# Problem of the Day:

Factor $(ab)^k$ as $xyz$ in all ways such that $y \neq \varepsilon$.

Assignment #2 is due on Friday.

Midterm: Wed. Oct. 26, in class.

Midterm tutorial: Mon. Oct 24, 6pm, room TBA.

Old midterms are available from our class web page.

$L = \{ a^n b^n : n \geq 0 \}$ is not a regular language.

Proof (by contradiction)

Assume L is regular.

Then L is accepted by a DFA M with k states for some integer k.

Since L is accepted by a DFA M with k states, the pumping lemma holds.

Let $w = a^r b^r$ where $r = \lceil k/2 \rceil$.

Consider all possibilities for y:

Case 1: $a^i \ (a^j)\ a^{r-i-j}\ b^r \qquad j \geq 1.$

Pump zero times: $a^{r-j}\ b^r \notin L$ since $r-j < r$.

Case 2: $a^{r-i}\ (\ a^i \ \ b^j\ )\ \ b^{r-j} \qquad i, j \geq 1.$

Pump 2 times: $a^{r-i}\, a^i \ \ b^j \ \ a^i \ \ b^j \ \ b^{r-j} \notin L$
because it is not of the form $a^* \, b^*$

Case 3: $a^r\ b^i \ \ (b^j)\ b^{r-i-j} \qquad j \geq 1.$

Pump zero times: $a^r\ b^{r-j} \notin$ since $r-j < r$.

Therefore, L is not regular.

$L = \{ a^n b^n : n \geq 0 \}$ is not a regular language.

Proof (by contradiction) Assume L is regular. Then L is accepted by a DFA M with k states for some integer k. Since L is accepted by a DFA M with k states, the pumping lemma holds. Let $w = a^k b^k$ (a more judicious choice for w).

Since w is in L and $|w| \geq k$, $\exists$ x, y, z such that

1. $w = x y z$,
2. $y \neq \varepsilon$,
3. $| x y | \leq k$, and
4. $x y^n z$ is in L for all $n \geq 0$.

For $w = a^k b^k$ (a more judicious choice for w):

Consider all possibilities for y:

Case 1: $a^i \ (a^j)\ a^{k-i-j}\ b^k$ $\quad j \geq 1$.

Pump zero times: $a^{k-j} b^k \notin L$ since $k-j < k$.

This covers all cases with $|xy| \leq k$.

Therefore, L is not regular.

# Using closure properties:

$L = \{w \in \{a, b\}^*$ : w has the same number of a's as b's$\}$ is not regular.

Proof (by contradiction)

Assume L is regular. The language $a^* b^*$ is regular since it has a regular expression. Because regular languages are closed under intersection, $L \cap a^* b^*$ is regular. But

$L \cap a^* b^* = \{ a^n b^n : n \geq 0\}$ which is not regular.

Therefore, L is not regular.

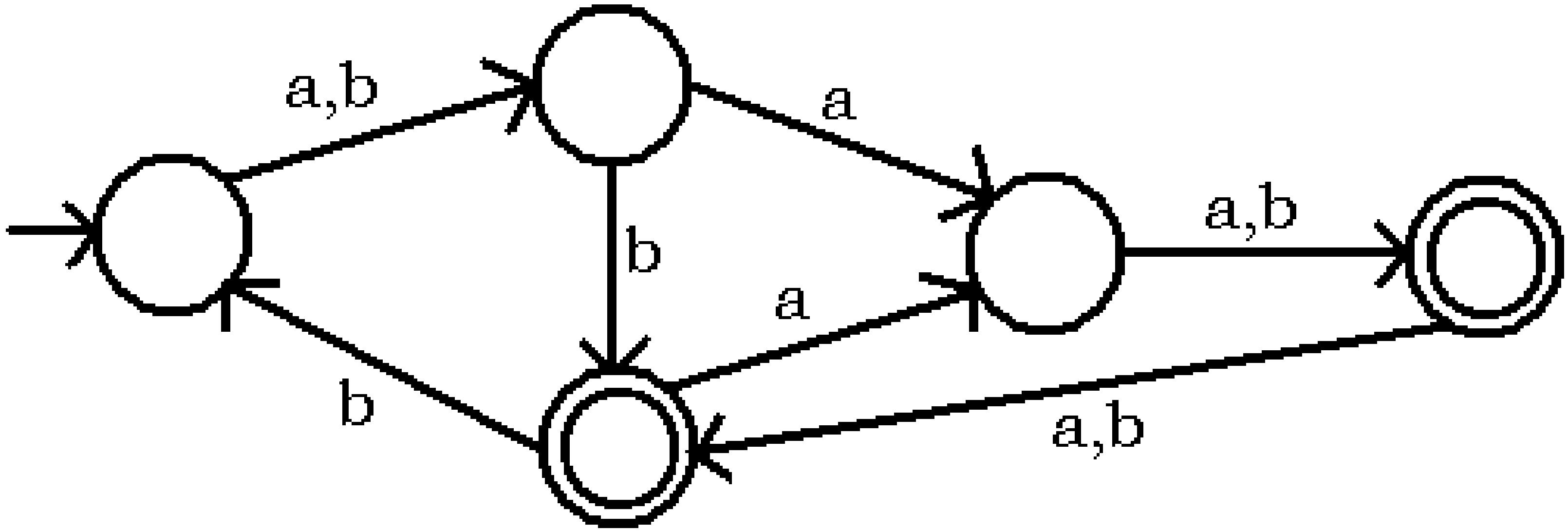
# Question from 2003 Midterm:

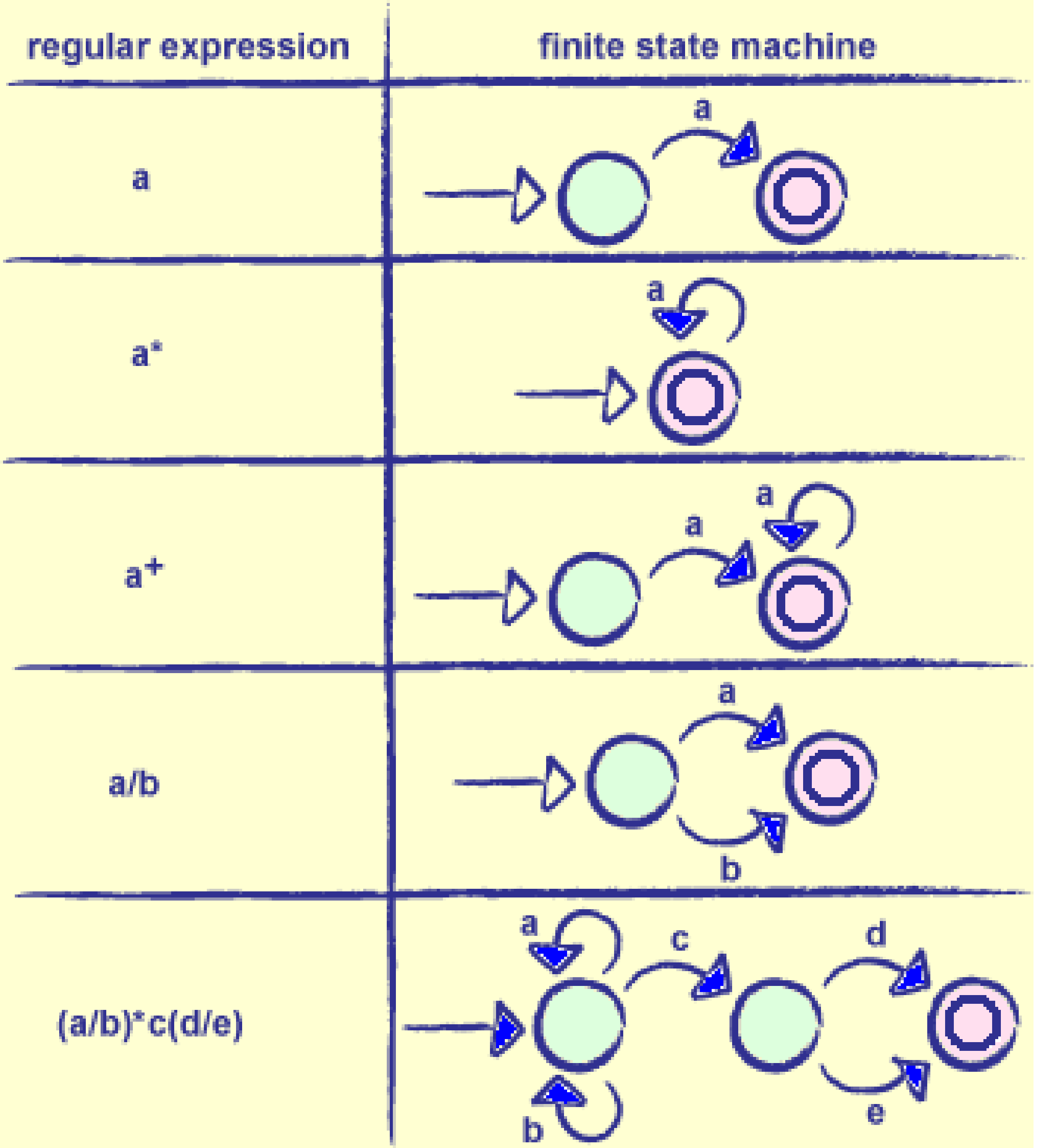
Apply the pumping lemma to $w = a^s\, b\, a^{s\hat{}4}$ to prove that $L = \{ a^n\, b\, a^r : n^2 \leq r \leq n^4 \}$ is not regular. All you may assume is that $s^4 + s + 1 \geq k$ where k is the number of states.

What is a more judicious choice for w and how does this change the proof?

Given the DFA below which accepts L, prove that $L^R = \{ u^R : u \in L\}$ is regular by designing a NDFA which accepts $L^R$.

For example, abaa is in L so $(abaa)^R$ = aaba is in $L^R$.






http://www.cgl.ucsf.edu/Outreach/bmi280/slides/swc/lec/re.html

~L 2006

# Algorithms to Answer Questions about Regular Languages

```
if (if23==23)
   x= -23.2e23-6;
```

The first step of a compiler is to break your program into tokens.

```
if (if23==23)
  x= -23.2e23-6;
```

Tokens:

Keywords: if

Brackets: ( )

Variables: if23 x

Assignment: =

Math Operator: -

Logical: ==

Delimiter: ;

Double: -23.2e23

Integers: 23 6

Keywords:

if U while U int U double U switch U ....

Variables: Not a Keyword but of the form:

(a-z U A-Z)(a-z U A-Z U 0-9 U _ )*

Non-negative Integers: N= (0 U (1-9)(0-9)*)

Numeric values:

(Φ* U - ) U N (Φ* U . (0-9)*)

(Φ* U (eUE)(Φ* U + U - ) N )

The pumping lemma and also closure properties are used to prove languages are not regular.

Regular languages are closed under:

- union
- concatenation
- Kleene star
- complement
- intersection
- exclusive or
- difference
- reversal

There are algorithms for the following questions about regular languages:

1. Given a DFA M and a string w, is $w \in L(M)$?

2. Given a DFA M, is $L(M) = \Phi$?

3. Given a DFA M, is $L(M) = \Sigma^*$ ?

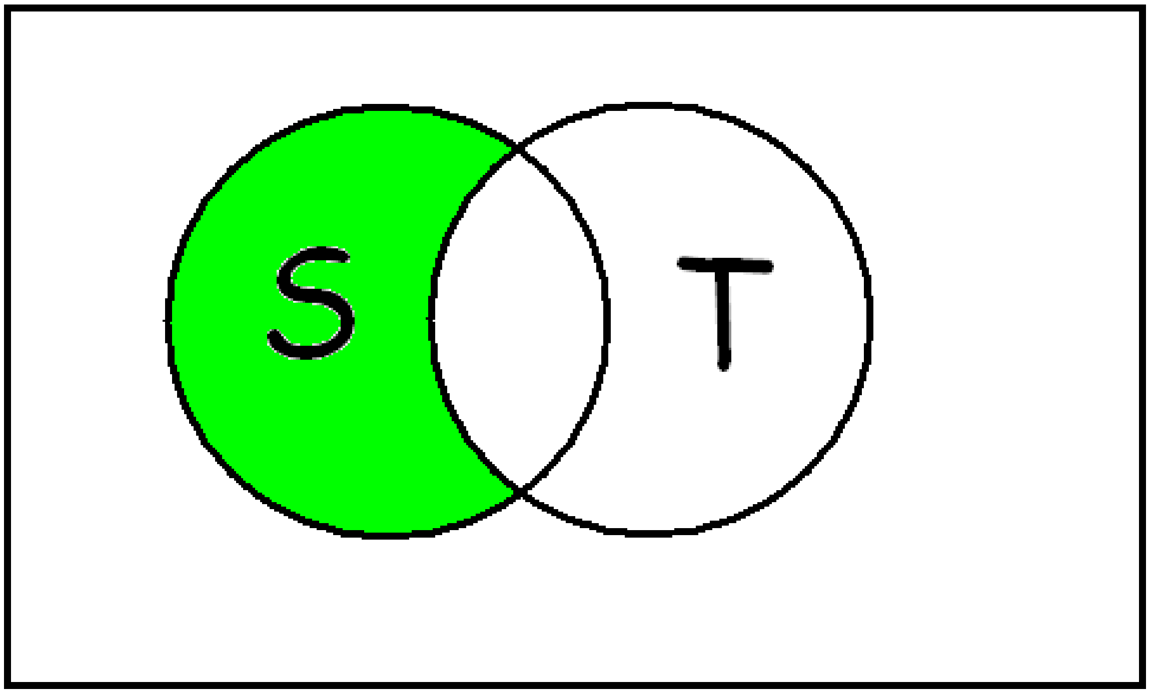
4. Given DFA's $M_1$ and $M_2$, is $L(M_1) \subseteq L(M_2)$?

5. Given DFA's $M_1$ and $M_2$, is $L(M_1) = L(M_2)$?

How can we use these to check correctness of student answers for the java tutorial?

# Java Regular expression tutorial:

S= student answer, T= teacher answer

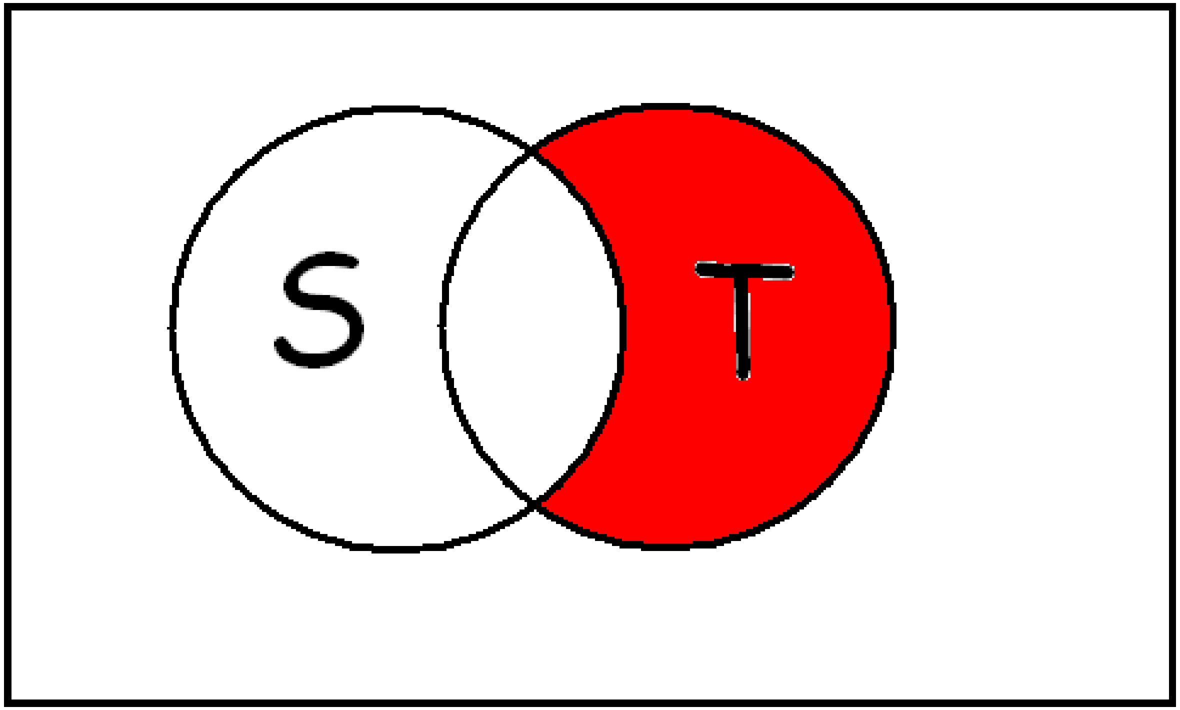



Strings student generates but should not.

Is S intersect the complement of T = $\Phi$?

# Java Regular expression tutorial:

S= student answer, T= teacher answer




Strings student should generate but does not.

Is T intersect the complement of S = Φ?