

$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$

1. Design a PDA that accepts  $L$ .

Hint: use four states, one for reading each different type of symbol.

2. Design a context-free grammar that generates  $L$ .

## Assignment #3: Due Fri. Oct. 21.

For Oct. 18/19: Some practice questions for Tutorial #5 have been posted.

Old midterms are available on the CSC 320 web page.

I'm away at a conference Oct. 19-21: No office hours. Ask assignment questions early.

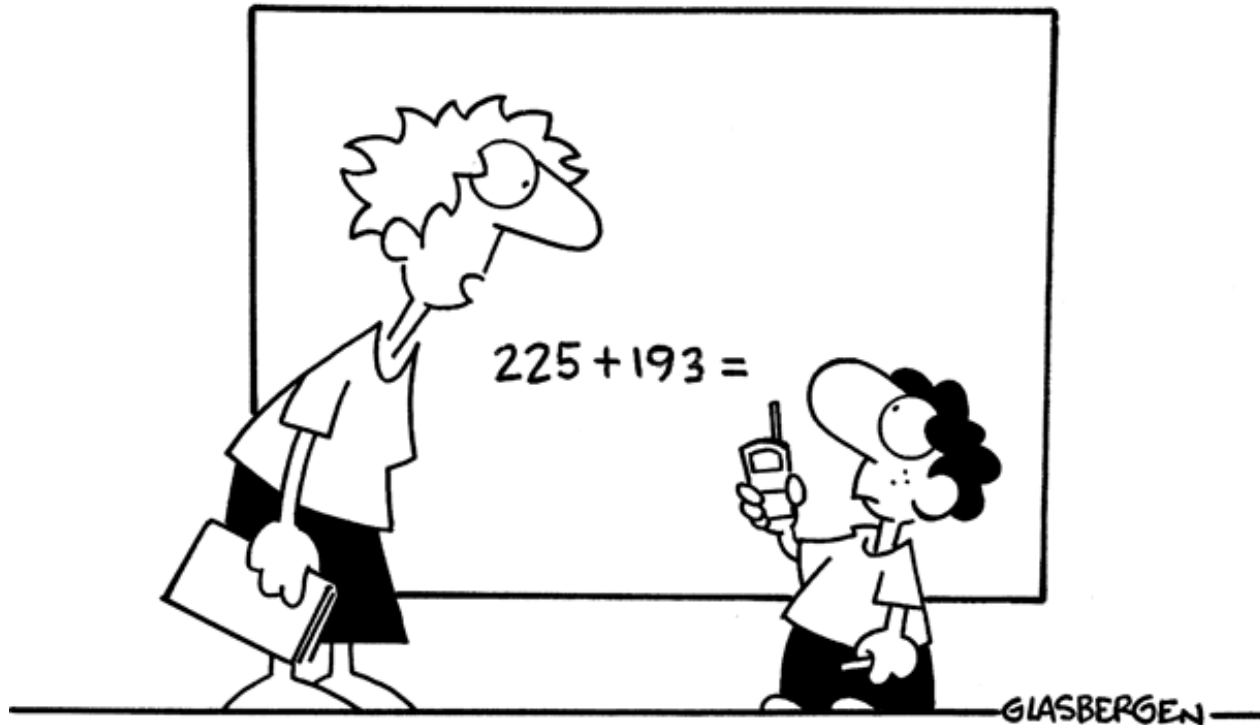
Our midterm is **Wed. Oct. 26** in class.

Midterm tutorial: 6pm, Mon. Oct. 24, ECS 116.

Assignment solutions are in the connex resources.

To get maximum benefit from homework and old exams, solve problems yourself instead of doing a web search for solutions.

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**“You have to solve this problem by yourself. You can’t call tech support.”**

A grammar for  $L_2$ :

$$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$$

Meaning:

S: match a with d

T: match a with c

U: match b with d

V: match b with c

$$L = \{ a^p b^q c^r d^s : p, q, r, s \geq 0, p+q = r+s \}$$

Meaning:

S: match a with d

T: match a with c

U: match b with d

V: match b with c

Start symbol: S

$$S \rightarrow a S d$$

$$S \rightarrow T$$

$$S \rightarrow U$$

$$S \rightarrow V$$

$$T \rightarrow a T c$$

$$T \rightarrow V$$

$$U \rightarrow b U d$$

$$U \rightarrow V$$

$$V \rightarrow b V c$$

$$V \rightarrow \varepsilon$$

Design a PDA which accepts the language  
 $\{ u u^R v v^R : u, v \in \{a,b\}^+ \}$

Hint: A bottom of the stack symbol could prove useful in helping to ensure that  $u$  matches with  $u^R$  and  $v$  matches with  $v^R$ .

$L = \{w \text{ in } \{a, b\}^* : w \text{ has the same number of } a\text{'s and } b\text{'s}\}$

Start state:  $s$

Final states:  $\{s\}$

| State | Input | Pop        | Next state | Push       |
|-------|-------|------------|------------|------------|
| $s$   | $a$   | $\epsilon$ | $s$        | $B$        |
| $s$   | $a$   | $A$        | $s$        | $\epsilon$ |
| $s$   | $b$   | $\epsilon$ | $s$        | $A$        |
| $s$   | $b$   | $B$        | $s$        | $\epsilon$ |

$L = \{w \text{ in } \{a, b\}^* : w \text{ has the same number of } a\text{'s and } b\text{'s}\}$

State state:  $s$ , Final states:  $\{f\}$

| State | Input         | Pop           | Next state | Push          |
|-------|---------------|---------------|------------|---------------|
| $s$   | $\varepsilon$ | $\varepsilon$ | $t$        | $X$           |
| $t$   | $a$           | $X$           | $t$        | $BX$          |
| $t$   | $a$           | $A$           | $t$        | $\varepsilon$ |
| $t$   | $a$           | $B$           | $t$        | $BB$          |
| $t$   | $b$           | $X$           | $t$        | $AX$          |
| $t$   | $b$           | $A$           | $t$        | $AA$          |
| $t$   | $b$           | $B$           | $t$        | $\varepsilon$ |
| $t$   | $\varepsilon$ | $X$           | $f$        | $\varepsilon$ |

A more deterministic solution:

Stack will never contain both  $A$ 's and  $B$ 's.

$X$ - bottom of stack marker.



# Languages which are context-free.

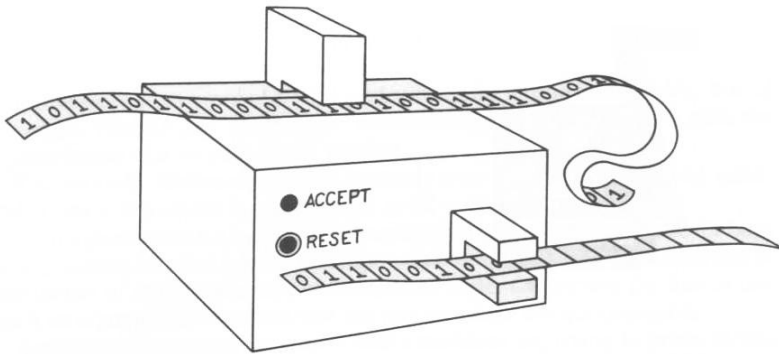
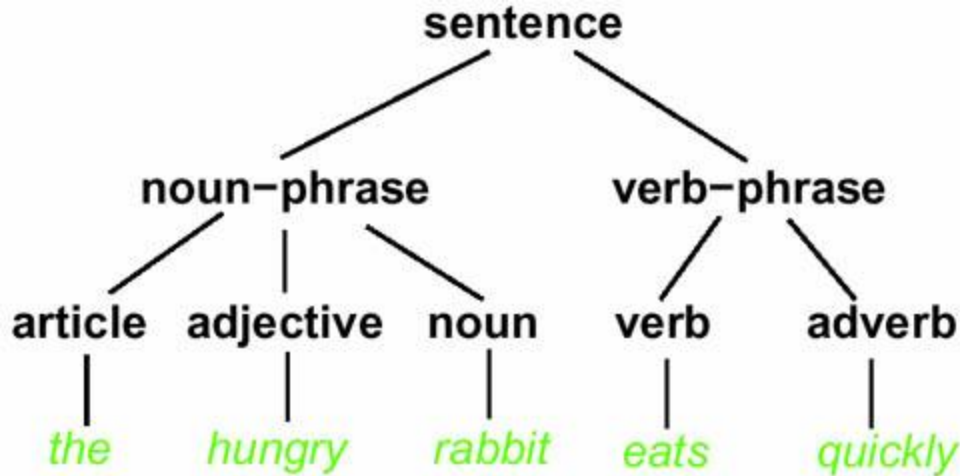
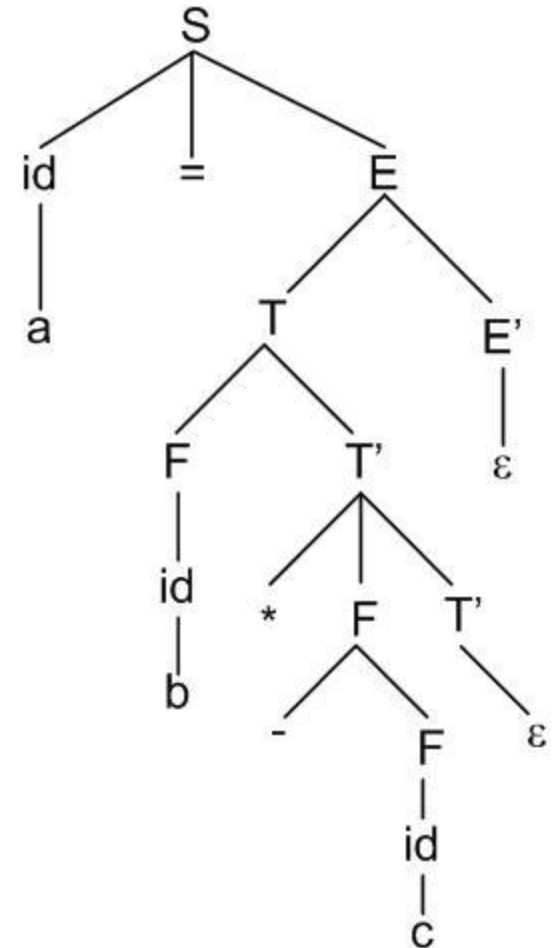


Figure 7.4 A push-down automaton



A Derivation Tree



Theorem:

If  $L$  is  $L(G)$  for some context-free grammar  $G$ , then there is a PDA  $M$  which accepts  $L$ .

Proof: By construction of a PDA which mimics derivations in the grammar.

A context-free grammar, start symbol  $S$ :

$$S \rightarrow \varepsilon$$

$$S \rightarrow A B A$$

$$A \rightarrow aa$$

$$B \rightarrow b S a$$

Apply construction to get a PDA.