Announcements

Assignment #2: Due at beginning of class Friday Oct. 7.

Midterm is in class on **Wed. Oct. 26.**

Make sure you sign the attendance sheet every class..

Theorem:

If L is a regular language, then L is L(M) for some DFA M.

Proof:

By showing how to construct a NDFA for L.

Last lecture, we proved by construction that for every NDFA, there is an equivalent DFA.

So this indirectly gives a construction for a DFA.

Regular expressions over Σ:

[Basis] 1. Φ and σ for each $\sigma \in \Sigma$ are regular expressions.

[Inductive step] If α and β are regular expressions, then so are:

2. $(\alpha\beta)$

3. $(\alpha \cup \beta)$ and

4. α^*

Note: Regular expressions are strings over

$\Sigma \cup \{ \, (\, , \,) \, , \, \Phi \, , \cup \, , \, * \, \}$

for some alphabet Σ.

Theorem: If L is accepted by a DFA M, then there is a regular expression which generates L.

There is a proof which constructs a regular expression from the DFA in the text (in the proof of Theorem 2.3.2). I expect you to know that this theorem is true but you are not responsible for the proof.

Conclusion: A language is regular if and only if it is accepted by a finite automaton.

The set of S= { L : L is L(M) for some DFA M} is closed under complement.

Let $M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$ accept L_1.

Proof: A construction for a new DFA M= $(K, \Sigma, \delta, s, F)$ which accepts the complement of L_1.

Regular languages are also closed for intersection (assignment #2).

Pigeons and the Pumping Lemma

Outline:

1. Another elementary proof technique- the pigeonhole principle. This is critical for proving the Pumping Lemma for regular languages (a tool for proving that a language is not regular)

2. Introduction to the pumping lemma.

The Pigeonhole Principle

Given two natural numbers *n* and *m* with *n* > *m*, if *n* items are put into *m* pigeonholes, then at least one pigeonhole must contain more than one item.

Picture from: Wikipedia, the free encyclopedia

If there's only one place (the pigeonhole) to put a number (the pigeon), it must go there.

The number 6 must go in the green square.

9	2							3
			3		4		2	
1	3			2		9		6
5		1				3		4
				6				
3		2				8		5
		6		1			3	8
	5		8		6			
8							9	7

OPEN: are there any uniquely completable squares with only 16 entries filled in?

From: Dan Rice's Sudoku Blog

Application:

Show that in any group of people there are at least two people with the same number of acquaintances.

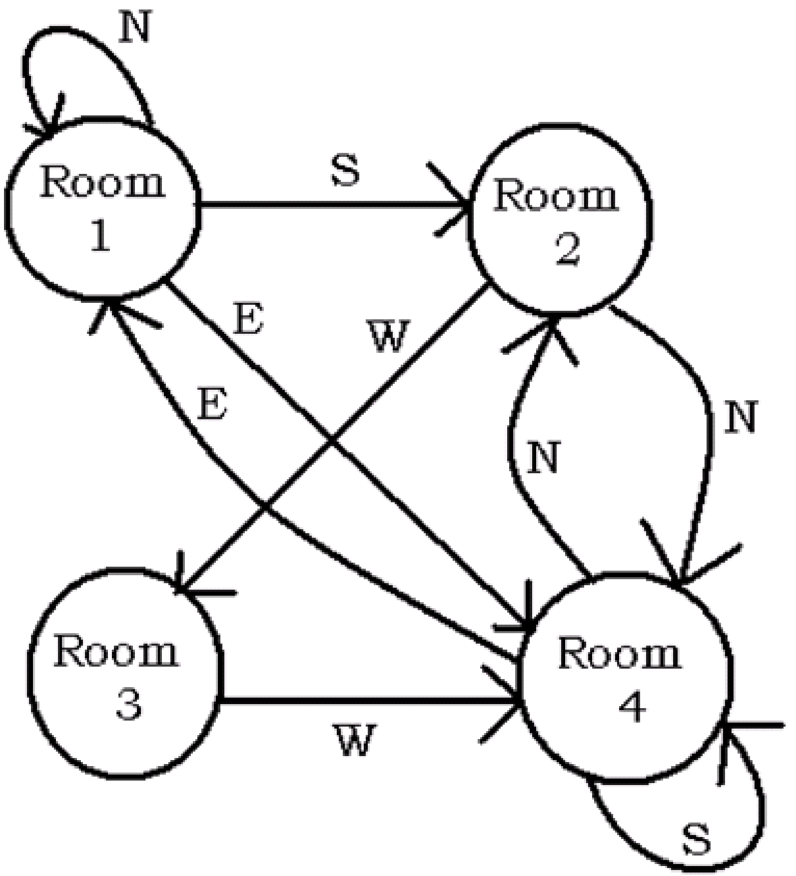
Note: we are assuming that if Sue is acquainted with Joe then Joe is acquainted with Sue.

Colossal Cave Adventure (from Wikipedia)
In the mid 1970s, programmer, caver, and role-player William Crowther developed a program called *Colossal Cave Adventure.* The game used a text interface to create an interactive adventure through a spectacular underground cave system. Crowther's work was later modified and expanded by programmer Don Woods, and Colossal Cave Adventure became wildly popular among early computer enthusiasts, spreading across the nascent ARPANET throughout the 1970s.

A big fan of Tolkien, Woods introduced additional fantasy elements, such as elves and a troll. Adventure was the first game to feature objects that could be picked up, used, and dropped (and that could be carried by a non-player character).

You are in a maze of twisty little passages, all alike.

If the maze has n rooms and each one has trails exiting to the N, S, W, E. How many trails must be traversed before some room is visited more than once?



The Pumping Lemma for Regular Languages:

If L is a language accepted by a DFA with k states, and $w \in L$, $|w| \geq k$, then there exists x, y, z such that

1. $w = x\,y\,z$,
2. $y \neq \varepsilon$,
3. $|\,x\,y\,| \leq k$, and
4. $x\,y^n z$ is in L for all $n \geq 0$.

Factor $a^r b^{3r}$ as x y z in all possible ways where $y \neq \varepsilon$.